

3 Identification based on the “recursiveness” assumption

Following CEE (2000), the systematic component of monetary policy is defined by assuming that in any period t monetary policymakers set the value of a policy instrument s_t as a (linear) function of the variables in their information set Ω_t , thereby following a *feedback rule* of the form:

$$s_t = f(\Omega_t) + \sigma_s v_t^s$$

where $\sigma_s v_t^s$ represents the monetary policy shock (with v^s normalized to have unit variance) and $f(\cdot)$ is the monetary policy feedback rule. The information set Ω_t contains contemporaneous and lagged variables to which monetary authorities react when setting the policy instrument.

The structural model is then specified as:

$$\mathbf{A} \begin{pmatrix} \mathbf{Y}_t \\ s_t \\ \mathbf{X}_t \end{pmatrix} = \mathbf{C}(L) \begin{pmatrix} \mathbf{Y}_{t-1} \\ s_{t-1} \\ \mathbf{X}_{t-1} \end{pmatrix} + \mathbf{B} \begin{pmatrix} \mathbf{v}_t^Y \\ v_t^s \\ \mathbf{v}_t^X \end{pmatrix}$$

where \mathbf{Y} contains k_1 non-policy variables, and \mathbf{X} contains k_2 policy indicators.

The identification scheme proposed by CEE is based on the following assumptions:

1. when setting s_t monetary authorities do not see the values of the variables in \mathbf{X}_t ;
2. the monetary policy shock v_t^s is orthogonal to the non-policy variables in \mathbf{Y}_t .

These two (sets of) assumptions imply the following structure of contemporaneous relations among the variables:

$$\mathbf{A} = \begin{pmatrix} \mathbf{a}_{YY} & \mathbf{0} & \mathbf{0} \\ (k_1 \times k_1) & (k_1 \times 1) & (k_1 \times k_2) \\ \mathbf{a}_{sY} & 1 & \mathbf{0} \\ (1 \times k_1) & (1 \times 1) & (1 \times k_2) \\ \mathbf{a}_{XY} & \mathbf{a}_{Xs} & \mathbf{a}_{XX} \\ (k_2 \times k_1) & (k_2 \times 1) & (k_2 \times k_2) \end{pmatrix}$$

Assumption (1) motivates the zero block in the central row of \mathbf{A} , so that the policy rule is specified as:

$$s_t = f(\mathbf{Y}_t, \mathbf{Y}_{t-1}, \dots, \mathbf{Y}_{t-q}, \mathbf{X}_{t-1}, \dots, \mathbf{X}_{t-q}) + \sigma_s v_t^s$$

where contemporaneous and lagged (up to q lags) values of the non-policy variables \mathbf{Y} and only lagged values of the variables in \mathbf{X} enter the policy rule. The two zero blocks in the upper k_1 -row submatrix of \mathbf{A} are motivated by assumption (2): the monetary policy shock v_t^s does not affect (contemporaneously) the variables in \mathbf{Y}_t . The zero k_1 -element vector in the middle rules out any *direct* effect on the macroeconomic variables, whereas the other $k_1 \times k_2$ zero block rules out any *indirect* effect of v_t^s on \mathbf{Y}_t through the (contemporaneous) effect on \mathbf{X}_t , measured by the elements in \mathbf{a}_{Xs} . In addition, matrix \mathbf{B} is assumed diagonal with the standard deviations of the structural disturbances in \mathbf{v} on the diagonal.

The assumptions on \mathbf{A} imply the following “*recursive*” structure on the model:

$$\begin{array}{ccccc}
 \mathbf{Y}_t & \longrightarrow & s_t & \longrightarrow & \mathbf{X}_t \\
 \text{predetermined} & & \text{policy} & & \text{variables directly} \\
 \text{variables} & & \text{instrument} & & \text{affected by policy}
 \end{array}$$

In the CEE empirical implementation of this identification strategy, \mathbf{Y}_t contains *goods market* variables (output, prices, commodity price index), s_t is either the Fed funds rate or the quantity of non-borrowed bank reserves, and the variables in \mathbf{X}_t are *money market* aggregates (bank reserves and the money stock). In summary, this identification scheme is based on the existence of a set of predetermined variables relative to the policy shock, and on the assumption that the only contemporaneous variables the Fed looks at when setting the policy instrument are the predetermined variables in \mathbf{Y}_t .

Under this “*recursiveness*” assumption the monetary policy shock v_t^s is identified by a simple OLS regression of the policy instrument on the predetermined variables (and lags of \mathbf{Y} and \mathbf{X}). However, CEE show that the recursiveness assumption is not sufficient to identify all elements of matrix \mathbf{A} (and \mathbf{B}), but is sufficient to identify the dynamic response (*impulse response functions*, IRF) of all variables in the system to the monetary policy shock v^s . In particular, there is a whole family of matrices \mathbf{A} (and \mathbf{B}) consistent with the recursiveness assumption and such that $\Sigma = \mathbf{A}^{-1}\mathbf{B}\mathbf{B}'\mathbf{A}^{-1'}$ (given the assumption that $E(\mathbf{v}_t\mathbf{v}_t') = \mathbf{I}$).² All those matrices generate the same IRF to a policy shock v^s . If \mathbf{A} is chosen to be lower triangular, then the IRF

²Note that matrix \mathbf{A}_0 in CEE (2000) corresponds to $\mathbf{B}^{-1}\mathbf{A}$ in our notation:

$$\underbrace{\mathbf{B}^{-1}\mathbf{A}}_{\mathbf{A}_0} \mathbf{u}_t = \mathbf{v}_t$$

Matrix \mathbf{B}^{-1} is simply a diagonal matrix with the reciprocal of the standard deviation of each structural disturbance on the main diagonal.

of all variables to v^s is invariant to the ordering of variables in \mathbf{Y} and \mathbf{X} (but the IRF of all variables to the non-policy shocks are sensitive to the ordering of variables in \mathbf{Y} and \mathbf{X}).

