

# 1 Exercises on 2nd order ODEs and singular points

**Exercise we started in class** Study the equation:

$$x(2-x)y''(x) + (2-x)y'(x) + 6y(x) = 0.$$

Classify the singular points. Notice that it has only 3 singularities, all Fuchsian.

Use the P-symbol method to write a basis of solutions around  $x = \infty$ , and its other singular points.

In case of degenerate indices, write at least one solution explicitly and discuss the form of the remaining one.

**Classification of singular points.** Study the singular points of the following equations.

If the P-symbol method can be applied, use it to write explicitly a basis of solutions to the equation around each of its singular points. If not, describe what is the expected form for the solution around the singular points. If the solutions admits a series expansion around the singular points, compute the first couple of terms.

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$$x^2y'' + xy' + (x^2 - a^2)y = 0.$$

(Bessel equation)

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$$(1 - x^2)y'' - 2xy' + a(a + 1)y = 0.$$

(Legendre equation).

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$$xy'' + (1 + a - x)y' + by = 0$$

(Laguerre equation),

where  $a, b \in \mathbb{C}$ .