Macroeconomic Analysis
Lecture Notes 6 - Unemployment
(answers to problem set)

PROBLEM 1. The wage rate is exogenously fixed by assumption. Therefore, the W function is simply:

$$W = \overline{w}$$

The job creation condition is:

$$\begin{aligned} y - w &= (r+s)\frac{c}{q(\theta)} \qquad \text{(JC)} \\ \text{with } q(\theta) &\equiv \frac{p(\theta)}{\theta}, \quad q'(\theta) < 0 \\ \text{and } p(\theta) &= \frac{m}{u}, \quad p'(\theta) > 0; \quad \theta \equiv \frac{v}{u} \end{aligned}$$

Assume that the matching function is Cobb-Douglas:

$$m(u,v) = u^{\alpha} v^{1-\alpha}$$

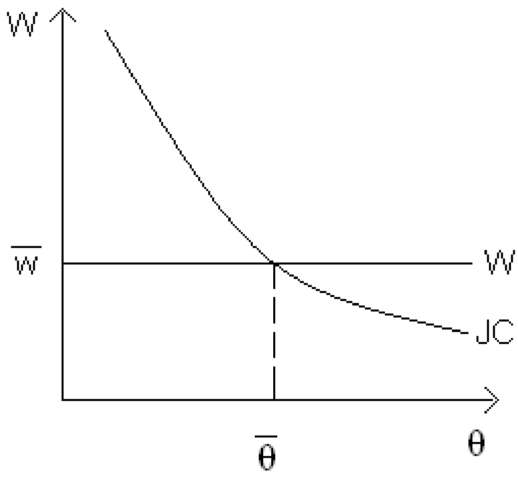
$$p(\theta) \equiv \frac{m(u,v)}{u} = \frac{u^{\alpha}v^{1-\alpha}}{u} = \left(\frac{v}{u}\right)^{1-\alpha} = \theta^{1-\alpha}$$

$$\eta \equiv \frac{p'(\theta)\cdot\theta}{p(\theta)} = \frac{(1-\alpha)\theta^{-\alpha}\cdot\theta}{\theta^{1-\alpha}} = \frac{(1-\alpha)\theta^{1-\alpha}}{\theta^{1-\alpha}} = 1-\alpha$$

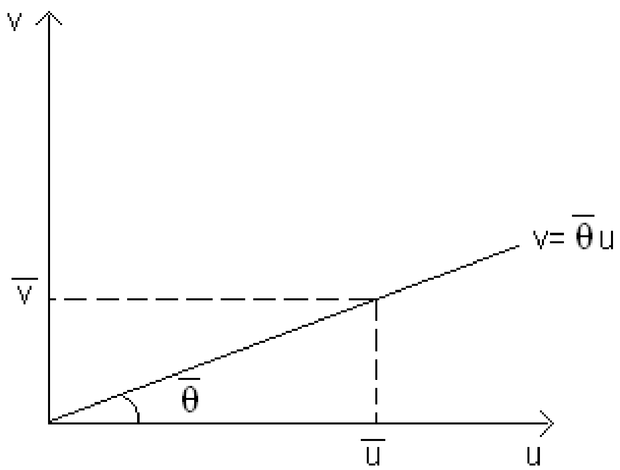
constant and not depending on θ,where

- $\theta \equiv$ measure of "tightness" of the labour market
- $p(\theta) \equiv \frac{m}{u} =$ probability of a match for the unemployed
- $q(\theta) \equiv \frac{p(\theta)}{\theta} =$ probability of a match for a firm

If $\theta \uparrow$, with y, r, and s constant, w should *decrease* in order for the equality above (JC) to hold. Therefore, there exists a negative correlation between w and θ.



Given $\overline{\theta}$, we can associate to each value of the unemployment rate only one value of v compatible with $\overline{\theta}$.



