Monetary Policy and Market Interest Rates

By Tore Ellingsen and Ulf Söderström*

Understanding the relationship between monetary policy and market interest rates is of utmost importance to bond traders and central bankers alike. Unanticipated changes in monetary policy strongly affect interest rates of almost all maturities, representing recurrent opportunities for traders to win or lose money. All serious bond analysts have their own quantitative model of the past relationship between policy moves and the yield curve. Policy makers on the other hand carefully watch the yield curve for news about market expectations. Academic economists are interested too: the effect of monetary policy on the real economy is one of our discipline's more controversial topics.

Given these efforts, our understanding of yield curve movements remains remarkably incomplete. True, there are some statistical regularities. It is empirically well established that monetary policy affects market interest rates, and that on average this relationship is positive: an increase in the central-bank rate leads to an increase in interest rates of all maturities. It is also well known, however, that there are many exceptions to the rule. For example, on a number of occasions in 1994 when the Federal Reserve announced an increase in its target rate, interest rates of long maturities fell. As noted by Tom Skinner and Jerónimo Zettelmeyer (1995), who study the interest rate response to monetary policy over long periods in four major economies, the fraction of such "abnormal" responses is considerable in all countries.[1]

At the moment, there is no coherent theory which tells us whether the yield curve will shift or rotate after a policy change. Some argue that the curve should always shift. For example, Timothy Cook and Thomas Hahn (1989), who first firmly established the positive empirical relationship between target rates and long rates, interpret their finding as supportive of the expectations theory of the term structure.[2] The expectations theory says that a long interest rate should be equal to the average of short interest rates over the same period of time plus a term premium; thus an increase in the first couple of short rates should drive up the long rate too, but by less. Christina D. Romer and David H. Romer (2000) disagree. To them, the positive movement in the long rate is inconsistent with standard monetary theory—a puzzle. According to received theory, they claim, an increase in short rates should reduce inflation, and hence reduce the level of sufficiently long rates. Romer and Romer suggest that the puzzle can be resolved if the central bank has access to private information about economic fundamentals, but they do not develop their argument formally.

In this paper, we provide a simple model within which each of the three mechanisms captured by the "standard" theory, the expectations hypothesis, and Romer and Romer (2000), respectively, are all at play. Our argument centers around the presumption that a change in monetary policy can come about for two distinct

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[1] Skinner and Zettelmeyer (1995) consider France, Germany, the United Kingdom, and the United States. Similar findings are reported by Ric Battellino et al. (1997) for Australia; Luigi Buttiglione et al. (1997) for Italy; Hans Lindberg et al. (1997) for Sweden.

[2] This view is echoed by, for example, Skinner and Zettelmeyer (1995) and Yash P. Mehra (1996).

reasons: either the monetary authorities respond to new and possibly private knowledge about the economy, or their policy preferences change. In the first case, policy is essentially *endogenous,* reflecting new input into a given objective function; in the second case, policy is *exogenous,* in the sense that the input is the same but the objective function is new. After an endogenous policy action, our model predicts that interest rates of all maturities move in the same direction as the policy innovation. After an exogenous policy action, on the other hand, short and long interest rates should move in opposite directions.

Let us now describe our approach in a little more detail. Our theoretical model is taken from Lars E. O. Svensson (1997b, 1999), and is quite simple, with reduced-form relationships for output and inflation. Key features of the economy are that shocks to output and inflation have persistent effects, and that monetary policy affects output and inflation with a lag. To this model we add an equation describing the term structure of interest rates. The central bank is assumed to control the one-period interest rate and to minimize a loss function which is quadratic in deviations of output and inflation from target. The simplified treatment of the economy allows us to derive the central bank's reaction function endogenously and to obtain a closed-form expression for the yield curve.

Assuming that the expectations hypothesis of the term structure holds, the model yields the following set of predictions. Suppose the central bank's objective function is known and stable. Whenever an economic shock is symmetrically observed by all agents, market interest rates respond immediately, and the change in the central-bank rate is fully anticipated. In this case, all interest rates move in the same direction (Proposition 1). *Unanticipated* changes in the central-bank rate can occur for two separate reasons. First, the central bank may have private (that is, advance) information about exogenous shocks to output and prices. In this case, an increase in the short interest rate could be interpreted by market participants as an indication of increased inflation, and as the central bank acts to squeeze inflation out of the economy, interest rates of all maturities go up (Proposition 3).[3] The existence of central-bank private information in the United States has been documented by Romer and Romer (2000), who suggest that the source of private information is likely to be superior data processing abilities at the Federal Reserve rather than earlier access to data. Second, the central bank's preferences may change. The policy preferences of the central bank are captured by the parameter λ, which measures the weight on output stabilization relative to inflation stabilization in the bank's objective function. Thus, if the short interest rate is increased and bond traders are confident that there has been no unanticipated change in the fundamentals, then they will typically infer that inflation stabilization has moved higher on the central bank's agenda. In this case, we show that sufficiently long interest rates will move in the opposite direction, because average inflation is reduced (Proposition 4). We also note that λ determines the magnitude of the interest rate response to fundamental shocks. For a given shock, short rates respond less and long rates more as we increase λ (Proposition 2).

