Macroeconomic Analysis
Lecture notes (6) on:
Unemployment: search and matching models of labour market dynamics

(F. Bagliano, 2017)

It is a common empirical observation that, in labour markets, the simultaneous processes of job creation and job destruction are typically very intense, even in the absence of marked changes in overall employment. Here, we briefly outline the modelling approach of an important strand of labour economics focused exactly on the determinants of the flows into unemployment (the phenomenon of job destruction) and out of unemployment (the phenomenon of job creation). The agents of these models are characterized by a high degree of heterogeneity: workers and firms are not all alike. The consequence is that the "matching" between unemployed workers and firms with vacancies cannot be ensured by the working of a "market" mechanism, but is achieved by a "search" process. Unemployed workers and firms willing to employ them are inputs in a "productive" process that generates employment; this process is given a stylized and very tractable formal representation in the model we describe below. This "*search and matching*" framework (due originally to the work of P. Diamond, D. Mortensen and C. Pissarides - and therefore often referred to as the Diamond-Mortensen-Pissarides model) is qualitatively realistic enough to offer practical implications for the dynamics of labour market flows, for the steady state of the economy, and for the dynamic adjustment process towards the steady state (see, among other references, Pissarides 2000, 2011).

1. Modelling unemployment

The importance of the gross flows justifies the fundamental economic mechanism on which the model is based: the *matching process* between firms and workers. Firms create job openings (*vacancies*) and unemployed workers search for jobs, and the outcome of a match between a vacant job and an unemployed worker is a productive job. Moreover, the matching process does not take place in a coordinated manner as in the traditional neoclassical model, where the labour market is perfectly competitive and supply and demand of labour are balanced instantaneously through an adjustment of the wage.

On the contrary, in the model considered here, firms and workers operate in a decentralized and uncoordinated manner, dedicating time and resources to the search for a partner. The probability that a firm or a worker meets a partner depends on the relative number of vacant jobs and unemployed workers: for example, a scarcity of unemployed workers relative to vacancies makes it difficult for a firm to fill its vacancy, while workers find jobs easily. Hence, there exists an externality between agents in the same market. Since this externality is generated by the search activity of the agents on the market, it is normally referred to as a *search externality*.

Formally, we define the labour force as the sum of the "employed" workers plus the "unemployed" workers, and we assume that it is constant and equal to L units. Equivalently, the total demand for labor is equal to the number of filled jobs plus the number of vacancies. The total number of unemployed workers and vacancies can therefore be expressed as uL e vL, respectively, where u denotes the unemployment rate and v denotes the ratio between the number of vacancies and the total labour force. In each unit of time, the total number of matches between an unemployed worker and a vacant firm is equal to mL (where m denotes the ratio between the newly filled jobs and the total labor force, the "matching rate"). The process of matching is summarized by a *matching function* which expresses the number of newly created jobs (mL) as a function of the number of unemployed workers (uL) and vacancies (vL):[1]

$$mL = m\,(uL, vL). \tag{1.1}$$

The function $m(\cdot)$, supposedly increasing in both arguments, is conceptually sim-

[1] The model is set in continuous time; therefore all endogenous variables are function of t. For ease of notation, the dependence on time is not shown here. It will be explicitly considered in the following section concerning the overall dynamics of the model.

ilar to the aggregate production function widely used in other fields of macroeconomics (e.g. growth theory and real business cycle theory). The creation of employment is seen as the outcome of a "productive process" and the unemployed workers and vacant jobs are the "productive inputs". Obviously, both the number of unemployed workers and the number of vacancies have a positive effect on the number of matches within each time period ($m_u > 0$, $m_v > 0$). Moreover, the creation of employment requires the presence of agents on both sides of the labour market ($m(0,0) = m(0, vL) = m(uL, 0) = 0$). Additional properties of the function $m(\cdot)$ are needed to determine the nature of the unemployment rate in a steady state equilibrium. In particular, for the unemployment rate to be constant in a growing economy $m(\cdot)$, needs to have *constant returns to scale*.[2] In that case, we can write:

$$m = \frac{m(uL, vL)}{L} = m(u, v). \tag{1.2}$$

The function $m(\cdot)$ determines the flow of workers who find a job and who exit the unemployment pool within each time interval.

Now consider the case of an unemployed worker: at each moment in time, the worker will find a job with probability $p = m(\cdot)/u$. With constant returns to scale for $m(\cdot)$ we may thus write:

$$\frac{m(u,v)}{u} = m\left(1, \frac{v}{u}\right) \equiv p(\theta) \qquad \text{an increasing function of } \theta \equiv \frac{v}{u}. \tag{1.3}$$

The instantaneous probability p that a worker finds a job is thus positively related to the *tightness* of the labour market which is measured by θ, the ratio between the number of vacancies and unemployed workers. An increase in θ, reflecting a relative abundance of vacant jobs relative to unemployed workers, leads to an increase in p (moreover, given the properties of m, $p''(\theta) < 0$).[3] Finally, the average length of an unemployment spell is given by $1/p(\theta)$, and is thus inversely related to θ. Similarly, the rate at which a vacant job is matched to a worker may

[2] Empirical studies of the *matching* technology confirm that the assumption of constant returns to scale is realistic.