The slope of this line is given by $\overline{\theta}$. The line $v = \overline{\theta} \cdot u$ is obtained combining W with JC (which, together, determine $\overline{\theta}$). Consider now the Beveridge Curve:

$$u = \frac{s}{s + p(\theta)}$$

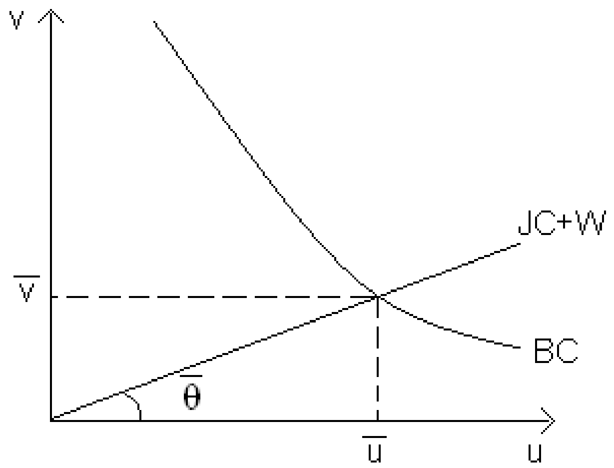
from which

$$p(\theta) = \frac{s(1-u)}{u}$$

Take:

$$\frac{\partial p(\theta)}{\partial u} = -\frac{s}{u^2} < 0$$

when u increases, $p(\theta)$ decreases $\Longrightarrow \theta \downarrow$ (given that $p'(\theta) > 0$). u an θ are negatively correlated. The Beveridge curve defines the level of vacancies ($\bar{v}$) that corresponds to the pair ($\bar{\theta}$, $\bar{u}$). Since $\theta = \frac{v}{u}$ and $\theta'(v) = \frac{1}{u} > 0$, a *reduction* in v implies a reduction in θ. v and θ are positively correlated. Conclusion: v and u are negatively correlated along the BC



Assume that:

$$\Delta y < 0 \Longrightarrow y \downarrow \quad \text{an aggregate shock.}$$

BC:

$$u = \frac{s}{s + p(\theta)} \quad \text{remains unchanged}$$

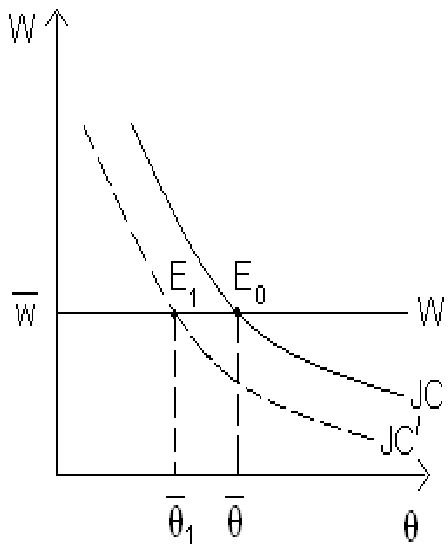
W:

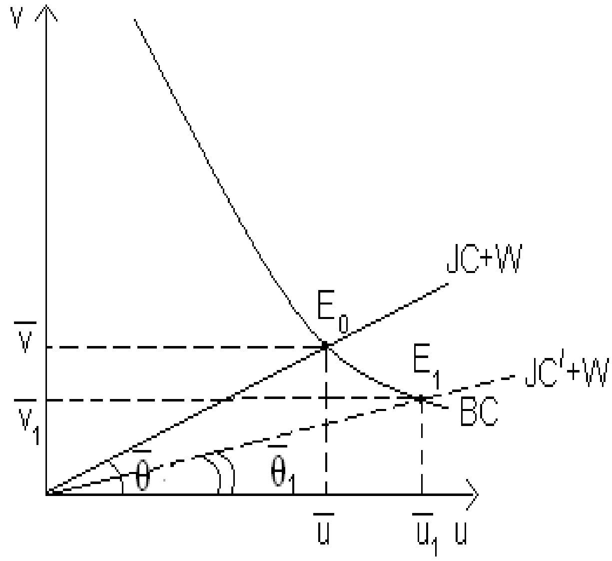
$$W = \overline{w} \quad \text{remains unchanged}$$