The early *VAR* studies of the monetary policy mechanism, adopting simple recursive (in many cases Choleski) identification schemes on a limited number of endogenous variables produced a series of results that were inconsistent with a plausible interpretation of how monetary policy works. Refinements of the identification strategy (or the adoption of a different approach, i.e. the “semi-structural” approach to be discussed below) provided at least partial solutions to the puzzles. In particular, two puzzles emerged from the empirical literature (mainly applied to US data):

1. the “**price puzzle**”, namely the empirical result of a restrictive monetary policy shock leading to an *increase* (instead of a decrease) of the price level

possible cause: misspecification of the monetary policy rule (since the central bank can act on the basis of more information than is captured by the few variables included in the *VAR*)

solution: extend the *VAR* to include proxies for expected inflation (e.g. a commodity price index), that can capture the additional information of the monetary policymaker

2. the “**liquidity puzzle**”, namely the absence of an impact fall of interest rates due to expansionary policy shocks

possible cause: confusion between money supply and money demand disturbances

solution: more careful modelling of the money market (on which monetary policy actions have their first impact effects) to separately identify supply and demand shocks. An example of this is the adoption of the “semi-structural” approach to identification of the monetary policy disturbance.

Response to One S.D. Innovations ± 2 S.E.

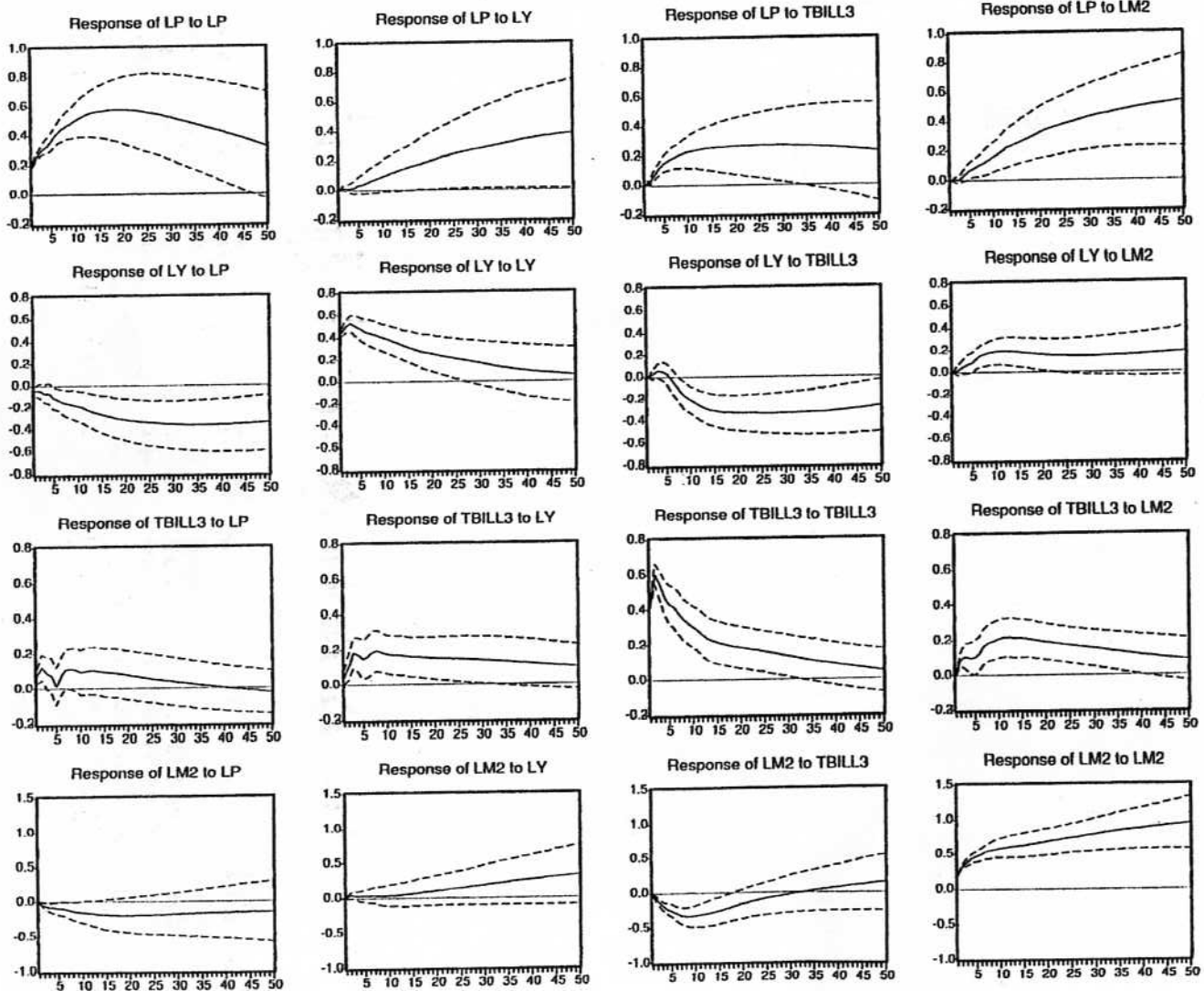
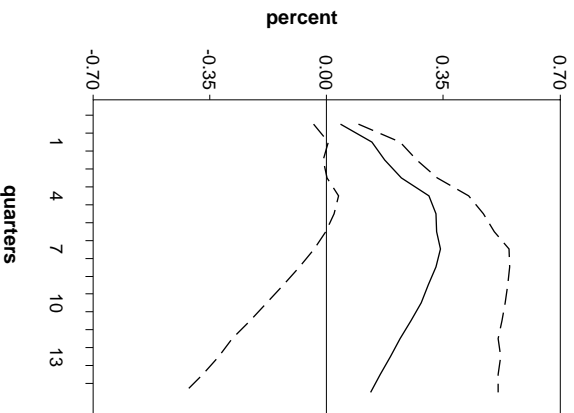