I. The Model

The model we use is taken from Svensson (1997b, 1999), and is a dynamic version of a simple aggregate supply-aggregate demand model, where we add an equation for the term structure of interest rates.[4] Monetary policy does not affect the inflation rate directly, but only through the level of aggregate demand. An important feature is the introduction of "control lags" in the response of the economy to monetary policy: policy affects aggregate demand after a lag of one period, and aggregate demand in turn affects the inflation rate in the subsequent period. This feature is consistent with the stylized facts about the response of output and inflation to monetary policy (see, for example, Ben S. Bernanke and Mark Gertler, 1995).

[3] However, it is not necessarily true that all future short rates go up. Because the initial increase in the short rate creates a reduction in output, it may have to be offset by future interest rate reductions.

[4] Laurence Ball (1999) develops a similar model, and most results apply also to his framework.

A. *Setup*

Let π_t be the deviation at time t of the inflation rate from its long-run average (given by the central bank's inflation target), and let y_t be the percentage deviation of real output from its "natural" level, that is, the output gap. The inflation process (the aggregate supply relationship) is governed by an accelerationist Phillips curve: the change in the inflation rate is positively related to the previous period's output gap according to

$$\pi_{t+1} = \pi_t + \alpha y_t + \varepsilon_{t+1}, \tag{1}$$

where $\alpha > 0$ and ε_t is an independently and identically distributed (i.i.d.) supply shock with mean zero. The output gap (or aggregate demand) is mean reverting and negatively related to the *ex ante* real short interest rate following

$$y_{t+1} = \hat{\beta} y_t - \gamma(i_t - \pi_{t+1|t}) + \eta_{t+1}, \tag{2}$$

where i_t is the deviation of the short interest rate (set by the central bank) from its long-run equilibrium level; $\pi_{t+1|t} \equiv E_t \pi_{t+1}$, that is, the period t expectation of inflation in period $t + 1$; $0 < \hat{\beta} < 1$; $\gamma > 0$; and η_t is an i.i.d. demand shock with mean zero. Taking expectations of equation (1) at t, and using in (2) then gives the output gap in terms of the *ex post* real interest rate as

$$y_{t+1} = \beta y_t - \gamma(i_t - \pi_t) + \eta_{t+1}, \tag{3}$$

where $\beta = \hat{\beta} + \alpha\gamma$, and we assume that $0 < \beta < 1$.

Our own contribution is to append a yield curve to this model. Bonds of different maturities are seen as imperfect substitutes, so the interest rate on a discount bond of maturity n at time t is set as the average of expected future short interest rates during the time to maturity plus a term premium,

$$i_t^n = \frac{1}{n} \sum_{s=0}^{n-1} i_{t+s|t} + \xi_t^n, \tag{4}$$

where $i_{t+s|t}$ is the expected short interest rate s periods ahead, and ξ_t^n is the term premium at time t for maturity n. Thus, in determining long rates, market participants will form (rational) expectations about the future path of the short central-bank rate.

B. *The Central-Bank Problem*

At each instant, the central bank is assumed to select the short interest rate i_t to minimize the intertemporal loss function

$$\mathscr{L}_t = E_t \sum_{s=0}^{\infty} \delta^s L(\pi_{t+s}, y_{t+s}), \tag{5}$$

where δ is a discount factor and the period loss function $L(\cdot)$ is quadratic in the deviations of inflation and the output gap from their zero targets,

$$L(\pi_t, y_t) = \tfrac{1}{2}[\pi_t^2 + \lambda y_t^2]. \tag{6}$$

The parameter $\lambda \geq 0$ is the weight of output stabilization relative to inflation stabilization. Although we will allow for shifts in this preference parameter, we assume that the central bank (as well as private agents) at any time view it as certain and constant. Any changes in λ are thus fully unanticipated, and seen as permanent.[5]

To solve the central bank's optimization problem, we can exploit the recursive structure of the model and treat the expected output gap $y_{t+1|t}$ as the control variable, following Svensson (1997b) and Ball (1999). We can then back out the optimal short interest rate from the relationship

$$y_{t+1|t} = \beta y_t - \gamma(i_t - \pi_t). \tag{7}$$

Thus the central bank's optimization problem can be expressed as the control problem

$$V(\pi_{t+1|t}) = \min_{y_{t+1|t}} \{\tfrac{1}{2}[\pi_{t+1|t}^2 + \lambda y_{t+1|t}^2] + \delta E_t V(\pi_{t+2|t+1})\}, \tag{8}$$

[5] If one allows the preference parameter to follow a martingale, so $E_t \lambda_{t+s} = \lambda_t$, certainty equivalence does not hold and the model becomes intractable.

subject to

$$\text{(9)} \qquad \pi_{t+2|t+1} = \pi_{t+1} + \alpha y_{t+1} = \pi_{t+1|t} + \varepsilon_{t+1} + \alpha(y_{t+1|t} + \eta_{t+1}).$$

The first-order condition associated with (8) and (9) is

$$\text{(10)} \qquad \lambda y_{t+1|t} + \alpha\delta E_t V_\pi(\pi_{t+2|t+1}) = 0.$$

Since the objective function is quadratic and the constraint is linear, the value function in (8) will be of the form

$$\text{(11)} \qquad V(\pi_{t+1|t}) = k_0 + \frac{k}{2}\pi^2_{t+1|t},$$

where the constant k remains to be determined. Then, using the law of iterated expectations, we can express the optimal expected output gap as a function of the expected inflation rate two periods ahead,

$$\text{(12)} \qquad y_{t+1|t} = -\frac{\alpha\delta k}{\lambda}\pi_{t+2|t},$$

where the unique positive solution for k is given by

$$\text{(13)} \qquad k = \frac{1}{2}\left[1 - \frac{\lambda(1-\delta)}{\alpha^2\delta} + \sqrt{\left(1 + \frac{\lambda(1-\delta)}{\alpha^2\delta}\right)^2 + \frac{4\lambda}{\alpha^2}}\right] \geq 1.$$

See Svensson (1997b) for details.