[3] More technically, the matching process is modelled as a Poisson process. The probability that an unemployed worker does *not* find employment within a time interval dt is thus given by $e^{-p(\theta)dt}$. For a small time interval this probability can be approximated by $1 - p(\theta)dt$. Similarly, the probability that the worker does find employment is $1 - e^{-p(\theta)dt}$ which can be approximated by $p(\theta)dt$.

be expressed as:

$$\frac{m(u,v)}{v} = m\left(1, \frac{v}{u}\right)\frac{u}{v} = \frac{p(\theta)}{\theta} \equiv q(\theta), \tag{1.4}$$

a decreasing function of the vacancy/unemployment ratio. An increase in θ reduces the probability that a vacancy is filled and $\frac{1}{q(\theta)}$ measures the average time that elapses before a vacancy is filled.[4] The dependence of p and q on θ captures the dual externality between agents in the labour market: an increase in the number of vacancies relative to unemployed workers increases the probability that a worker finds a job ($\partial p(\cdot)/\partial v > 0$), but at the same time it reduces the probability that a vacancy is filled ($\partial q(\cdot)/\partial v < 0$).

1.1. The dynamics of unemployment

Changes in unemployment result from a difference between the flow of workers who lose their job and become unemployed, and the flow of workers who find a job. The inflow into unemployment is determined by the "separation rate" which we take as given: at each moment in time a fraction s of jobs (corresponding to a fraction $1-u$ of the labor force) is hit by a shock that reduces the productivity of the match to zero: in this case the worker loses her job and returns to the pool of unemployed, while the firm is free to open up a vacancy in order to bring employment back to its original level. Given match destruction rate s, jobs therefore remain productive for an average period of $1/s$.

Given these assumptions we can now describe the dynamics of the number of unemployed workers as the result of inflows into unemployment (determined by s) minus outflows out of unemployment (determined by $p(\theta)$). Since L is constant, $d(uL)/dt = \dot{u}L$ and hence:

$$\begin{aligned} \dot{u}\,L &= \underbrace{s\,\underbrace{(1-u)\,L}_{\text{employed}}}_{\text{inflow}} - \underbrace{p(\theta)\,\underbrace{u\,L}_{\text{unempl.}}}_{\text{outflow}} \\ &\Rightarrow \quad \dot{u} = s\,(1-u) - p(\theta)\,u \end{aligned} \tag{1.5}$$

[4] To complete the description of the functions p and q, we define the elasticity of p with respect to θ as $\eta(\theta)$. We thus have:

$$\eta(\theta) = \frac{p'(\theta)\theta}{p(\theta)}$$

From the assumption of constant returns to scale we know that $0 \leq \eta(\theta) \leq 1$. Moreover, the elasticity of q with respect to θ is equal to $\eta(\theta) - 1$.

The dynamics of the unemployment rate depend on the tightness of the labor market θ: at a high value for the ratio of vacancies to unemployed workers, workers easily find a job leading to a large flow out of unemployment. From equation (1.5) we can immediately derive the steady state relationship between the unemployment rate and θ, setting $\dot{u} = 0$:

$$u = \frac{s}{s + p(\theta)} \tag{1.6}$$

Since $p'(\cdot) > 0$, the properties of the matching function determine a negative relation between θ and u: a higher value of θ corresponds to a larger flow of newly created jobs. In order to keep unemployment constant, the unemployment rate must therefore decrease to generate an offsetting increase in the flow of destroyed jobs. The steady-state relationship (1.6) is illustrated graphically in the left panel of Figure 1: to each value of θ corresponds a unique value for the unemployment rate. Moreover, the same properties of $m(\cdot)$ ensure that this curve is convex. For points above or below $\dot{u} = 0$, the unemployment rate tends to move towards the stationary relationship: keeping θ constant at θ_0, a value $u > u_0$ causes an increase in the flow out of unemployment and a decrease of the flow into unemployment, bringing u back to u_0. Moreover, given u and θ, the number of vacancies is uniquely determined by $v = \theta u$, where v denotes the number of vacancies as a proportion of the labor force. The picture on the right hand side of the figure shows the curve $\dot{u} = 0$ in (v, u) space. This locus is known as the *Beveridge curve,* and identifies the level of vacancies v_0 that corresponds to the pair (θ_0, u_0) of the left hand panel.