FIG. 6.4. Impulse responses in a four-variables VAR of the MTM

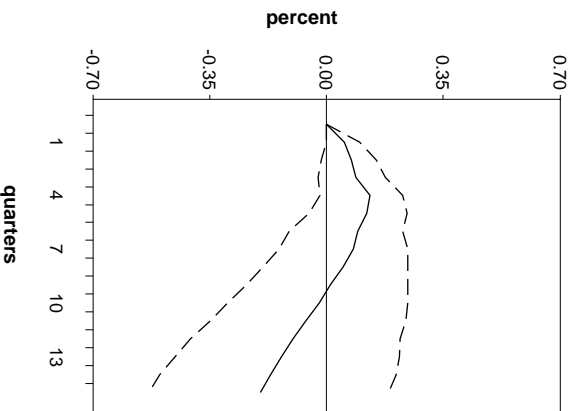
Fed Funds Model, MP first: No Pcom

MP Shock => P



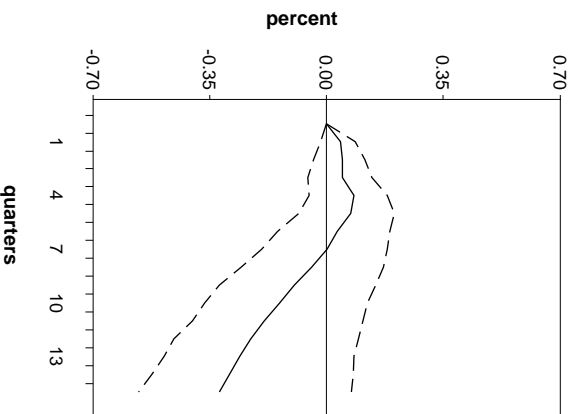
Fed Funds Model: No Pcom

MP Shock => P



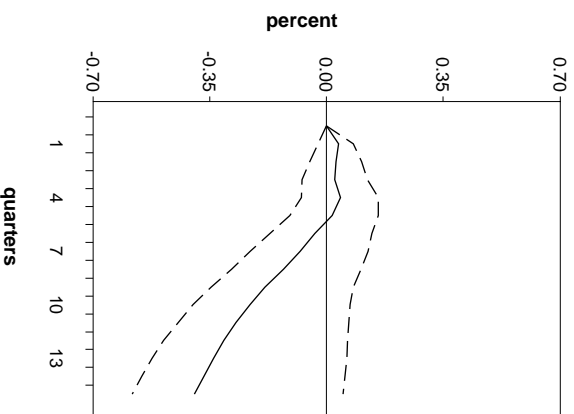
Fed Funds Model: Pcom after MP

MP Shock => P



Fed Funds Model with M1

MP Shock => P



4 The “semi-structural” approach

This approach (introduced by Bernanke and Mihov, *QJE* 1998, henceforth BM) focuses on the policy block of the system \mathbf{P} , using a standard model for the market of US commercial bank reserves (appropriate to the institutional setting prevailing until the end of the 1990s) and the Fed’s operating procedures. Behavioral relationships among innovations in the relevant reserve aggregates and interest rates are specified as follows (with TR : total reserves; BR : borrowed reserves; NBR : nonborrowed reserves; FF : Fed funds rate; time subscripts ignored):

$$u_{TR}^d = -\alpha u_{FF} + v^d \quad (4)$$

$$u_{BR}^d = \beta(u_{FF} - u_{DISC}^d) + v^b \quad (5)$$

$$u_{NBR}^s = \phi^d v^d + \phi^b v^b + v^s \quad (6)$$

where u_{DISC} denotes innovations to the Fed discount window rate and, given the very high predictability of discount rate changes, will be assumed equal to zero, and the equation for the supply of nonborrowed reserves (6) captures the behavior of the Fed (potential reaction to the total reserves demand shock v^d , and to the borrowed reserves demand shock v^b , plus the policy shock v^s).