Given the optimal expected output gap $y_{t+1|t}$ from (12), we back out the optimal interest rate from (7) as

$$\text{(14)} \qquad i_t - \pi_t = \frac{\alpha\delta k}{\gamma\lambda}\pi_{t+2|t} + \frac{\beta}{\gamma}y_t,$$

and leading (1) two periods and taking expectations gives the expected inflation rate two periods ahead as

$$\text{(15)} \qquad \pi_{t+2|t} = \pi_t + \alpha(1+\beta)y_t - \alpha\gamma(i_t - \pi_t).$$

Combining (14) and (15) we then have

$$\text{(16)} \qquad i_t - \pi_t = \frac{\alpha\delta k}{\gamma\lambda}\pi_t + \frac{\beta\lambda + \alpha^2\delta k(1+\beta)}{\gamma\lambda}y_t - \frac{\alpha^2\delta k}{\lambda}(i_t - \pi_t) = A\pi_t + By_t,$$

where

$$\text{(17)} \qquad A = \frac{\alpha\delta k}{\gamma(\lambda + \alpha^2\delta k)} > 0,$$

$$\text{(18)} \qquad B = \frac{\beta}{\gamma} + \alpha A > 0.$$

Thus, the optimal interest rate for the central bank is an increasing function of the current inflation rate and output gap,

$$\text{(19)} \qquad i_t = (1+A)\pi_t + By_t,$$

so the central bank follows a rule similar to that proposed by John B. Taylor (1993).

To aid intuition, three observations are useful. First, the model is formulated in deviations of inflation and output from their average levels (normalized to zero for convenience), and so is the interest rate in equation (19). Therefore a negative shock to inflation or output will lead to negative values of the short interest rate. Second, since monetary policy affects inflation via output, and with a lag of two periods, the way to dampen the inflationary effects of a positive shock is to create a recession. Svensson (1997b) shows that the response of the central bank to both inflation and output shocks is decreasing in the preference parameter λ. Thus, a central bank more prone to output stabilization will respond less to any shock. In particular, after a positive shock a central bank with a higher λ will choose to create a smaller recession, regardless of whether the initial shock is to inflation or output. Third, observe that although the optimal policy rule (19) depends only on current inflation and output, monetary policy is inherently forward looking: since the central bank can only affect inflation with a two-period lag, it sets its policy instrument depending on its forecast of inflation two periods ahead [see equation (14) or Svensson, 1997b]. The presence of current inflation and output in the policy rule only reflects the fact that these are important in predicting future inflation and output.

At this low level of complexity, our formulation appears to be a close approximation to monetary policy makers' view of the world (see, for example, Alan S. Blinder, 1997), and it fits the macroeconomic facts rather well (Glenn D. Rudebusch and Svensson, 1999). The main criticism to be directed at the model is the lack of forward-looking behavior expressed in the aggregate relationships (1) and (2). A plausible extension would therefore be to include forward-looking behavior in the determination of both inflation and output, following, for example, Julio J. Rotemberg and Michael Woodford (1997), Richard Clarida et al. (1999), or Bennett T. McCallum and Edward Nelson (1999).

What is going to be essential for our results is that monetary policy has lasting effects, since there is inertia in both inflation and output. In a purely forward-looking model, monetary policy merely has a contemporaneous effect, because only expectations about future output and inflation enter the basic supply and demand relationships; past output and inflation do not matter. On the other hand, as long as there is some inertia in inflation and output, even a large dose of forward-looking behavior does not destroy our qualitative results: they typically continue to hold in hybrid models (including both forward-looking and backward-looking behavior), such as the extension proposed by Svensson (1997a).[6] Since purely forward-looking models appear to be at odds with the data in a variety of ways, as shown by Arturo Estrella and Jeffrey C. Fuhrer (1998, 1999), we are not too worried about the fact that our results break down at the extreme case.

[6] These hybrid models generally do not have analytical solutions, but can be solved numerically, using the methods developed by Olivier Jean Blanchard and Charles M. Kahn (1980), David K. Backus and John Driffill (1986), and others; see Paul Söderlind (1999).