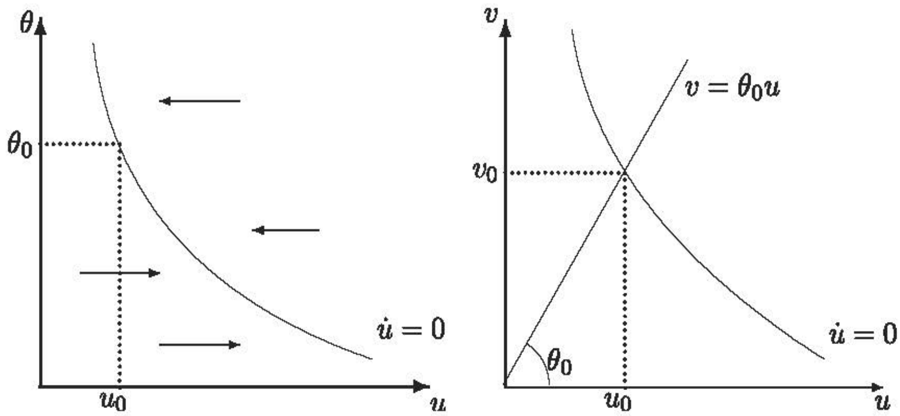

Figure 1. Dynamics of the unemployment rate and the "Beveridge curve"

In the sequel we will use both graphs to illustrate the dynamics and the comparative statics of the model. At this stage it is important to note that variations in the labour market tightness are associated with a *movement along the curve* $\dot{u} = 0$, while changes in the separation rate s or the efficiency of the matching process (captured by the properties of the matching function) correspond to *movements of the curve* $\dot{u} = 0$. For example, an increase in s or a decrease in the matching efficiency cause an upward shift of $\dot{u} = 0$. Equation (1.6) gives a first steady state relationship between u and θ. To find the actual equilibrium values, we need to specify a second relationship between these variables. This relationship can be derived from the behavior of firms and workers on the labour market.

1.2. Supply of vacancies and wage determination

The crucial decision of firms concerns the supply of vacancies on the labour market. The decision of a firm whether to create a vacancy depends on the expected future profits over the entire time horizon of the firm, which we assume to be infinite. Formally, each individual firm solves an intertemporal optimization problem taking as given the aggregate labour market conditions which are summarized by θ, the labour market tightness. Individual firms therefore disregard the effect of their decisions on θ, and consequently on the matching rates $p(\theta)$ and $q(\theta)$ (the externality effects that we referred to above).

To simplify the analysis, we assume that each firm can offer at most *one* job. If the job is filled, the firm receives a constant flow of output equal to y. Moreover, it pays a wage w to the worker and it takes this wage as given. On the contrary, if the job is not filled, the firm incurs a flow cost c, which reflects the time and resources invested in the search for suitable workers. Firms therefore find it attractive to create a vacancy as long as its value, measured in terms of expected profits, is positive; in the opposite case, the firm will not find it attractive to offer a vacancy and will exit the labour market. The value that a firm attributes to a vacancy (denoted by V) and to a filled job (J) can be expressed using the following equations. Given a constant real interest rate r, we can express these values as

$$r\,V(t) = -c + q(\theta(t))\,(J(t) - V(t)) + \dot{V}(t), \tag{1.7}$$

$$r\,J(t) = (y - w(t)) + s\,(V(t) - J(t)) + \dot{J}(t), \tag{1.8}$$

which are explicit functions of time. In both equations, the left-hand side represent the (instantaneous) opportunity cost for a firm of having a vacancy (rV) and having a filled job (rJ). The right-hand sides represent the (instantaneous) returns of a vacancy and a job. In particular, in (1.7) the vacancy return is equal to a negative cost component ($-c$), plus the capital gain in case the job is filled with a worker ($J - V$), which occurs with probability $q(\theta)$, plus the change in the value of the vacancy itself ($\dot{V}$). Similarly, the right-hand side of (1.8) defines the instantaneous return of a filled job as the value of the flow output minus the wage ($y - w$), plus the capital loss ($V - J$) in case the job is destroyed, which occurs with probability s, plus the change in the value of the job ($\dot{J}$).

Now, if we focus on steady state equilibria we can impose $\dot{V} = \dot{J} = 0$ in equations (1.7) and (1.8). Moreover, we assume free entry of firms and as a result $V = 0$: new firms continue to offer vacant jobs until the value of the marginal vacancy is reduced to zero. Substituting $V = 0$ in (1.7) and (1.8) and combining the resulting expressions for J, we get:

$$\left.\begin{array}{r} J = c/q(\theta) \\ J = (y-w)/(r+s) \end{array}\right\} \Rightarrow y - w = (r+s)\,\frac{c}{q(\theta)}. \tag{1.9}$$

Equation (1.7) gives us the first expression for J. According to this condition the equilibrium value of a filled job is equal to the expected costs of a vacancy, that is the instantaneous cost of a vacancy c times the average duration of a vacancy $1/q(\theta)$. The second condition for J can be derived from (1.8): the value of a filled job is equal to the value of the constant profit flow $y - w$. These instantaneous returns are discounted at rate $r+s$ to account for both impatience (r) and the risk that the match breaks down (s). Equating these two expressions yields the final solution (1.9), which gives the marginal condition for employment in a steady state equilibrium: the marginal productivity of the worker (y) needs to compensate the firm for the wage w paid to the worker and for the instantaneous cost of opening a vacancy. The latter is equal to the product of the discount rate $r + s$ and the expected costs of a vacancy $c/q(\theta)$.

