Boundary layer 1 .

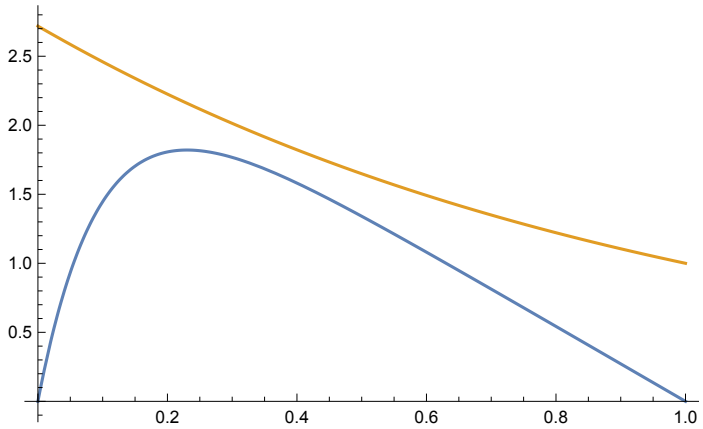
Equazione:

$\epsilon\, y''(x) + (1+\epsilon)\, y'(x) + y(x) = 0$

In[ ]:=
```
ϵ = 0.1;
```

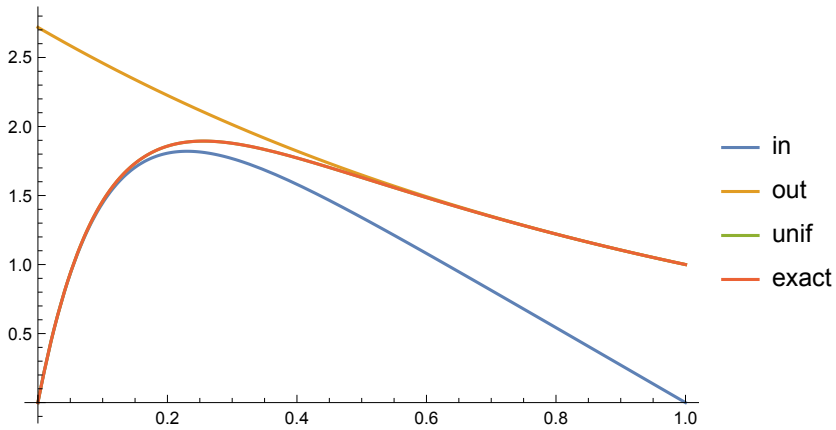
In[ ]:=
```
Plot[{ E (1 - E^(-x / ϵ) ) - E x , E^(1 - x)}, {x, 0, 1}, PlotRange → All]
```

Out[ ]=



In[ ]:=
```
Plot[{ E (1 - E^(-x / ϵ) ) - E x , E^(1 - x), E^(1 - x) - E^(1 - x / ϵ),
   (E^(-x) - E^(-x / ϵ)) / (1 / E - E^(-1 / ϵ) ) }, {x, 0, 1},
 PlotRange → All, PlotLegends → {"in", "out", "unif", "exact"}]
```

Out[ ]=



Boundary layer 2 .

Equazione:

$\epsilon\, y''(x) + (1+x)\, y'(x) + y(x) = 0$

In[1]:=
```
sY = DSolve[ eps y''[x] + (1 + x) y'[x] + y[x] == 0, y[x], x] // Flatten
```

Out[1]=
$$\left\{y[x] \to e^{-\frac{x+\frac{x^2}{2}}{\text{eps}}}\, c_2 + e^{-\frac{1}{2\,\text{eps}}-\frac{x+\frac{x^2}{2}}{\text{eps}}}\, \sqrt{\text{eps}}\, \sqrt{\frac{\pi}{2}}\, c_1\, \text{Erfi}\left[\frac{1+x}{\sqrt{2}\,\sqrt{\text{eps}}}\right]\right\}$$

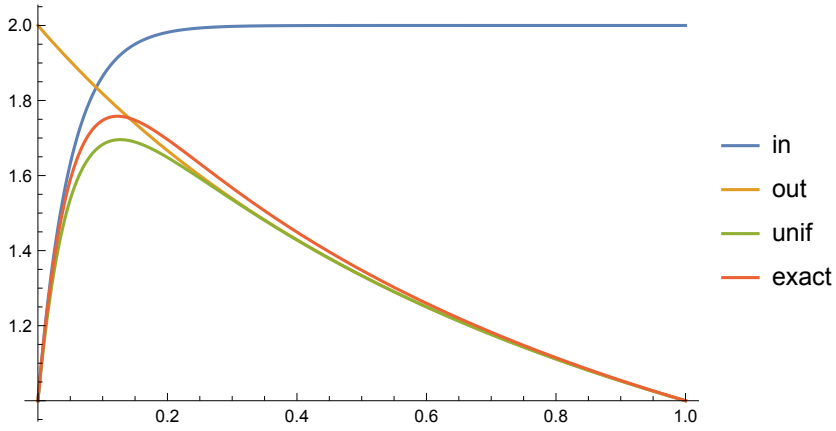
In[2]:=
```
scs = Solve[{y[x] /. sY /. x → 1, y[x] /. sY /. x → 0} == {1, 1}, {c2, c1}] // Simplify //
  First
```

Out[2]=
$$\left\{c_2 \to \frac{e^{\frac{3}{2}/\text{eps}}\, \text{Erfi}\left[\frac{1}{\sqrt{2}\,\sqrt{\text{eps}}}\right] - \text{Erfi}\left[\frac{\sqrt{2}}{\sqrt{\text{eps}}}\right]}{\text{Erfi}\left[\frac{1}{\sqrt{2}\,\sqrt{\text{eps}}}\right] - \text{Erfi}\left[\frac{\sqrt{2}}{\sqrt{\text{eps}}}\right]},\ c_1 \to \frac{e^{\frac{1}{2}/\text{eps}}\left(-1 + e^{\frac{3}{2}/\text{eps}}\right)\sqrt{\frac{2}{\pi}}}{\sqrt{\text{eps}}\left(-\text{Erfi}\left[\frac{1}{\sqrt{2}\,\sqrt{\text{eps}}}\right] + \text{Erfi}\left[\frac{\sqrt{2}}{\sqrt{\text{eps}}}\right]\right)}\right\}$$

In[5]:=
```
ϵ = 0.05;
```

In[6]:=
```
Plot[{1 + (1 - E^(-x / ϵ)), 2 / (1 + x) ,
  2 / (1 + x) - E^(-x / ϵ), (y[x] /. sY /. scs /. eps → ϵ)}, {x, 0, 1},
 PlotRange → All, PlotLegends → {"in", "out", "unif", "exact"}]
```

Out[6]=



In[3]:=
```
ϵ = 0.01;
```

In[4]:=
```
Plot[{1 + (1 - E^(-x / ϵ)), 2 / (1 + x) ,
  2 / (1 + x) - E^(-x / ϵ), (y[x] /. sY /. scs /. eps → ϵ)}, {x, 0, 1},
 PlotRange → All, PlotLegends → {"in", "out", "unif", "exact"}]
```

Out[4]=