Monetary Economics II - Exam
(F.Bagliano)

Time available: **90 minutes**

1. (**VAR models**) Given the following dynamic model capturing the relationships among a set of n endogenous variables (collected in vector $\mathbf{y}$) written in *structural* form (with standard notation):

$$\mathbf{A}\,\mathbf{y}_t = \mathbf{C}(L)\,\mathbf{y}_{t-1} + \mathbf{v}_t$$

where $\mathbf{v}_t$ is the vector of structural disturbances with $E(\mathbf{v}_t\mathbf{v}_t') = \mathbf{D}$ diagonal,

(a) derive the *reduced form* (VAR) of the model, discussing the identification problem and stating formally what is the *necessary condition* for the identification of structural shocks and why;

(b) now suppose that the endogenous variables are: m_t (nominal quantity of money), R_t (long-term nominal interest rate), g_t (GDP), r_t (short-term nominal interest rate) and p_t (price level). If your aim is to study the *monetary policy transmission mechanism* using the variables described above, what is a plausible identification strategy based on the "recursiveness" assumption? Clearly specify what is the monetary policy instrument, what variables are predetermined with respect to the policy instrument, and what is the form of the systematic component of the monetary policy rule. In your opinion, is a recursive identification scheme appropriate in this case? if not, explain why and suggest an alternative identification strategy.

2. (Term structure of interest rates) Consider the following $VAR(1)$ model for the levels of the long-term rate R^L and the short-term rate R, both non-stationary $I(1)$ variables:

$$
\begin{aligned}
R_t &= a_{11}R_{t-1} + a_{12}R^L_{t-1} + e^R_t \\
R^L_t &= a_{21}R_{t-1} + a_{22}R^L_{t-1} + e^L_t
\end{aligned}
$$

(a) Rewrite the system in first differences and define matrix $\mathbf{\Pi}$ (containing long-run information on the series) in terms of the a_{ij} coefficients;

(b) discuss the implications that the expectations theory of the term structure (with no risk premium) imposes on $\mathbf{\Pi}$ and describe a possible empirical strategy to formally test those implications;

(c) if the previous test gives results favourable to the theory, outline a strategy to derive the common stochastic trend driving both R and R^L.

3. (CCAPM) Consider the intertemporal utility-maximization and portfolio allocation problem of a representative agent endowed with the following (period) utility function:

$$
u(C_t) = \log(C_t)
$$

Suppose that there exist one risk-free asset with certain return (for a generic time $t+i$) r^f_{t+i+1}, and one risky asset with stochastic return r_{t+i+1}, unknown by the agent at time $t+i$. This stochastic return has a *positive* covariance with the agent's consumption level.

(a) Find the first-order condition (Euler equation) of the problem; explain its economic meaning using the concept of "stochastic discount factor";

(b) find the *equity premium* in this case and provide an economic interpretation;

(c) with reference to your answers to (a) e (b), explain the meaning of the *riskfree rate puzzle*.

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1. (**VAR models**) Consider two macroeconomic variables with the following properties:
$$x_t \sim I(1)\,, \ y_t \sim I(1)$$

   (a) Write the appropriate reduced form (VAR) of the bivariate model for x_t and y_t in order to apply the Blanchard-Quah identification strategy (explain under what condition this methodology is applicable);

   (b) write the relation between the variance-covariance matrix of VAR innovations ($\mathbf{u}_t$) and structural disturbances ($\mathbf{v}_t$), and derive the restrictions available for the identification of the structural shocks. Point out how many additional restrictions are needed to achieve *just-identification* of the system;

   (c) suppose that you want to impose the restrictions that each structural shock has a "permanent" effect on one variable and a purely "transitory" effect on the other. Write the appropriate restrictions in this case.

2. (**Term structure of interest rates**) Suppose that R^L (long-term interest rate) and R (short-term rate) are both $I(1)$ variables, generated by the following bivariate VAR:

$$
\begin{aligned}
R_t &= R_{t-1} + e_t^R \\
R_t^L &= a\,R_{t-1} + b\,R_{t-1}^L + e_t^L
\end{aligned}
$$

ehere R follows a *random walk*.

(a) Rewrite the system in first differences (with ΔR_t and ΔR_t^L on the left-hand side) and define matrix $\mathbf{\Pi}$, capturing the long-run properties of the series;

(b) what rank can $\mathbf{\Pi}$ have? what are the implications for the long-run relationship between the two rates R and R^L?

(c) if the "expectations theory" is true, what is the form of the cointegrating relation (vector $\boldsymbol{\beta}$) and of the vector of adjustment coefficients ($\boldsymbol{\alpha}$)?

3. (**CCAPM**) Consider the intertemporal utility maximization problem with portfolio allocation of an infinitely-lived agent endowed with the following utility function U_t:

$$U_t = E_t \sum_{i=0}^{\infty} \left(\frac{1}{1+\rho} \right)^i u(C_{t+i})$$

where $u(C_{t+i})$ is utility in any period $t + i$. Suppose that there are three financial assets: one riskless with certain rate of return (for period $t + i$) r_{t+i+1}^f, one risky with rate of return r_{t+i+1}^1 with *positive* covariance with the agent's consumption level, and onle risky with rate of return r_{t+i+1}^2 with *negative* covariance with the agent's consumption level. The rates of return on the two risky assets are not known by the agent at the beginning of the period.

(a) Derive and discuss the first-order conditions (Euler equations) of the problem for the three assets using the concept of "stochastic discount factor";

(b) obtain the equilibrium relative levels of the riskless rate r^f and the expected returns on the risky assets, $E(r^1)$ and $E(r^2)$, providing a clear economic interpretation.