This last term is just like an adjustment cost for the firm's employment level. It introduces a wedge between the marginal productivity of labour (y) and the wage rate (w), the size of which is endogenous and depends on the aggregate conditions on the labour market. In equilibrium, the size of the adjustment costs depend on the unemployment rate and on the number of vacancies, which are summarized at the aggregate level by the value of θ. If, for example, the value of output minus wages ($y - w$) increases, then vacancy creation will become profitable ($V > 0$)

and more firms will offer jobs. As a result, θ will increase, leading to a reduction in the matching rate for firms and an increase in the average cost of a vacancy and both these effects tend to bring the value of a vacancy back to zero.

Finally, notice that equation (1.9) still contains the wage rate w. This is, in principle, an additional endogenous variable. Hence the "*job creation condition*" (1.9) is not yet the steady state condition which together with (1.6) would allow us to solve for the equilibrium values of u and θ. To complete the model we need to specify a process of wage determination. To simplify the model (but without losing its general insights) we assume that the wage rate is exogenously fixed at a constant level $\bar{w} < y$ (an extreme form of real wage rigidity; Hall. 2005 takes a similar approach). An alternative assumption often adopted in the literature derives w endogenously by a decentralized bargaining process for each firm-worker pair, after a match has occurred (a "Nash-bargaining" assumption).

2. Steady state equilibrium

The assumption on wage determination completes the description of the steady state equilibrium of the model. The equilibrium can be summarized by (1.6) and (1.9), and by the additional equation simply setting the wage exogenously: $w = \bar{w}$. We will refer to the three relations as BC (Beveridge curve), JC (job creation condition) and W (wage level):

$$u = \frac{s}{s + p(\theta)} \qquad (BC) \qquad (2.1)$$

$$y - w = (r + s)\,\frac{c}{q(\theta)} \qquad (JC) \qquad (2.2)$$

$$w = \bar{w} \qquad (W). \qquad (2.3)$$

The system can be solved recursively for the variables u, θ and w. In fact, combining JC and W we obtain the values of w (exogenous here at level $\bar{w}$) and the market tightness measure θ_0, as shown in the left-hand panel of Figure 2. Given θ_0, the BC relation gives the steady-state equilibrium level of the unemployment rate u_0. Finally, simply using the definition of $\theta = v/u$ we can obtain the equilibriun value of the vacancy rate v_0 which equates the flows into and out of unemployment, as displayed in the right-hand panel of Figure 2.

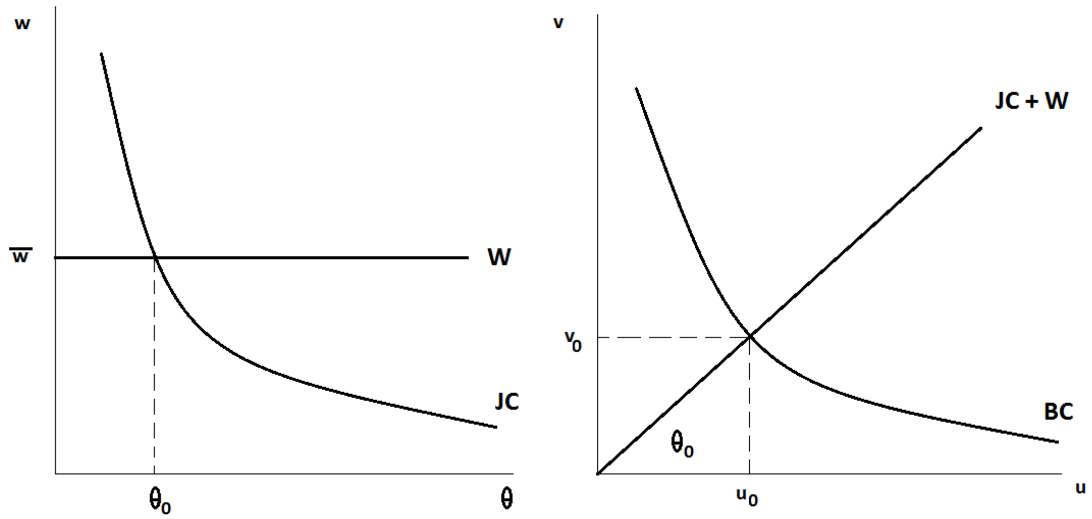

Figure 2. Steady state equilibrium in the labour market

This dual graphical representation facilitates the static comparative analysis, which is intended to study the effect of changes in the parameters on the steady state equilibrium (the analysis of the dynamic adjustment towards a new steady state will be the subject of the next section). In some cases, parameter changes have an unambiguous effect on all of the endogenous variables. In other instances, the effects are more complex and not always of unambiguous sign. Consider, for example, the effects of the following two types of shocks which may be at the root of cyclical variations in overall unemployment: the first is an "aggregate" disturbance. This is represented by a variation in the productivity of labor y which affects all firms at the same time and with the same intensity. The second shock is a "sectoral" disturbance, represented by a change in the separation rate s. This shock hits individual firms independently of the aggregate state of the economy (captured by labor productivity y).