C. *The Term Structure of Interest Rates*

Knowing the short rate at each point in time, it is now relatively straightforward to compute the economy's yield curve as a function of current inflation and output. The n-period interest rate is set as an average of future short rates plus a term premium,

$$i_t^n = \frac{1}{n} \sum_{s=0}^{n-1} i_{t+s|t} + \xi_t^n, \tag{20}$$

so we first need to find the expected path of future short rates in order to evaluate rates of longer maturities. Leading the interest rate rule (19) s periods and taking expectations gives

$$i_{t+s|t} = (1 + A)\pi_{t+s|t} + By_{t+s|t}. \tag{21}$$

The expected output gap $s \geq 1$ periods from now is obtained by leading the inflation and output relationships (1) and (3), taking expectations, and using (21),

$$\begin{aligned} y_{t+s|t} &= \beta y_{t+s-1|t} - \gamma(i_{t+s-1|t} - \pi_{t+s-1|t}) \\ &= -\gamma A \pi_{t+s-1|t} \\ &\quad + (\beta - \gamma B) y_{t+s-1|t} \\ &= -\gamma A \pi_{t+s|t}, \end{aligned} \tag{22}$$

since $\beta - \gamma B = -\alpha\gamma A$. Likewise, the expected future inflation rate $s \geq 2$ periods ahead is

$$\begin{aligned} \pi_{t+s|t} &= \pi_{t+s-1|t} + \alpha y_{t+s-1|t} \\ &= (1 - \alpha\gamma A)\pi_{t+s-1|t}. \end{aligned} \tag{23}$$

It is then easily established by repeated substitution that expected inflation and output $s \geq 1$ periods ahead will follow the geometric series

$$\pi_{t+s|t} = (1 - \alpha\gamma A)^{s-1}[\pi_t + \alpha y_t] \tag{24}$$

and

$$y_{t+s|t} = -\gamma A(1 - \alpha\gamma A)^{s-1}[\pi_t + \alpha y_t]. \tag{25}$$

Using these relations in (21), the expected future short interest rate $s \geq 1$ periods ahead is given by

$$i_{t+s|t} = [1 + A(1 - \gamma B)] \times (1 - \alpha\gamma A)^{s-1}[\pi_t + \alpha y_t], \tag{26}$$

and its sum is obtained, using the formula for geometric series, as

$$\sum_{s=1}^{n-1} i_{t+s|t} = [1 + A(1 - \gamma B)]X_n[\pi_t + \alpha y_t], \tag{27}$$

where

$$X_n = \frac{1 - (1 - \alpha\gamma A)^{n-1}}{\alpha\gamma A}. \tag{28}$$

Finally, using the interest rate rule (19) and the sum (27) in the definition (20), the market interest rate of maturity n is given by

$$i_t^n = \frac{1}{n}\{(1 + A)\pi_t + By_t + [1 + A(1 - \gamma B)]X_n[\pi_t + \alpha y_t]\} + \xi_t^n. \tag{29}$$

As promised, this is our closed-form expression for the economy's yield curve.

II. Monetary Policy and the Term Structure of Interest Rates

We are now ready to examine how the term structure of interest rates is affected by monetary policy actions. From the central-bank reaction function (19), we see that current monetary policy is entirely determined by current inflation, output, and the preferences of the central bank. Consequently, it is straightforward to separate endogenous monetary policy, responding to the development of inflation and output, from exogenous policy moves, due to shifts in the preference parameter λ.

In a first scenario, we examine how market interest rates vary when all parameters and shocks are symmetrically observed by all agents. In this scenario, interest rates respond to supply and demand shocks directly, with the magnitude depending on the central bank's preference parameter, since the response of the monetary authorities is perfectly predicted by market participants. The actual policy actions of the central bank then add no new information, and so will not affect the term structure of interest rates.

We next turn to a scenario where the central bank has access to advance information about either the supply or demand shock, or about its own preferences.[7] In this case, the central bank's policy actions contain information about the unobservable variable. Consequently, interest rates will react to the actual policy moves, as market participants use this information to revise their beliefs about future monetary policy. Most importantly, the reaction of interest rates to endogenous policy is markedly different from the reaction to exogenous policy moves.[8]

The way we have chosen to model it, the central bank and market rates respond instantaneously to new information, so there is no clear distinction between the two responses. It is still enlightening to consider a more realistic setting, where market rates respond more quickly to new information than the central bank. Table 1 clarifies the timing of the two responses. At

[7] For simplicity, we will assume throughout that only one variable at a time is unobservable to market participants. We thus do not have a proper signal extraction problem for private agents. We choose to concentrate on the simple perfect-inference case here, to illustrate our mechanisms in a transparent way.

[8] Note that in this private information setting, market interest rates respond only to the unanticipated component of monetary policy. Our terminology may be slightly confusing: endogenous and exogenous policy moves do *not* coincide with anticipated and unanticipated policy, respectively. We refer to endogenous policy as responding to information (possibly private) about the economy, and exogenous policy as independent of the economic development and due to central-bank preference shifts.

TABLE 1—TIMING WITHIN A PERIOD

Stage	Symmetric Information	Asymmetric Information: Shocks	Asymmetric Information: Preferences
1	Shock hits the economy, observable to all	Shock hits the economy, unobservable to market	Shock hits the economy, observable to all
2	Market responds to shock, given observed λ	No market response	Market responds to shock, given beliefs of λ
3	Central bank responds to shock	Central bank responds to shock	Central bank responds to shock
4	No market response	Market infers shock, responds to shock	Market infers λ, responds to shock

the first stage, the economy is hit by a shock. Next, to the extent that the shock is observable to market participants, market interest rates respond, given participants' beliefs of how the central bank will react. Under symmetric information, the central-bank response in the third stage is perfectly anticipated by the market, so there is no additional response of interest rates to the actual central-bank move. However, when the central bank has access to information not available to market participants, the market will watch the bank's move to infer the unobservable variable, and respond to the new information.