This set of relations among VAR innovations relies on a simple structural model of the US money market where banks’ reserve demand and reserve supply interact to determine the (overnight) Federal Funds (FF) rate in equilibrium, denoted by FF^* in Figure 1 below. Total reserves (TR) demand is negatively related to the FF market rate, reflecting the opportunity cost for banks of keeping reserves at the Federal Reserve (Fed). A stochastic disturbance v^d is added to capture variations in income and other factors inducing fluctuations in bank’s deposits demands (and therefore affecting the amount of reserves banks have to maintain). The supply of reserves is partly determined by the Fed’s regular interventions onto the market with open-market operations, providing liquidity to banks (“nonborrowed reserves”, NBR). In deciding the amount of NBR to offer, the Fed can respond to contemporaneous disturbances to total reserve demand and to borrowed reserve demand through the policy parameters ϕ^d and ϕ^b . The general specification adopted above for the supply of NBR encompasses various operating procedures, e.g. “federal funds rate targeting” and “non-borrowed reserves targeting”, characterized by different values of the policy parameters. A stochastic component v^s reflects deliberate actions by the Fed aimed at changing the supply of reserves available to the market (in a way that does not depend on contemporaneous reserve demand shocks) and, ultimately, affect the market rate for federal funds. Moreover, banks can borrow at their discretion from the

Fed (from the so-called “discount window”) at the discount rate, set by the Fed and traditionally kept below the market rate (at least in the institutional setting prevailing until the end of the 1990s, and appropriate for the analysis of the Bernanke-Mihov paper).³ This component of the reserve supply is therefore determined by the banks’ demand for borrowed reserves (BR); such demand depends on the spread between the cost of borrowing (the discount rate) and the return on reserves offered by the market (the prevailing FF rate). The coefficient β measures the elasticity of the demand for borrowed reserves to the spread; such elasticity is heavily influenced by the “moral suasion” of the Fed, inducing banks not to fully exploit the arbitrage opportunities offered by a positive FF -discount rate spread. Finally, the borrowed reserves demand is influenced by stochastic factors modifying banks’ behavior captured by v^b .

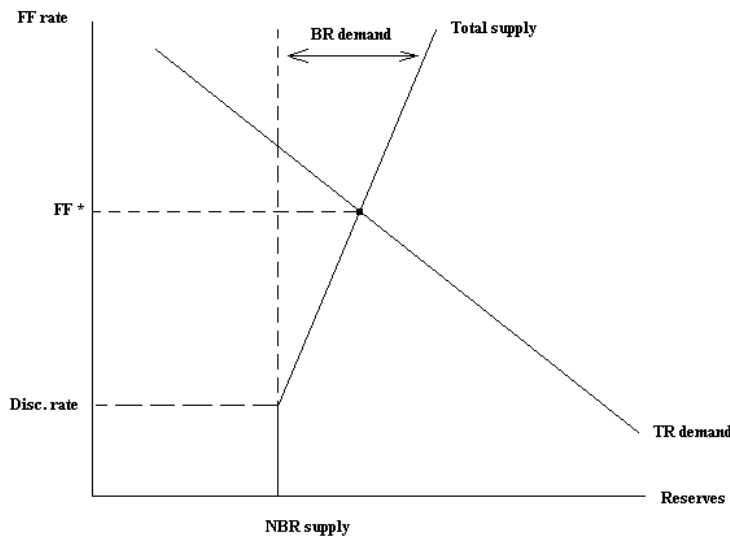


Figure 1: The bank reserves market

³From 2002, the US Federal Reserve has changed the role of the discount window. Since then, the discount rate has become a penalty rate for borrowing from the Fed and therefore its level is set above the FF rate, thus eliminating potential arbitrage opportunities for the banks, and the consequent need for non-price ("moral suasion") rationing by the Fed. The relevant empirical work referred to in these notes concerns the working of the reserve market before the recent changes, and therefore correctly assumes a positive FF -discount rate spread.

The equilibrium condition on the reserve market equates (innovations in) total reserve demand and supply: $u_{TR}^d = u_{BR}^d + u_{NBR}^s$ (note that the demand for borrowed reserves yields an increased supply of reserves). In terms of our general formulation of the model we have (using $u_{BR} \equiv u_{TR} - u_{NBR}$):

$$\begin{pmatrix} 1 & -\frac{1}{\beta} & \frac{1}{\beta} \\ \alpha & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_{FF} \\ u_{TR} \\ u_{NBR} \end{pmatrix} = \begin{pmatrix} -\frac{1}{\beta} & 0 & 0 \\ 0 & 1 & 0 \\ \phi^b & \phi^d & 1 \end{pmatrix} \begin{pmatrix} v^b \\ v^d \\ v^s \end{pmatrix}$$

$$\mathbf{A} \mathbf{u} = \mathbf{B} \mathbf{v}$$

from which:

$$\mathbf{u} = \mathbf{A}^{-1} \mathbf{B} \mathbf{v}$$

$$\begin{pmatrix} u_{FF} \\ u_{TR} \\ u_{NBR} \end{pmatrix} = \begin{pmatrix} -\frac{1+\phi^b}{\alpha+\beta} & \frac{1-\phi^d}{\alpha+\beta} & -\frac{1}{\alpha+\beta} \\ \alpha \frac{1+\phi^b}{\alpha+\beta} & \frac{\beta+\alpha\phi^d}{\alpha+\beta} & \frac{\alpha}{\alpha+\beta} \\ \phi^b & \phi^d & 1 \end{pmatrix} \begin{pmatrix} v^b \\ v^d \\ v^s \end{pmatrix} \quad (7)$$

All VAR residuals u are expressed as linear combinations of the structural disturbances v : no innovation to a particular variable in the policy block may be interpreted as (at least proportional to) the policy shock v^s before specifying appropriate identifying restrictions. Note from (7) that the responses of FF , TR and NBR to the policy shock v^s do not depend on the policy parameters ϕ^d and ϕ^b , whereas the responses of reserve aggregates and the funds rate to the non-policy shocks depend on the policy parameters. The relationship between VAR innovations and structural disturbances yields a measure of the “*liquidity effect*” of the monetary policy shock, capturing the negative response of the Federal funds rate to a positive (i.e. expansionary) realization of v^s : this effect is $-\frac{1}{\alpha+\beta}$ and depends on the two behavioural parameters in the model (α and β).