A reduction in y moves the JC schedule downwards. This results in a reduction of θ at an unchanged (exogenous) wage $\bar{w}$. Since the curve BC does not shift, the unemployment rate must increase while the number of vacancies v is reduced. In the case of a sectoral shock we observe a similar inward shift of the JC relation.

This results in a decrease of the labour market tightness θ, as in the case of the aggregate shock. At the same time, however, the curve BC shifts to the right. Hence, while the unemployment rate increases unambiguously, it is in general not possible to determine the effect on the number of vacancies. In reality, however, v appears to be procyclical and this suggests that aggregate shocks are a more important source of cyclical movements on the labour market than sectoral disturbances.

3. Labour market dynamics

Until now all the relationships we derived referred to the steady state equilibrium of the system. In this section we will now analyze the evolution of unemployment and vacancies along the adjustment path towards the steady state equilibrium.

The discussion of the flows into and out of unemployment in section 1 already delivered the law of motion for unemployment. This equation is repeated here (making the time dependence of the endogenous variables explicit):

$$\dot{u}(t) = s\,(1 - u(t)) - p(\theta(t))\,u(t). \tag{3.1}$$

The dynamics of u are due to the flow of separations and the flow of newly created jobs resulting from the matches between firms and workers. The magnitude of the flow out of unemployment depends on aggregate labor market conditions, captured by θ, via its effect on $p(\cdot)$. Outside a steady state equilibrium the path of θ will influence unemployment dynamics in the economy. Moreover, given the definition of θ as the ratio of vacancies to unemployed workers this will also affect the value of the labor market tightness. In order to give a complete description of the adjustment process towards a steady state equilibrium we therefore need to study the dynamics of θ. This requires an analysis of the job creation decisions of firms.

3.1. Market tightness dynamics

At each moment in time firms exploit all opportunities for the profitable creation of jobs. Hence, in a steady state equilibrium as well as along the adjustment path $V(t) = 0\ \forall t$, and outside a steady state equilibrium $\dot{V}(t) = 0$, $\forall t$. The value of a filled job for the firm can de derived from (1.7) and (1.8). From the first equation, setting $V(t) = \dot{V}(t) = 0$, we get the following relation:

$$J(t) = \frac{c}{q(\theta(t))} \tag{3.2}$$

Equation (3.2) is identical to the steady state expression derived before. Firms continue to create new vacancies, thereby influencing θ, until the value of a filled job equals the expected cost of a vacancy. Since entry into the labor market is costless for firms (the resources are used to maintain open vacancies), equation (3.2) will hold at each instant during the adjustment process. Outside steady state the dynamics of J needs to satisfy the differential equation (1.8), with $V(t) = 0$:[5]

$$\dot{J}(t) = (r + s)J(t) - (y - \bar{w}) \tag{3.3}$$

We now have all the elements needed to determine the dynamics of θ. Differentiating (3.2) with respect to time, where by definition $q(\theta) \equiv p(\theta)/\theta$, yields:

$$\dot{J}(t) = \frac{cp(\theta(t)) - c\theta(t)p'(\theta(t))}{p(\theta(t))^2}\,\dot{\theta}(t) = \frac{c}{p(\theta(t))}\left[1 - \eta(\theta(t))\right]\,\dot{\theta}(t), \tag{3.4}$$

where $0 < \eta(\theta) < 1$ (defined in footnote 4 above) denotes the elasticity of $p(\theta)$ with respect to θ. To simplify the derivations we henceforth assume that $\eta(\theta) = \eta$ is constant (which is satisfied for instance if the matching function $m(\cdot)$ is of the Cobb-Douglas type). Substituting (3.3) for $\dot{J}$ and using the expression $J = c\frac{\theta}{p(\theta)}$ we can rewrite equation (3.4) as:

$$\dot{\theta}(t)\,\frac{c}{p(\theta(t))}\,(1 - \eta) = (r + s)\frac{c\,\theta(t)}{p(\theta(t))} - (y - \bar{w}) \tag{3.5}$$

Finally, after rearranging terms we get the final form for the dynamic equation of θ:

$$\dot{\theta}(t) = \frac{r + s}{1 - \eta}\theta(t) - \frac{p(\theta(t))}{c\,(1 - \eta)}\,(y - \bar{w}) \tag{3.6}$$

Changes in θ depend (in addition to all the parameters of the model) only on the value of θ itself. The labour market tightness does not in any independent way depend on the unemployment rate u. In $(\theta,\, u)$ space the curve $\dot{\theta} = 0$ can thus be represented by a horizontal line at height $\bar{\theta}$, which defines the unique steady

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[5] The solution of (3.3) shows that the value $J(t)$ is the present discounted value (in continuous time) of future values of net productivity $(y - \bar{w})$, discounted using the real interest rate (r) and the rate of separation (s):

$$J(t) = \int_t^{\infty} (y - \bar{w})\mathrm{e}^{-(r+s)\tau}\, d\tau.$$

Therefore, J is a forward-looking variable, and will adjust immediately to any change in expected future levels of y, $\bar{w}$, r and s.

state equilibrium value for the ratio between vacancies and unemployed. This is illustrated in the left panel of Figure 3. Once we have determined $\bar{\theta}$, we can obtain for each value of the unemployment rate the level of v that is compatible with a stationary equilibrium. For instance, in the case of u_0 this is equal to v_0.