JC:

$$y - w = (r + s)\frac{c}{q(\theta)} \quad \text{does change}$$

If $y \downarrow$, the LHS $\downarrow$. For the RHS to decrease in order for the equality to continue to hold, with given w, r, s, and c, $q(\theta)$ must increase, which implies a decrease in θ (since $q'(\theta) < 0$). With W constant and θ decreasing, JC shifts downwards - since $\theta \downarrow$ the slope of JC + W decreases as well. In sum: θ decreases from $\overline{\theta}$ to $\overline{\theta}_1$, with $\overline{\theta}_1 < \overline{\theta}$. u increases and v decreases. W remains constant at the fixed level w.





Intuitively, a reduction in labor productivity reduces the expected profits of a filled job. Hence, firms have an incentive to open less vacancies, with an increase in the unemployment rate and a decrease in θ.

Suppose now that:

$$\Delta s > 0 \Longrightarrow s \uparrow \quad \text{sectoral shock}$$

W:

$$W = \overline{w} \quad \text{remains unchanged}$$

JC:

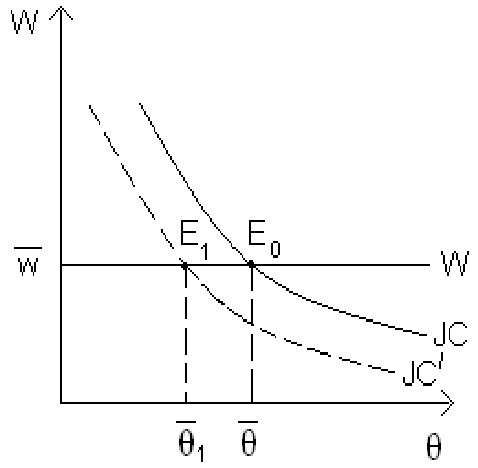
$$y - w = (r + s)\frac{c}{q(\theta)} \qquad \text{does change}$$

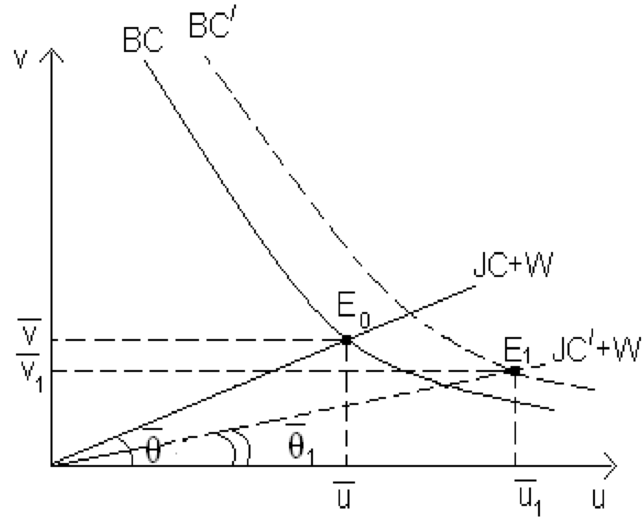
If s↑, the RHS↑. With y, w, r, and c given, $q(\theta)$ must increase for the equality to continue to hold.$\Longrightarrow \theta \downarrow$, with $q'(\theta) < 0$. With w given and θ decreasing, JC shifts downwards and the (JC + W) curve has a lower slope. Notice that:

$$BC \rightarrow u = \frac{s}{s + p(\theta)}$$

$$\frac{\partial u}{\partial s} = \frac{p(\theta)}{[s + p(\theta)]^2} > 0$$

If $s \uparrow \Longrightarrow p(\theta) \uparrow \Longrightarrow \theta \uparrow \Longrightarrow v \uparrow$, since $\theta'(v) = \frac{1}{u} > 0$, u given. If, with u given, $v \uparrow$, then the BC shifts outwards.