All along, we will assume that the term premium is independent of all relevant variables, that is, that the expectations hypothesis of the term structure holds.[9] This simplifying assumption serves to streamline the results below.

A. *Symmetrically Observed Shocks*

When all variables are publicly observable, we see directly from equation (29) how market interest rates are affected by supply and demand shocks as well as by shifts in the preference parameter λ, the weight the central bank places on output stabilization.

Differentiating equation (29) with respect to ε_t, the interest rate of maturity n will respond to a supply shock according to

$$\frac{di_t^n}{d\varepsilon_t} = \frac{1}{n}\{1 + A + [1 + A(1 - \gamma B)]X_n\}. \tag{30}$$

Likewise, the interest rate will respond to a demand shock η_t according to

$$\frac{di_t^n}{d\eta_t} = \frac{1}{n}\{B + \alpha[1 + A(1 - \gamma B)]X_n\}. \tag{31}$$

Our first result is that these two derivatives are positive.

PROPOSITION 1: *Under symmetric information, interest rates of all maturities are positively related to both supply and demand shocks, with the magnitude diminishing with maturity. Thus all interest rates (including the central-bank rate) move in the same direction in response to a shock.*

PROOF:
See the Appendix.

This result seems quite intuitive, but it turns out not to be as straightforward as it looks. In particular, following an inflationary shock (to output or inflation), the short interest rate is always raised in the same period, but may be set either above or below the initial level in future periods. Future short interest rates are pushed below the initial level if the central bank is sufficiently averse to inflation (if λ is small). In this case the central bank responds to an inflationary shock by creating a large recession. This will bring inflation close to its target, but leave a need for a lower interest rate in order to close the (negative) output gap. What our proof of

[9] While we agree that the term premium could vary in a systematic way with inflation, output, or the monetary policy stance, it is noteworthy that a noisy term premium coupled with active monetary policy may account for some of the alleged empirical failures of the expectations hypothesis (see N. Gregory Mankiw and Jeffrey A. Miron, 1986; McCallum, 1994).

Proposition 1 shows is that any negative response of future short interest rates is dominated by the positive response in the first period. Thus long interest rates of all maturities increase.

A second implication of the model is that the response of all interest rates to a shock is linear, since the terms on the right-hand sides of (30) and (31) are constant, for a given n. Consequently, the relationship between any two interest rates will also be linear.

We next investigate how the magnitude of the preference parameter λ (the weight on output stabilization in the central bank's objective function) affects the response of interest rates to a given shock. As λ increases, the central bank becomes less inflation averse, and more prone to stabilizing output. For a given shock, the optimal interest rate policy is less fierce, and the central-bank rate is changed by a smaller amount, since both A and B are decreasing in λ. In the long run, however, a given shock will remain for longer in the economy, so future short rates are expected to be higher than if the central bank had neutralized a larger portion of the shock in the initial move. Therefore, central banks with a larger value of λ will see a larger effect on long rates for a given shock, since the central-bank rate is expected to differ from the initial level for a longer period of time.

PROPOSITION 2: *Under symmetric information, a higher value of λ makes the short interest rate respond less, and interest rates on bonds of sufficiently long maturity respond more, to a given shock. Consequently, the ratio of the response of rates on bonds of sufficiently long maturity to the change in the short rate is greater.*

PROOF:

See the Appendix.

We can now summarize our first set of results. When all shocks are observable to all agents, all interest rates move in the same direction in response to a shock that leads the public to revise their expectations of future monetary policy. For a more inflation-averse central bank, short rates will respond more, but long rates less to a given shock.

B. *Asymmetric Information*

For efficient bond markets to respond to the actual policy moves of the central bank, these moves must contain some information not previously available to market participants. Or, in other words, the central bank must have access to private information about relevant variables in the economy.[10] In our model, this information can be of two kinds: information about shocks to the inflation or output paths, or information about the central bank's preferences. We will study the two kinds of central-bank private information separately, to see how the presence of private information affects the determination of interest rates.

We begin by considering the case where the central bank has private (or advance) information about the current realization of either the supply or the demand shock. If only one of the shocks is unobservable at a time, the realization of this shock is easily inferred by market participants after observing the reaction of the central bank by inverting the policy rule (19). Thus, when the current realization of the supply shock ε_t is unobservable, it is inferred as

$$\hat{\varepsilon}_t(i_t) = \frac{1}{1+A}\, i_t - (\pi_{t-1} + \alpha y_{t-1}) - \frac{B}{1+A}\, y_t, \tag{32}$$

where all variables on the right-hand side are observable at time t. Similarly, when the central bank has private information about the demand shock η_t, its current realization is inferred as

$$\hat{\eta}_t(i_t) = \frac{1}{B}\, i_t - \frac{1+A}{B}\, \pi_t - [\beta y_{t-1} - \gamma(i_{t-1} - \pi_{t-1})]. \tag{33}$$

[10] Throughout, private information refers to information the central bank obtains earlier than private agents within a given period. An alternative interpretation is that all agents get access to information at the same time, but the central bank is better at processing this information. For evidence on these issues, see Romer and Romer (2000).