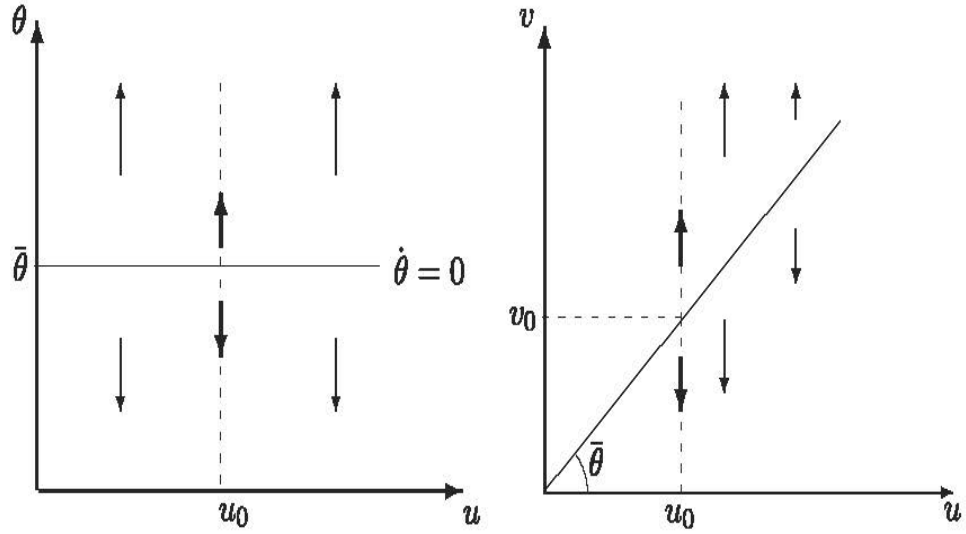

Figure 3. Dynamics of market tightness θ and supply of vacancies v

Moreover, equation (3.6) also indicates that for points above or below the curve $\dot{\theta} = 0$, θ tends to move away from its equilibrium value. Formally, one can show this by calculating[6]

$$\left.\frac{\partial \dot{\theta}}{\partial \theta}\right|_{\dot{\theta}=0} = r + s > 0$$

from (3.6). The apparently "unstable" behavior of θ is due to the nature of the job creation decision of firms. Looking at the future, firms' decision whether to open a vacancy today is based on the expected future values of θ. For example, if firms expect a future increase in θ due to an increase in the number of vacant jobs, they will anticipate an increase in the future costs to fill a vacancy. As a result, firms have an incentive to open vacancies immediately to anticipate the

[6] Note that this derivative is computed at a steady state equilibrium point (on the $\dot{\theta} = 0$ locus). Hence, we may use (1.9), and replace $y - \bar{w}$ with $(r + s)\frac{c\theta}{p(\theta)}$ to obtain the expression in the text.

increase in the cost. At the aggregate level this induces an immediate increase of v (and of θ) in anticipation of further increases in the future. Hence, there is an obvious analogy between the variations of v and the movement of forward-looking asset prices (such as stocks): expectations of a future increase in the price cause an increase in current prices.

As a result of forward-looking behavior on the part of firms, both v and θ are "jump" variables. Their value is not predetermined: in response to changes in the exogenous parameters (even if these changes are only expected in the future and have not yet materialized) v and θ may exhibit discrete changes. The unemployment rate, on the contrary, is a "predetermined" variable. The dynamics of the unemployment rate are governed by (3.1) and u adjusts gradually to changes in θ, even in case of a discrete change in the labour market tightness. An unanticipated increase in v and θ leads to an increase of the flow out of unemployment, resulting in a reduction of u. However, the positive effect of the number of vacancies on unemployment is mediated via the stochastic matching process on the labour market. The immediate effect of an increase in θ is an increase in the matching rate for workers $p(\theta)$ and this translates only gradually in an increase in the number of filled jobs. The unemployment rate will therefore only start to decrease after some time.

The aggregate effect of the decentralized decisions of firms (each of which disregards the externalities of its own decision on aggregate variables) consists of changes in the degree of labour market tightness θ and, as a result, in changes in the speed of adjustment of the unemployment rate. The dynamics of u are therefore intimately linked to the presence of the externalities that characterize the functioning of the labour market in the search and matching approach.