In this case:

- w remains unchanged
- θ decreases from $\overline{\theta}$ to $\overline{\theta}_1$, $\overline{\theta}_1 < \overline{\theta}$.
- u increases unambiguously
- The effect on v is ambiguous

An increase in s increases $\dot{u}$:

$$\dot{u} = s(1-u) - p(\theta)u$$

and makes the market tightness (θ) lower. The effect on the firms' willingness to open new vacancies is ambiguous

PROBLEM 2 The two dynamic equations of interest are:

$$\dot{u} = s(1-u) - p(\theta)u$$

for the unemployment rate, and

$$\dot{\theta} = \frac{r+s}{1-\eta} \cdot \theta - \frac{p(\theta)}{c(1-\eta)}(y - \bar{w})$$

for the degree of tightness of the labour market. η is defined as:

$$\eta \equiv \frac{p'(\theta) \cdot \theta}{p(\theta)}$$

For simplicity, assume η constant and not depending on θ. (as is the case for a Cobb-Douglas type matching function). The $\dot{u} = 0$ locus:

$$\dot{u} = 0 \Longrightarrow \qquad p(\theta) = \frac{s(1-u)}{u}, \quad \theta \equiv \frac{v}{u}$$

Take:

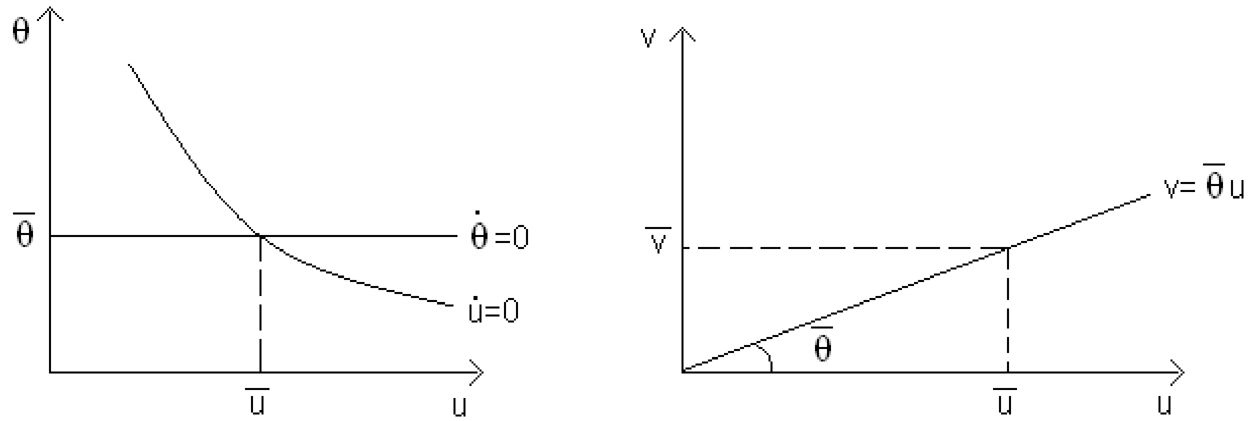
$$\frac{\partial p(\theta)}{\partial u} = \frac{-s}{u^2} < 0$$

when u increases, with s given, $p(\theta)$ decreases. Since $p'(\theta) > 0 \Longrightarrow \theta \downarrow \Longrightarrow$ negative correlation between θ and u along the $\dot{u} = 0$ locus. The $\dot{\theta} = 0$ locus:

$$\dot{\theta} = 0 \Longrightarrow \frac{r+s}{1-\eta} \cdot \theta = \frac{p(\theta)}{c(1-\eta)}(y - \bar{w})$$

In the $\dot{\theta} = 0$ locus there is no independent role for u - it depends only on θ. In other words, the locus $\dot{\theta} = 0$ appears graphically as a horizontal line in the space (θ, u) at the steady-state value $\bar{\theta}$.

Given $\bar{\theta}$ we can find out the unique value for the unemployment rate $(\bar{u})$ compatible with $\bar{\theta}$.