Inverting (7), structural shocks may be expressed in terms of VAR residuals as follows:

$$\begin{pmatrix} v^b \\ v^d \\ v^s \end{pmatrix} = \begin{pmatrix} -\beta & 1 & -1 \\ \alpha & 1 & 0 \\ \beta\phi^b - \alpha\phi^d & -(\phi^b + \phi^d) & 1 + \phi^b \end{pmatrix} \begin{pmatrix} u_{FF} \\ u_{TR} \\ u_{NBR} \end{pmatrix} \quad (8)$$

In particular, the policy shock v^s is expressed as a linear combination of the VAR innovations in FF , TR and NBR and can be recovered once an appropriate identification scheme, based on the operating procedures followed by the Fed, is assumed and the remaining free parameters are estimated.

Before proceeding to the illustration of alternative identification schemes reflecting different policy regimes, let us write the complete VAR system in structural form as in (1), including the non-policy, macroeconomic variables (here the logs of GDP , the consumer price index P , the IMF index of world commodity price Pcm), and the policy block (with the Fed funds rate FF , total reserves TR and nonborrowed reserves NBR):

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ a_{21} & 1 & 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & 1 & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & 1 & -\frac{1}{\beta} & \frac{1}{\beta} \\ a_{51} & a_{52} & a_{53} & \alpha & 1 & 0 \\ a_{61} & a_{62} & a_{63} & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} GDP_t \\ P_t \\ Pcm_t \\ FF_t \\ TR_t \\ NBR_t \end{pmatrix} = \mathbf{C}(L) \begin{pmatrix} GDP_{t-1} \\ P_{t-1} \\ Pcm_{t-1} \\ FF_{t-1} \\ TR_{t-1} \\ NBR_{t-1} \end{pmatrix} \quad (9)$$

$$+ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\beta} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \phi^b & \phi^d & 1 \end{pmatrix} \begin{pmatrix} v_{1t}^{NP} \\ v_{2t}^{NP} \\ v_{3t}^{NP} \\ v_t^b \\ v_t^d \\ v_t^s \end{pmatrix}$$

where the first 3 structural disturbances are generically denominated as *non-policy* (NP) shocks v_i^{NP} ($i = 1, 2, 3$) since no economic structural interpretation is given for them (a simple recursive structure is adopted for the non-policy block of the system).

4.1 Alternative identification schemes

The BM theoretical model of the reserves market contains seven parameters (α , β , ϕ^d , ϕ^b and σ_d^2 , σ_b^2 , σ_s^2) to be estimated from the variance-covariance matrix of the VAR innovations in the policy block (u_{FF} , u_{TR} , u_{NBR}), which yields six estimated values. One restriction is then needed for just-identification of the model; additional restrictions over-identify the model and allow for overidentification tests. Also considering the complete model in (9) we confirm the need for at least one additional restriction: there are 22 parameters to be estimated (12 coefficients a_{ij} , 6 variances of the structural shocks and the 4 structural parameters α , β , ϕ^d and ϕ^b) but the estimated elements of the variance-covariance matrix of VAR innovations are only 21 (6 variances and 15 covariances).

Several sets of restrictions on the model's parameters have been proposed in order to identify monetary policy shocks, allowing for different monetary policy regimes (i.e. Fed operating procedures). Of the five different schemes

analyzed by BM, two (each imposing one over-identifying restriction, therefore allowing for a test) have proved particularly relevant: the “federal funds rate targeting” (whereby the Fed sets the amount of *NBR* in order to keep the *FF* rate at the chosen level, offsetting the disturbances to total and borrowed reserve demand), and the “non-borrowed reserve targeting” (whereby the Fed targets the quantity of *NBR* reserves in the market, not reacting to reserve demand shocks). The specific operating procedure adopted by the Fed is essential in providing values for the policy parameters consistent with the actual behavior of the central bank. To clarify this point, Figure 2 shows the effect on the equilibrium *FF* rate of a positive shock to total reserve demand ($v^d > 0$) that shifts the *TR* curve. The effect on the *FF* rate depends crucially on the response of the Fed. Under a (perfect) *FF* rate targeting regime the appropriate reaction to keep the rate unchanged at the pre-shock level is to accommodate entirely the disturbance, increasing the supply of *NBR* by v^d , therefore setting $\phi^d = 1$, whereas no reaction to the shock ($\phi^d = 0$) occurs under a *NBR* targeting regime, allowing the demand disturbance to affect the equilibrium *FF* rate.