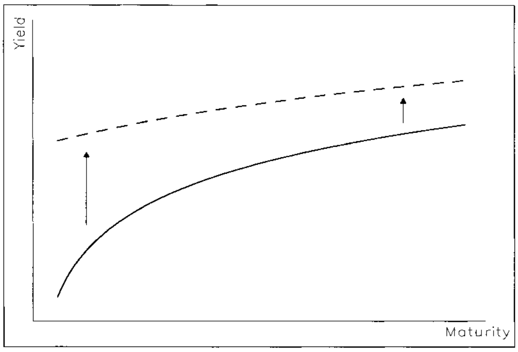

FIGURE 1. YIELD CURVE RESPONSE TO AN ENDOGENOUS POLICY CONTRACTION

In this simplistic setup, when the realization of the unobservable shock is perfectly inferred by bond markets, the results from the previous section remain. Now, however, market interest rates will react to the policy actions of the central bank, since these reveal information about the realized shocks, and thus about the future path of monetary policy. Consequently, although the results below are simple corollaries of Propositions 1 and 2 above, they have quite distinct interpretations for the response of interest rates to monetary policy.

First, when the supply or demand shock is unobservable to the public, Proposition 1 implies that all interest rates will move in the same direction as the central-bank rate, as market participants infer the realization of the unobservable shock:

PROPOSITION 3: *When the central bank has private information about either the supply or the demand shock, market interest rates will be positively related to the central-bank rate. This relationship becomes weaker as the interest rate's maturity increases.*

PROOF:
Follows immediately from Proposition 1.

A graphical representation of this result is given in Figure 1. A monetary tightening leads the public to infer that a positive inflationary shock has hit the economy, and the entire yield curve shifts upwards, with the reaction decreasing with maturity. For a surprise expansion of policy, the reaction is the opposite.

Most interesting, however, is the response of interest rates to an unexpected shift in the preferences of the central bank. We now assume that all shocks are observable, but that the current value of the preference parameter λ (the central bank's weight on output stabilization) is known only to the central bank itself. After a given shock has hit the economy, the public expects the central bank to act according to the rule (19), given their beliefs about the parameter λ. Any unexpected policy response is then interpreted as a (permanent) change in λ, leading the public to revise their expectations about the future path of the central-bank rate.

Since a central bank with a lower value of λ will set a higher interest rate (in absolute terms) for a given shock, but keep the interest rate away from the initial level for a shorter period of time, an unexpectedly large tightening leading to a revision downwards in the public's perception of λ will lead to rising short rates but falling long interest rates. This is the basic intuition behind our final result:

PROPOSITION 4: *When the central bank's preferences are unobservable to the public, interest rates on bonds of sufficiently long maturity will move in the opposite direction to the innovation in the central-bank rate. Thus, the yield curve will tilt as a response to unexpected monetary policy: an unexpectedly high central-bank rate tilts the yield curve clockwise; an unexpectedly low rate tilts it counterclockwise.*

PROOF:
See the Appendix.

This response is shown in Figure 2. When a positive shock realizes, the yield curve shifts up in anticipation of the central bank's response (1). If the central bank acts as expected, market interest rates will not move at all when the central-bank rate is adjusted. If, however, the central bank sets a higher interest rate than was expected, the public realizes that the bank has become more inflation averse (that is, λ has decreased). Then short rates rise, but longer rates fall, leading to a clockwise tilt of the yield curve (2). Similarly, if the central bank responds with a lower rate than expected, the yield curve tilts counterclockwise. As shown in the proof, this result applies not only to new shocks that

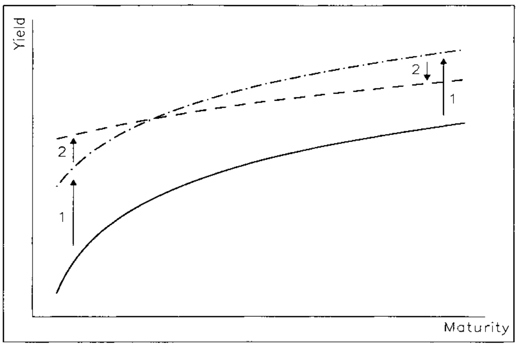

FIGURE 2. YIELD CURVE RESPONSE TO AN EXOGENOUS POLICY CONTRACTION

hit the economy, but also to preference changes that occur out of steady state, as the central bank is offsetting the effects of previous shocks.

It should be stressed that the inertia of inflation and output is crucial for these results, since changes in the current stance of monetary policy have long-lasting effects on the economy, and thus affect the future policy stance: a looser policy today leads to higher inflation, and thus a need for tighter policy, in the future. In the purely forward-looking model of Rotemberg and Woodford (1997), Clarida et al. (1999), and others, current policy has no such effects on future policy via the state of the economy, since there is no inertia in inflation and output, but only in the shock processes themselves. In that model, after a persistent inflationary shock, an increase in the weight on output stabilization λ leads to a looser policy (a lower interest rate) in all future periods, and thus to a downward shift of the entire yield curve. As mentioned above, however, only a small degree of inertia is needed to produce our results.