3.2. Steady state and dynamics

We are now in a position to characterize the system graphically, using the differential equations (3.1) and (3.6) for u and θ. In both panels of Figure 4 we have drawn the stationary curves $\dot{\theta} = 0$ and $\dot{u} = 0$. Moreover, for each point outside the unique steady state equilibrium we have shown the movement of θ and u using horizontal (for u) and vertical (for θ and v) arrows.

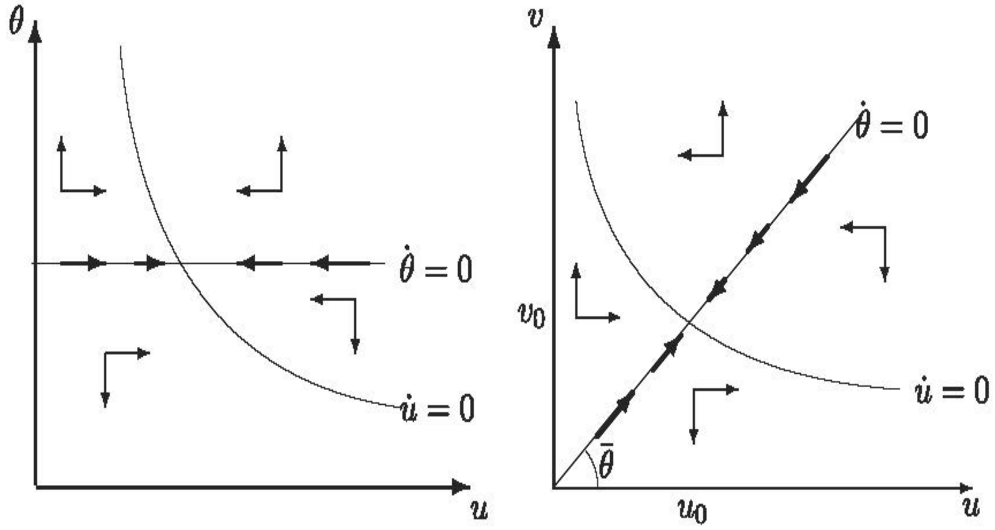

Figure 4. Joint dynamics of unemployment and vacancies

Similar to what we have seen in the analysis of dynamic models of real-financial interactions, the combination of a single state variable (u) and a single jump variable (θ) implies that there is only one *saddlepath* that converges to the steady state equilibrium. In this model, since the expression for $\dot{\theta} = 0$ does not depend on u, the saddlepath coincides with the stationary curve $\dot{\theta} = 0$: all the other points are located on paths that diverge from the curve $\dot{\theta} = 0$ and never reach the steady state, violating the transversality conditions. Hence, as a result of the forward looking nature of the vacancy creation decisions of firms, the labour market tightness θ will jump immediately to its long run value and remain there during the entire adjustment process.

Let us now analyze the adjustment process of the labour market in response to permanent and temporary reductions in labor productivity y, the latter shock capturing a cyclical downturn in aggregate activity (i.e. a recessionary period).

Figure 5 illustrates the dynamics following an unanticipated *permanent reduction* in *productivity* ($\Delta y < 0$) at date t_0. In the graph on the left, the curve $\dot{\theta} = 0$ shifts downward while $\dot{u} = 0$ does not change. In the new steady state equilibrium (point C) the unemployment rate is higher and labour market tightness is lower. Moreover, from the graph on the right it follows that the number of vacancies has also diminished. The Figure also illustrates the dynamics of the variables: at date t_0 the economy jumps to the new saddle path which coincides with the new curve $\dot{\theta} = 0$. Given the predetermined nature of the unemployment rate, the whole adjustment is performed by v and θ, which make a discrete jump downwards as shown by B in the two graphs. From t_0 onwards both unemployment and the number of vacancies increase gradually keeping θ fixed until the new steady state equilibrium is reached.

The permanent reduction in labor productivity reduces the expected profits of a filled job (see the solution of the dynamic equation for the value of a job $J(t)$ in footnote 5). Hence, from t_0 onwards firms have an incentive to open less vacancies. Moreover, initially the number of vacancies v falls below its new equilibrium level because firms anticipate that the unemployment rate will rise. In the future it will therefore be easier to fill a vacancy. As a result, firms prefer to reduce the number of vacancies at the beginning of the adjustment process, increasing their number gradually as the unemployment rate starts to rise.