Given $\bar{\theta}$ and $\bar{u}$, the number of vacancies $(\bar{v})$ is uniquely determined

It is possible to introduce the $\dot{u} = 0$ locus in the space (v, u). To do this, we know that:

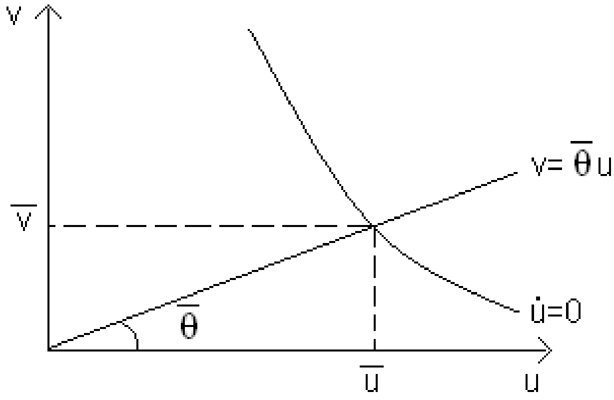
$$\dot{u} = 0 \Longrightarrow \quad p(\theta) = \frac{s(1-u)}{u}$$

$$\frac{\partial p(\theta)}{\partial u} = \frac{-s}{u^2} < 0$$

when u increases, with given s, $p(\theta)$ decreases. This implies a reduction in θ, since $p'(\theta) > 0$. But since:

$$\theta = \frac{v}{u}, \ \theta'(v) > 0 \Longrightarrow v \downarrow$$

In sum, when $u \uparrow$ then $v \downarrow \Longrightarrow$ negative correlation between v and u along the $\dot{u} = 0$ locus.



Dynamics:
1.

$$\begin{aligned} \frac{\partial \dot{u}}{\partial u} &= -s - [-p'(\theta) \cdot u \cdot \frac{v}{u^2} + p(\theta)] \\ &= -s + p'(\theta)\theta - p(\theta) \end{aligned}$$

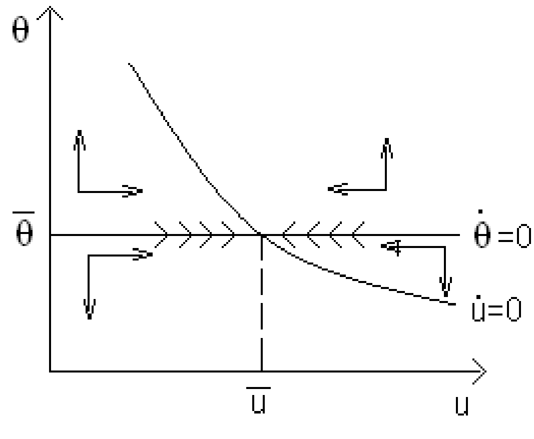
Divide both sides by $p(\theta)$:

$$\frac{\partial \dot{u}}{\partial u}/p(\theta) = -\frac{s}{p(\theta)} + \eta - 1 < 0, \quad 0 \le \eta \le 1$$

$$\implies \quad \frac{\partial \dot{u}}{\partial u} < 0 \quad \text{stable locus}$$

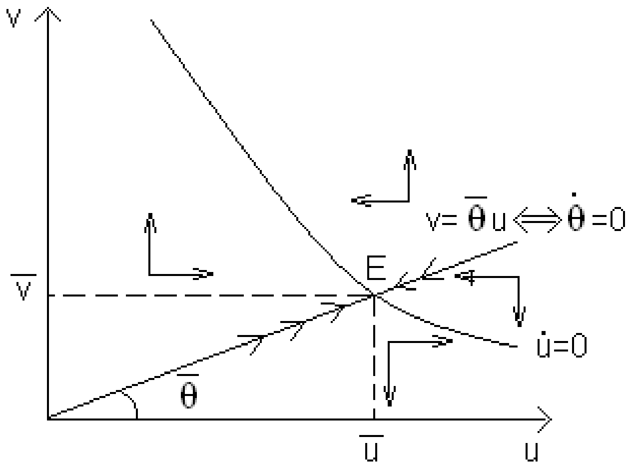
2. It is possible to show that:

$$\frac{\partial \dot{\theta}}{\partial \theta} > 0 \ \ \text{unstable locus}$$



Given $\bar{\theta}$ and $\bar{u}$, we can determine uniquely the value of v ($\bar{v}$) compatible with $\bar{\theta}$

The system converges towards the equilibrium $(\bar{v}, \bar{u})$, along the $\dot{\theta} = 0$ locus.