III. Final Remarks

As mentioned in the introduction, there is some confusion in the literature as to what should be the "normal" response of long interest rates to monetary policy. Some argue that long rates should increase as monetary policy is tightened, mainly via the expectations hypothesis of the term structure. Others think that a monetary tightening should increase short rates but decrease long rates, as inflation expectations fall. Our results suggest that these differing views are two sides of the same coin. When long rates satisfy the expectations hypothesis, they may rise or fall after a policy tightening, depending on market participants' interpretation of the policy move.

Our theoretical analysis might be extended in a number of directions. Perhaps the most natural extension is to allow the central bank to have a time-varying inflation target. It can be shown that a more ambitious (lower) inflation target qualitatively has the same effect as a lower weight on output stabilization.[11] (That model has the additional desirable feature that certainty equivalence holds also when the target follows a martingale.)

In their study of the 1974–1979 funds rate targeting regime in the United States, Cook and Hahn (1989) show that when the Federal Reserve moved its target level for the federal funds rate, interest rates of all maturities on average moved in the same direction as the target. Interpreting this finding, and similar results for other countries, in the light of our model indicates that monetary policy actions are driven more often by economic developments than by preference shifts. Skinner and Zettelmeyer (1995) present results that lend support to Proposition 2: long interest rates respond more to short rates in the United States and the United Kingdom than in Germany and France. Accepting the conventional wisdom that the central banks of Germany and France have been more inflation averse than the Federal Reserve and the Bank of England, this is exactly what our model would predict.

In order to test our model more rigorously on time-series data, it is necessary to classify changes in monetary policy according to whether they reflect unanticipated changes in policy preferences or in economic developments. To construct such a classification scheme, we would ideally like to know how

[11] A related issue is whether the perceived inflation target is related to observed inflation. In that case, a time-varying inflation target could also explain the large effects of monetary policy on the very long end of the yield curve; empirical results indicate that forward rates respond strongly to policy innovations more than ten years into the future, a feature which is not consistent with our model, but could be due to changes in the perceived inflation target as market participants observe (or infer) shocks to current inflation.

financial investors perceive each policy event, since it is the investors' beliefs that determine the interest rate response. Obviously, this ideal is unattainable. In a companion paper (Ellingsen and Söderström, 2000) we attempt to classify policy events over one decade in the United States with the help of newspaper reports, finding some support for our theoretical predictions. In that paper, we also discuss alternative approaches to empirical tests of our model.

Appendix

PROOF OF PROPOSITION 1:

(i) $di_t^n/d\varepsilon_t$ and $di_t^n/d\eta_t > 0$.

For a supply shock, the expression in braces in equation (30) is

$$\text{(A1)} \quad 1 + A + [1 + A(1 - \gamma B)]X_n = 1 + A + X_n + (1 - \beta - \alpha\gamma A)AX_n.$$

Note that

$$0 < \alpha\gamma A = \frac{\alpha^2\delta k}{\lambda + \alpha^2\delta k} \leq 1,$$

which implies that

$$0 < \alpha\gamma AX_n = 1 - (1 - \alpha\gamma A)^{n-1} \leq 1$$

for all n. Consequently,

$$\alpha\gamma A^2X_n \leq A,$$

which, since $\beta < 1$, implies that the right-hand side of equation (A1) and thus the derivative (30) are positive. For a demand shock in (31), the expression in braces,

$$B + \alpha[1 + A(1 - \gamma B)]X_n = \frac{\beta}{\gamma} + \alpha[A + X_n + (1 - \beta - \alpha\gamma A)AX_n],$$

is, by the same argument, also positive.

(ii) $di_t^n/d\varepsilon_t$ and $di_t^n/d\eta_t$ fall with maturity n.

From equation (26), setting $s = 1$,

$$\frac{di_{t+1|t}}{d\varepsilon_t} = 1 + A - \gamma AB < \frac{di_t}{d\varepsilon_t} = 1 + A,$$

since $\gamma AB > 0$, and

$$\frac{di_{t+1|t}}{d\eta_t} = \alpha(1 + A - \gamma AB) < \frac{di_t}{d\eta_t} = B,$$

since

$$1 + A - \gamma AB < \frac{B}{\alpha} = \frac{\beta}{\alpha\gamma} + A,$$

since $\beta = \hat{\beta} + \alpha\gamma > \alpha\gamma$ and $\gamma AB > 0$.

Also using equation (26), note that for $s \geq 2$,

$$\frac{di_{t+s|t}}{d\varepsilon_t} = (1 - \alpha\gamma A)\,\frac{di_{t+s-1|t}}{d\varepsilon_t}$$

and

$$\frac{di_{t+s|t}}{d\eta_t} = (1 - \alpha\gamma A)\,\frac{di_{t+s-1|t}}{d\eta_t}.$$

Since $0 < \alpha\gamma A \leq 1$, the response of expected future short rates to a current shock is nonincreasing over time (in absolute terms).

Thus, since long rates are an average of expected short rates, and every new term is smaller than the previous term, the average will decrease with maturity n.

PROOF OF PROPOSITION 2:

That the short end of the yield curve responds less to a given shock as λ increases follows from the optimal interest rate rule

$$i_t = (1 + A)\pi_t + By_t,$$

where A and B are decreasing in λ.