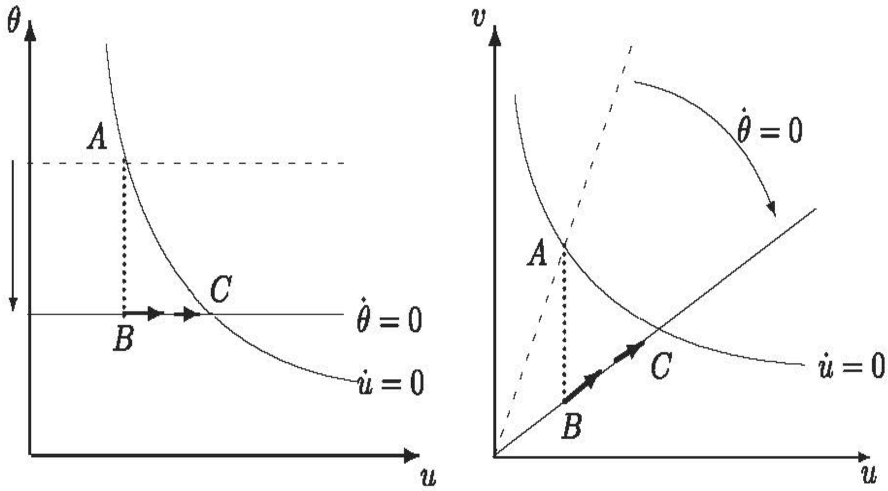

Figure 5. Permanent reduction in labour productivity

Finally, let us consider the case of a *temporary reduction* of *productivity*: differently from before, agents now anticipate at t_0 that productivity will return to its higher initial value at some future date t_1. Given the temporary nature of the shock the new steady state equilibrium coincides with the initial equilibrium (point A in the graphs of Figure 6). At the time of the change in productivity, t_0, the immediate effect is a reduction in the number of vacancies which causes a discrete fall in θ. However, this reduction is smaller than the one that resulted in the case of a permanent change and it moves the equilibrium from the previous equilibrium A to a new point B'. From t_0 onwards, the unemployment rate and the number of vacancies increase gradually but not at the same rate: as a result, their ratio θ increases, following the diverging dynamics that lead towards the new and lower stationary curve $\dot{\theta} = 0$. To obtain convergence of the steady state equilibrium at A, the dynamics of the adjustment need to bring θ to its equilibrium level at t_1 when the shock ceases and productivity returns to its previous level (point B''). In fact, convergence to the final equilibrium can only occur if the system is located on the saddle path, which coincides with the stationary curve for θ, at date t_1. After t_1 the dynamics concerns only the unemployment rate u and the number of vacancies v, which decrease in the same proportion until the system reaches its initial starting point A.

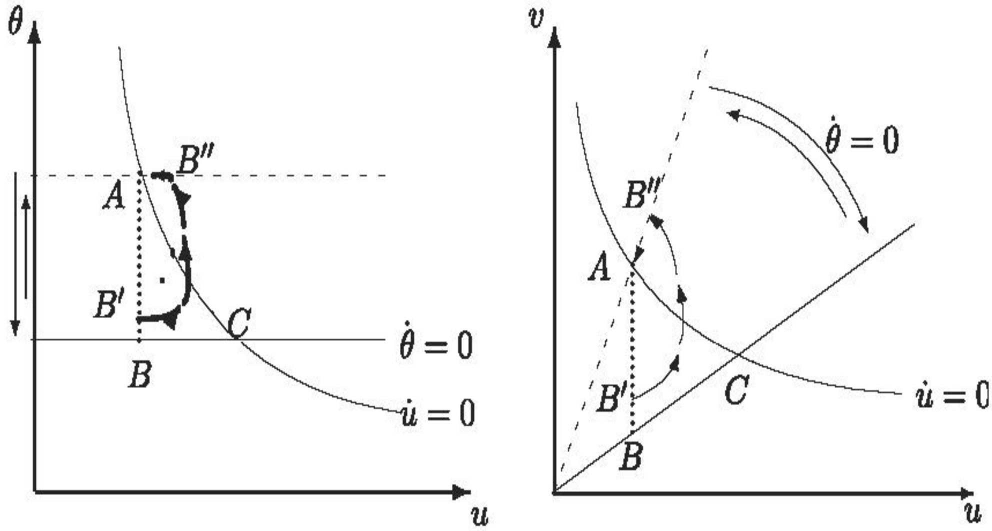

Figure 6. Temporary reduction in labour productivity

The graph on the right hand side of Figure 6 also illustrates that cyclical variations in productivity give rise to a counter-clockwise movement of unemployment and vacancies around the Beveridge curve. This is coherent with empirical data for the changes in unemployment and vacancies during recessions, which are approximated here by a temporary reduction in productivity.

References

[1] Hall R. (2005), "Employment fluctuations with equilibrium wage stickiness", *American Economic Review,* March

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Problems

1. Using the steady state equations for the (u, θ) relation (BC), the job creation condition (JC) and assuming a fixed wage rate $\bar{w}$,compare the effects on the steady state levels of u, v and θ of a smaller labour productivity ($\Delta y < 0$, an *aggregate* shock) and of a higher separation rate ($\Delta s > 0$, a *sectoral* shock). [Formal derivation is not required; graphical analysis with economic explanation is sufficient]

2. Using the two dynamic equations for the unemployment rate u and the degree of tightness of the labour market θ, derive the effect on the steady state of the economy of an *anticipated* future increase in the wage rate $\bar{w}$ (firms know at time t_0 that the wage rate will increase permanently at a future date t_1). Describe the dynamic adjustment of u, v and θ towards the new steady state equilibrium.