Consider an *anticipated future increase* in the exogenous wage rate: $\bar{w} \uparrow$.
Consider the two loci:

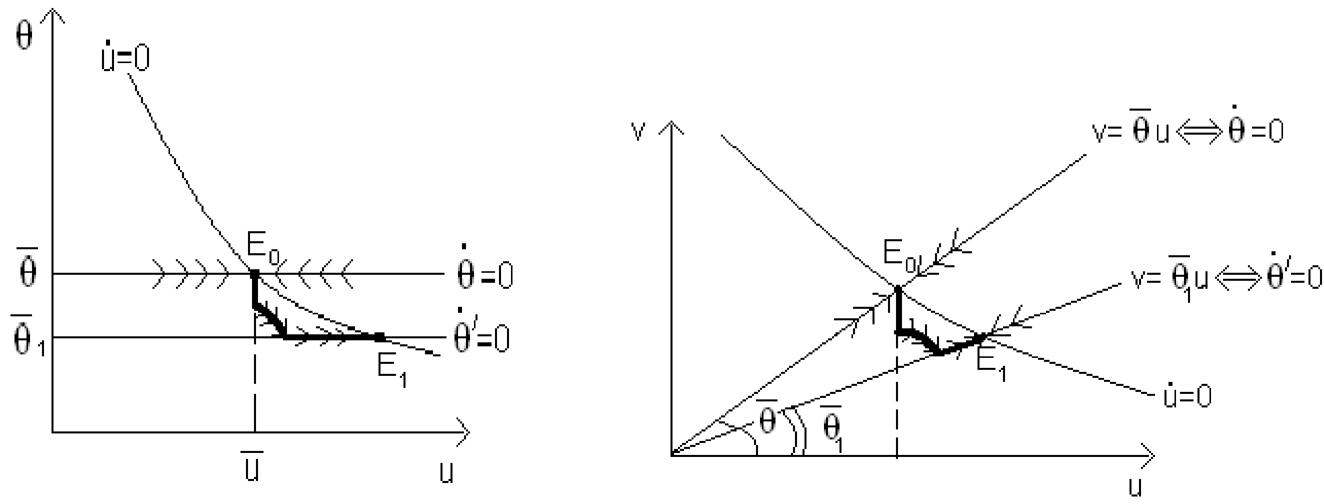
$$\dot{u} = 0 \Longrightarrow \quad p(\theta) = \frac{s(1-u)}{u} \quad \text{does not change}$$

$$\dot{\theta} = 0 \Longrightarrow \quad \frac{r+s}{1-\eta} \cdot \bar{\theta} = \frac{p(\bar{\theta})}{c(1-\eta)}(y - \bar{w})$$

when $\bar{w}$ increases, the RHS decreases. With r, s, η, c and y given, the only way for the LHS to decrease (and to keep equating the RHS) is to reduce $\bar{\theta}$. In sum:

$$\bar{w} \uparrow \Longrightarrow \bar{\theta} \downarrow$$

the $\dot{\theta} = 0$ locus shifts downwards.



The saddle path is represented by the $\dot{\theta} = 0$ locus. The jump variables are v and θ: in response to changes in the exogenous parameters, v and θ exhibit discrete changes. The state variable is u, which adjusts gradually to changes in θ.
At t_0 firms anticipate the future increase in the wage rate $\bar{w}$ and *immediately* reduce the number of vacancies: v and θ fall by a discrete amount. Between t_0 and t_1, the dynamics are governed by the differential equations associated with the initial steady-state (E_0). v and θ continue to decrease (while the unemployment rate increases) until they reach the new saddle path at t_1. From t_1 onwards, u and v increase in the same proportion, leaving θ unchanged.