Showing that the long end responds more to a given shock with a higher λ is more complicated. After a supply shock, the interest rate of maturity n reacts according to

$$\frac{di_t^n}{d\varepsilon_t} = \frac{1}{n}\{1 + A + [1 + A(1 - \gamma B)]X_n\}.$$

As the central-bank preference parameter λ changes, this reaction changes by

$$\frac{d}{d\lambda}\left[\frac{di_t^n}{d\varepsilon_t}\right] = \frac{d}{dA}\left\{\frac{1}{n}[1 + A + X_n + (1 - \alpha\gamma A - \beta)AX_n]\right\}\frac{dA}{d\lambda}$$

$$= \frac{1}{n}\left\{1 + \frac{dX_n}{dA} - \alpha\gamma AX_n + [1 - \alpha\gamma A - \beta]\frac{d(AX_n)}{dA}\right\}\frac{dA}{d\lambda}.$$

Define $\rho = 1 - \alpha\gamma A$, implying that

$$X_n = \frac{1 - \rho^{n-1}}{1 - \rho}; \quad AX_n = \frac{1 - \rho^{n-1}}{\alpha\gamma};$$

$$\alpha\gamma AX_n = 1 - \rho^{n-1},$$

and

$$\frac{dX_n}{dA} = \frac{dX_n}{d\rho}\frac{d\rho}{dA}$$

$$= -\alpha\gamma\frac{-(n-1)(1-\rho)\rho^{n-2} + (1 - \rho^{n-1})}{(1-\rho)^2}$$

$$= \frac{\rho^{n-2}[(n-1)(1-\rho) + \rho] - 1}{(1-\rho)A}.$$

Then

$$\text{(A2)} \quad \frac{d}{d\lambda}\left[\frac{di_t^n}{d\varepsilon_t}\right]$$

$$= \frac{1}{n}\left\{1 + \frac{dX_n}{dA} - \alpha\gamma AX_n + (\rho - \beta)\frac{d(AX_n)}{dA}\right\}\frac{dA}{d\lambda}$$

$$= \frac{1}{n}\left\{\rho^{n-1} + \frac{\rho^{n-2}[(n-1)(1-\rho) + \rho] - 1}{(1-\rho)A} + (\rho - \beta)(n-1)\rho^{n-2}\right\}\frac{dA}{d\lambda}.$$

Multiplying by $(1 - \rho)A \geq 0$ and rearranging, the term in braces is

$$\text{(A3)} \quad \rho^{n-2}(n-1)(1-\rho)[A(\rho - \beta) + 1] + \rho^{n-1}[(1-\rho)A + 1] - 1.$$

As n increases indefinitely, both ρ^{n-1} and $(n - 1)\rho^{n-2}$ tend to zero, making the term in (A3) negative. Since $dA/d\lambda$ is negative, the entire derivative (A2) is then positive for a sufficiently large n.

After a demand shock, the reaction of long rates is

$$\frac{di_t^n}{d\eta_t} = \frac{1}{n}\{B + \alpha[1 + A(1 - \gamma B)]X_n\}$$

$$= \frac{1}{n}\left\{\frac{\beta}{\gamma} + \alpha[A + (1 + A(1 - \gamma B))X_n]\right\}.$$

Consequently,

$$\frac{d}{d\lambda}\left[\frac{di_t^n}{d\eta_t}\right] = \alpha\frac{d}{d\lambda}\left[\frac{di_t^n}{d\varepsilon_t}\right],$$

so the reaction of long rates to a given demand shock is affected by changes in λ in the same direction as the reaction to a supply shock.

PROOF OF PROPOSITION 4:

For a new shock, the proof follows directly from the previously stated Proof of Proposition 2. For an old shock being worked out by the central bank, note that (21) implies that the sensitivity of the central-bank rate in period $t + s$ to a supply shock in period t is

$$\frac{di_{t+s}}{d\varepsilon_t} = (1 + A)\frac{d\pi_{t+s}}{d\varepsilon_t} + B\frac{dy_{t+s}}{d\varepsilon_t}.$$

Since $d\pi_{t+s}/d\varepsilon_t$ and $dy_{t+s}/d\varepsilon_t$ depend only on the initial λ, and so are not affected by the preference shift at $t + s$, and since $dB/d\lambda = \alpha dA/d\lambda$, the derivative of $di_{t+s}/d\varepsilon_t$ with respect to λ is, using (24) and (25),

$$\frac{d}{d\lambda}\left[\frac{di_{t+s}}{d\varepsilon_t}\right] = \left[\frac{d\pi_{t+s}}{d\varepsilon_t} + \alpha\,\frac{dy_{t+s}}{d\varepsilon_t}\right]\frac{dA}{d\lambda}$$

$$= (1 - \alpha\gamma A)^s\,\frac{dA}{d\lambda}$$

$$= (1 - \alpha\gamma A)^s\,\frac{d}{d\lambda}\left[\frac{di_t}{d\varepsilon_t}\right].$$

After s periods, only a fraction $(1 - \alpha\gamma A)^s$ of the shock from time t remains in the system. Thus, the qualitative effects of a preference shift in period $t + s$ are the same as those of a change in period t, and the same applies to all long rates.

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