

This file is to highlight some incorrect statements that I made in the lectures or previous versions of the notes. Please note these corrections.

- **Asymptotic vs convergent series.** In the lecture on the definition of asymptotic series, it was stated that, when a function admits a convergent series as well as an asymptotic series of the same form, for example  $f(x) = \sum_{n=0}^{\infty} a_n x^n$  for  $|x| < r$ ,  $r > 0$ , and  $f(x) \sim \sum_{n=0}^{\infty} b_n x^n$  for  $x \rightarrow 0$ , then they should coincide, i.e.  $a_n = b_n \forall n$ .

**However, this was not correct: convergent series and asymptotic series are different concepts, and they do not need to coincide, even when both exist with the same form. It is better to keep the concepts separate. What is true, is that two asymptotic series of the same form have to coincide: i.e., the coefficients of an asymptotic expansion are unique.**

- **2nd order ODE: behaviour at a Fuchsian singularity in the resonant case.**

Consider a Fuchsian singularity  $x_0$  for a 2nd order linear ODE, with indices  $\rho_1, \rho_2$ , with  $\rho_1 - \rho_2 \in \mathbb{N}$  (the *resonant* case).

In this case, the ODE admits two solutions of the form:

$$y_1(x) = (x - x_0)^{\rho_1} \sum_{n=0}^{\infty} c_n (x - x_0)^n, \quad (\text{standard form}) \quad (0.1)$$

where  $c_n$  are coefficients, and

$$y_2(x) = A \log(x - x_0) \times y_1(x) + (x - x_0)^{\rho_2} \sum_{n=0}^{\infty} d_n (x - x_0)^n, \quad (0.2)$$

where  $d_n$  and  $A$  are coefficients.

**Notice that  $A$  could also, in some cases, be zero, in that case we would have the “standard form” also for the second independent solution. In the original version of the notes, it was assumed that  $A$  is always nonzero, so it was rescaled to 1. This was not correct.**

- In the first version of “NOTES Part 5 - Heat Equation”, the part on the separation of variables (pages 25 - 40) contained a mistake in the time-dependence of the temporal part of a factorised solution. The time dependence for a solution of wave number  $k$  was written as  $e^{-k^2 t / \alpha}$ , instead it should be  $e^{-k^2 t \alpha}$ . This has now been corrected.