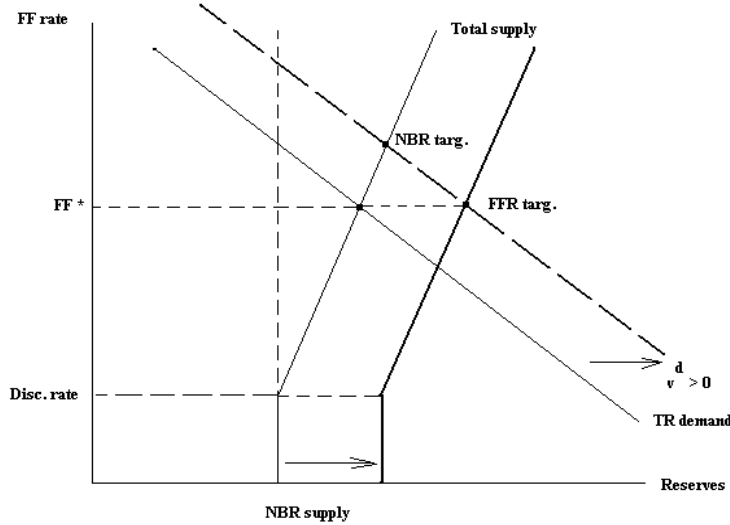


Figure 2: A shock to *TR* demand

When the disturbance comes from the borrowed reserves ($v^b > 0$), the positively sloped portion of the supply curve shifts, as in Figure 3. Again, the effects on market equilibrium depends on the prevailing operating procedures: if the Fed targets *NBR* there is no reaction to the shock ($\phi^b = 0$) and the

market FF rate falls, whereas under a FF rate targeting regime the Fed completely offsets the shock ($\phi^b = -1$) shifting the supply curve back to its original position.

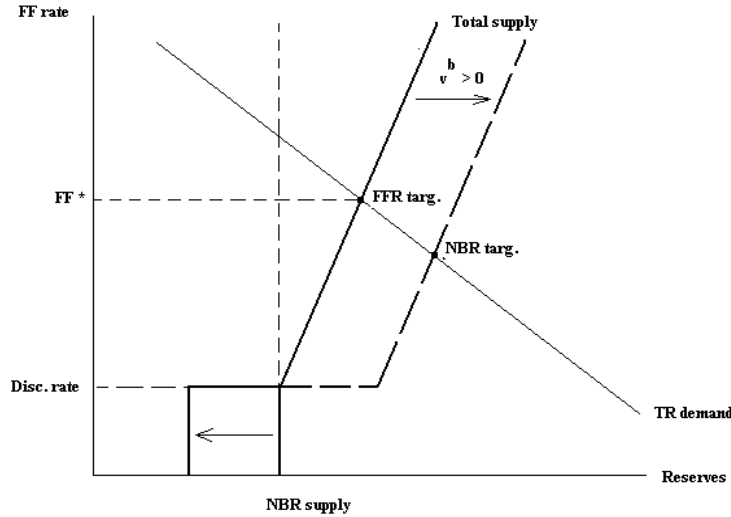


Figure 3: A shock to BR demand

The two different operating procedures that the Fed might adopt place different sets of restrictions on the policy parameters and therefore on the structural model of the reserves market, giving rise to two specific models:

1. *Federal funds rate model*: the Fed targets the Federal funds rate, fully offsetting shocks to TR and BR . Therefore: $\phi^d = 1$ and $\phi^b = -1$.

$$\begin{pmatrix} u_{FF} \\ u_{TR} \\ u_{NBR} \end{pmatrix} = \begin{pmatrix} 0 & 0 & -\frac{1}{\alpha+\beta} \\ 0 & 1 & \frac{\alpha}{\alpha+\beta} \\ -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} v^b \\ v^d \\ v^s \end{pmatrix}$$

and (inverting), the policy shock v^s can be found as

$$v^s = -(\alpha + \beta) u_{FF}$$

2. *Nonborrowed reserves model*: the Fed targets the quantity of nonborrowed reserves, allowing the Federal funds rate to fluctuate in the face

of reserve demand disturbances. Therefore, nonborrowed reserves show no response to reserve demand shocks; then innovations in nonborrowed reserves reflect shocks to policy. Under this operating procedure the following restrictions must hold: $\phi^d = \phi^b = 0$,

$$\begin{pmatrix} u_{FF} \\ u_{TR} \\ u_{NBR} \end{pmatrix} = \begin{pmatrix} -\frac{1}{\alpha+\beta} & \frac{1}{\alpha+\beta} & -\frac{1}{\alpha+\beta} \\ \frac{\alpha}{\alpha+\beta} & \frac{\beta}{\alpha+\beta} & \frac{\alpha}{\alpha+\beta} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v^b \\ v^d \\ v^s \end{pmatrix}$$

and the policy shock becomes simply

$$v^s = u_{NBR}$$

As a final note on the importance of carefully modelling the reserves market in order to identify the monetary policy shock taking into account the operating procedure of the central bank and on the dangers of mixing several different policy regimes in estimation, let us look at the “*liquidity effect*” of a policy shock, i.e. the impact of v^s on the FF rate. As noted above, the theoretical model in (7) predicts that the liquidity effect is $-\frac{1}{\alpha+\beta}$. Therefore, consistently with basic intuition about the effect of policy actions on the monetary market, the liquidity effect should be negative (in response to expansionary policy shocks), with a magnitude given by banks’ behaviour (captured by α and β). However, if the system is estimated ignoring the potential role of different policy regimes and simply identifying the innovation to nonborrowed reserves (u_{NBR}) as the policy shock (v^s), the estimate of the liquidity effect is biased when the operational procedure followed by the central bank is different from nonborrowed reserves targeting. To see the point, assume a recursive identification scheme of the policy block, with NBR ordered *before* FF : the estimated liquidity effect is therefore given by the regression coefficient of u_{FF} onto u_{NBR} . Given the general structural model in (7) such coefficient may be computed as:

$$\begin{aligned} \frac{cov(u_{FF}, u_{NBR})}{var(u_{NBR})} &= \frac{cov\left(-\frac{1+\phi^b}{\alpha+\beta} v^b + \frac{1-\phi^d}{\alpha+\beta} v^d - \frac{1}{\alpha+\beta} v^s, \phi^b v^b + \phi^d v^d + v^s\right)}{var(\phi^b v^b + \phi^d v^d + v^s)} \\ &= -\frac{1}{\alpha+\beta} \left(1 + \frac{\phi^b \sigma_b^2 - \phi^d \sigma_d^2}{(\phi^b)^2 \sigma_b^2 + (\phi^d)^2 \sigma_d^2 + \sigma_s^2}\right) \end{aligned}$$

If the central bank in fact follows a *nonborrowed reserves targeting* regime, $\phi^b = \phi^d = 0$ and the liquidity effect is correctly estimated; however, if a different operational procedure is implemented, the estimate is biased. For

example, under a *Federal funds rate targeting* (whereby $\phi^d = 1$ and $\phi^b = -1$) the estimated liquidity effect would be

$$-\frac{1}{\alpha + \beta} \left(1 - \frac{\sigma_b^2 + \sigma_d^2}{\sigma_b^2 + \sigma_d^2 + \sigma_s^2} \right) > -\frac{1}{\alpha + \beta}$$

implying an underestimation of the magnitude of the effect (not an uncommon result in the empirical literature).

5 Identification based on long-run restrictions

A different way of identifying structural disturbances (and proceed to impulse response and forecast error variance decomposition analysis) has been proposed by Blanchard and Quah (BQ, *AER* 1989) and relies on restrictions on the long-run responses of endogenous variables to structural shocks rather than on their contemporaneous reactions.

This methodology, applied by Bernanke and Mihov (Carn. Roch 1998) to the long-run real effects of monetary policy, has been implemented on small sets of macroeconomic variables to separate the effects of permanent from temporary disturbances. Here, we cast the argument in terms of a bivariate model including one non-stationary ($I(1)$) variable y and one stationary ($I(0)$) variable z . Since, in order to apply the BQ identification procedure, only stationary variables must enter the VAR, we specify the vector of endogenous variables as $(\Delta y \quad z)'$. Note that if the two variables of interest were both non-stationary, the BQ methodology could be applied to the vector of first differences of both variables provided that they are not cointegrated.

Let us start from the reduced (VAR) form of the bivariate system:

$$\begin{pmatrix} \Delta y_t \\ z_t \end{pmatrix} = \mathbf{C}^*(L) \begin{pmatrix} \Delta y_{t-1} \\ z_{t-1} \end{pmatrix} + \begin{pmatrix} u_t^Y \\ u_t^Z \end{pmatrix} \quad (10)$$

where the innovations are defined as u^Y and u^Z . The relation between the VAR residuals u and the structural disturbances is now specified as

$$\begin{pmatrix} u_t^Y \\ u_t^Z \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} v_t^1 \\ v_t^2 \end{pmatrix} \quad (11)$$

where the two structural disturbances are given the economic interpretation of a shock having a *permanent* effect on the non-stationary variable y (the “permanent” disturbance) and a shock having only a *transitory* effect on y (the “transitory” disturbance). Both shocks have only temporary effects on the stationary variable z . The two structural disturbances are orthogonal and, for simplicity, their variance is normalized to unity, so that $E(\mathbf{v}\mathbf{v}') = \mathbf{I}$.

In (11) the VAR innovations for Δy and z are linear combinations of the permanent and transitory disturbances. In order to retrieve the structural disturbances v^1 and v^2 from estimation of the VAR innovations u^Y and u^Z (and of their covariance matrix), identification of the four elements b_{ij} ($i, j = 1, 2$) is necessary. The BQ methodology achieves identification by imposing long-run restrictions on the impulse response function of Δy with respect to the transitory shock.

From (11) we can write the covariance matrix of VAR innovations in terms of the coefficients b_{ij} :

$$\begin{aligned} E(\mathbf{u}\mathbf{u}') &= \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} E(\mathbf{v}\mathbf{v}') \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}' \\ \Rightarrow \begin{pmatrix} s_Y^2 & s_{YZ} \\ s_{YZ} & s_Z^2 \end{pmatrix} &= \begin{pmatrix} b_{11}^2 + b_{12}^2 & b_{11}b_{21} + b_{12}b_{22} \\ b_{11}b_{21} + b_{12}b_{22} & b_{21}^2 + b_{22}^2 \end{pmatrix} \end{aligned}$$

since $E(\mathbf{v}\mathbf{v}') = \mathbf{I}$. Given the estimated values for the variances and covariance of VAR innovations (denoted by s_Y^2 , s_Z^2 and s_{YZ}), the above relation delivers the following three equations to obtain the b_{ij} coefficients:

$$b_{11}^2 + b_{12}^2 = s_Y^2 \quad (12)$$

$$b_{21}^2 + b_{22}^2 = s_Z^2 \quad (13)$$

$$b_{11}b_{21} + b_{12}b_{22} = s_{YZ} \quad (14)$$

One additional restriction is needed for (just)identification. This additional restriction is derived by obtaining the vector bivariate moving average form from the VAR representation of the system in (10):

$$\begin{aligned} \begin{pmatrix} \Delta y_t \\ z_t \end{pmatrix} &= [\mathbf{I} - \mathbf{C}^*(L)L]^{-1} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} v_t^1 \\ v_t^2 \end{pmatrix} \\ &= \begin{pmatrix} \pi_{11}(L) & \pi_{12}(L) \\ \pi_{21}(L) & \pi_{22}(L) \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} v_t^1 \\ v_t^2 \end{pmatrix} \\ &= \begin{pmatrix} d_{11}(L) & d_{12}(L) \\ d_{21}(L) & d_{22}(L) \end{pmatrix} \begin{pmatrix} v_t^1 \\ v_t^2 \end{pmatrix} \end{aligned} \quad (15)$$

In (15) the polynomials $d_{ij}(L)$ trace out the responses of Δy and z to the permanent and transitory structural disturbances. The restriction that the transitory shock, say v^2 , does not affect y in the long run amounts to imposing:

$$d_{12}(1) = 0$$

which can be expressed in terms of the b_{ij} 's and the $\pi_{ij}(1)$'s as

$$\pi_{11}(1)b_{12} + \pi_{12}(1)b_{22} = 0 \quad (16)$$

From estimation of the reduced form of the system the values of $\pi_{11}(1)$ and $\pi_{12}(1)$ are obtained, so that (16) can be added to the three equations in (12)-(14) to form the set of four just-identifying restrictions needed for retrieving the structural disturbances from VAR innovations. Once the structural shocks are obtained, the impulse response functions of Δy and z to the permanent (v^1) and the transitory (v^2) shocks can be computed and the forecast error variance of the endogenous variables can be decomposed into the fractions attributable to the two structural disturbances.

6 Identification based on (non-VAR) high-frequency financial data

All the identification schemes presented so far rely on particular assumptions on either contemporaneous or long-run responses of some variables in the system to other variables. Such assumptions allow identification of the structural disturbances (and particularly the monetary policy shock) from the estimated VAR residuals.

A different approach to identification tries to derive measures of the unanticipated monetary policy actions exploiting information not embedded in the VAR system used to explore the dynamic effects of monetary policy. Deriving such measures from non-VAR information bypasses the identification problem, providing an “exogenous” variable, capturing the monetary policy shock, that can be used in the VAR system to derive the impulse response functions of the endogenous variables to the policy disturbance. Non-VAR measures of monetary policy shocks have been proposed by Rudebusch (*International Economic Review* 1998), Bagliano-Favero (*European Economic Review* 1998 and 1999), Brunner (*Journal of Money, Credit and Banking* 2000) and briefly discussed also in CEE (2000).

The proposed measures generally exploit information on expected monetary policy actions obtained from financial markets, in particular from prices of future contracts or from interest rates on short-term bills. Once a measure of this kind is derived, it can be included in the reduced form VAR system (1) as a contemporaneous exogenous variable:

$$\mathbf{y}_t = \mathbf{C}^*(L) \mathbf{y}_{t-1} + \mathbf{G}^* x_t + \mathbf{u}_t$$

where \mathbf{G}^* is a n -dimensional vector capturing the contemporaneous (reduced form) impacts of the monetary policy shock x_t on the endogenous variables in the model. The dynamic response (*IRF*) of the variables in \mathbf{y} to x_t can be obtained from the *VMA*(∞) representation of the VAR:

$$\begin{aligned} \mathbf{y}_t &= (\mathbf{I} - \mathbf{C}^*(L)L)^{-1} \mathbf{G}^* x_t + (\mathbf{I} - \mathbf{C}^*(L)L)^{-1} \mathbf{u}_t \\ &= (\mathbf{I} + \mathbf{\Psi}_1 L + \mathbf{\Psi}_2 L^2 + \dots)^{-1} \mathbf{G}^* x_t + (\mathbf{I} + \mathbf{\Psi}_1 L + \mathbf{\Psi}_2 L^2 + \dots)^{-1} \mathbf{u}_t \end{aligned}$$

This non-VAR identification strategy may be particularly useful when the identification problem is difficult to solve on the basis of “recursive” or even “structural” assumptions, for example because there is an obvious simultaneous feedback between the monetary policy variable and other endogenous variables in the system, such as long-term interest rates and exchange rates.

Examples of non-VAR measures of monetary policy shocks (applied in the literature to US data) include:

- the difference between the Federal funds rate at month t and the one-month Federal funds *future* at month $t-1$, taken as an unbiased predictor of the next-month effective rate; this measure is directly comparable to the reduced form VAR innovation in the Federal funds rate u_{FF} ;
- the change of the 3-month Treasury bills on the days of policy announcements;
- the difference between effective overnight rates on the days immediately after FOMC policy meetings and the expected overnight rates for those days derived from estimation of the curve of instantaneous forward rates on the days immediately preceding the meetings.

Faust, Swanson and Wright (*JME* 2004) recently proposed to use high-frequency data from financial markets (in particular from the Federal funds future market) in a novel way to identify the response of VAR variables to monetary policy shocks.