Macroeconomic Analysis

Lecture notes (3) on: Nominal rigidities, rational expectations and stabilization policy

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The existence of nominal rigidities in the process of price and wage setting has important consequences on the validity of the NCM policy ineffectiveness proposition, even in models with rational expectations. These notes outline two basic models that introduce, in a framework typical of the NCM, rigidities in *nominal wage* setting, and analyze the conditions under which monetary policy retains its effectiveness in stabilizing output fluctuations around its natural level. These models, combining "keynesian" elements such as nominal rigidities with the macroeconomic structure of the NCM models, are considered typical of the first generation of models in a strand of literature known as the *new keynesian macroeconomics*.

Here we present simplified versions of the models by S. Fischer (1977) and J. Taylor (1979), that share the common assumption that nominal wage are set by contracts (and are not perfectly flexible prices). Details of the wage setting scheme are different: nominal wages are "predetermined" in Fischer (1977) and "fixed" in Taylor (1979)

1. A model with *predetermined* nominal wages (Fischer 1977)

The key assumption of the model is that nominal wages are set by contracts involving workers and firms at specific dates, and cannot be changed during the period of validity of the contract. In this setting, the potential output stabilization power of systematic, feedback, monetary policy rules is studied. In a modified version of Fischer (1977), we make different assumptions on the length of the contracts.

1.1. One-period contracts

Suppose that the nominal wage is set at the beginning of *each* period, during which it remains fixed. This wage level applies to *all* workers in the economy and is set at the beginning of the period with the aim of keeping the (expected) real wage constant. During each period, with the nominal wage fixed at the level set by contract, employment is chosen by firms at the profit-maximizing level, depending on the real wage (i.e. on their labor demand function). The price level is perfectly flexible and adjusts during each period to clear the goods market.

A simple formalization of this economy follows (as usual, all variables are in logarithms). The nominal wage for period t, w_t , is set at the beginning of the period so as to maintain the real wage, expected on the basis of available information at t - 1, $w_t - E_{t-1}p_t$, at a constant level, that we set for simplicity equal to zero (in logs). Then we have:

$$w_t = E_{t-1} p_t \tag{1.1}$$

Labor demand by firms l^D (from profit maximization with only one input, labor) depends negatively on the real wage:¹

$$l_t^D = -(w_t - p_t) (1.2)$$

Since, after the nominal wage w_t is set, employment is determined by firms $(l_t = l_t^D)$, then output y_t is a simple increasing function of l^D and therefore depends negatively on the real wage:²

$$y_t = -(w_t - p_t) (1.3)$$

Finally, the model is closed by a simple aggregate demand function:

$$y_t = (m_t - p_t) + v_t (1.4)$$

¹For algebraic convenience, a unitary (negative) elasticity of labor demand to the real wage is assumed.

 $^{^{2}}$ Again, for simplicity, the coefficient on the real wage is set to one and no supply shock is allowed.

where v_t is an aggregate demand shock. During each period t, the nominal wage w_t is fixed (e.g. because the bargaining process is costly and firms and workers prefer not to change the wage even if $p_t \neq E_{t-1}p_t$), whereas p_t and y_t adjust to their equilibrium values. We note that in this model there is no market-clearing on the labor market, unlike in Friedman's and NCM schemes. Indeed, in Friedman's model, even though workers have imperfect information and act on the basis of an expected price level which differs from the actual price level observed by firms, the equality between labor supply and demand always occurs in equilibrium; instead, in Frischer's model employment is determined by labor demand only.

The model can now be solved to find the equilibrium levels of output and prices and to evaluate the potential role for stabilization monetary policy in the presence of nominal rigidities in the wage setting process. Substituting (1.1) into (1.3) and equating aggregate demand and supply we get:

$$p_t = \frac{1}{2} \left(m_t + E_{t-1} p_t + v_t \right) \tag{1.5}$$

Computing $E_{t-1}p_t$ and substituting it into (1.5) we obtain the price level p_t :

$$p_t = (E_{t-1}m_t + E_{t-1}v_t) + \frac{1}{2}(m_t - E_{t-1}m_t) + \frac{1}{2}(v_t - E_{t-1}v_t)$$
(1.6)

The first term in brackets collects the components of m_t and v_t expected at the beginning of the period, whereas the last two terms are the "unanticipated" components, that constitute the price level "surprise" at time t:

$$p_t - E_{t-1}p_t = \frac{1}{2}\left(m_t - E_{t-1}m_t\right) + \frac{1}{2}\left(v_t - E_{t-1}v_t\right)$$
(1.7)

Finally, substituting (1.7) into (1.3), and recalling that $w_t = E_{t-1}p_t$, we obtain the equilibrium output as:

$$y_t = \frac{1}{2} \left(m_t - E_{t-1} m_t \right) + \frac{1}{2} \left(v_t - E_{t-1} v_t \right)$$
(1.8)

Two conclusions stand out: (i) only unanticipated changes in m can have real effects: therefore, the main conclusion of NCM models on the ineffectivenes of systematic monetary policies applies here; (ii) real effects of the unanticipated components of m and v last only for one period: no persistent fluctuations are generated (whereas persistence is an important feature of observed business cycles).

1.2. Two-period contracts

Let us now introduce two-period contracts for a fraction 2k (with 0 < k < 0.5) of the whole workforce. Such contracts determine the nominal wage for two periods and may set the wage at different levels in each of the two periods; once set, the wages cannot be adjusted in response to unexpected changes in the price level (we rule out here any form of wage indexation). In each period, half of those "longterm" contracts (concerning a fraction k of the workers) expire and are renewed. The remaining fraction 1 - 2k of workers has "short-term" contracts lasting oneperiod, as in the model of the preceding section). The timing of contract renewal in this economy is shown in Figure 1.

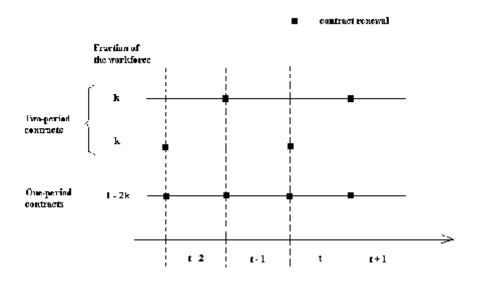


Figure 1

In setting the nominal wage w, all workers try to maintain a constant real wage. Since both long- and short-term contracts are present, in each period t there will be two different levels of the nominal wage:

(i) the workers with short-term contracts (a fraction 1 - 2k), renewed in each period, and the workers under long-term contracts that expire at the end of period t - 1 (a fraction k) set the wage for period t as follows

$$w_{t-1,t} = E_{t-1}p_t \tag{1.9}$$

where $w_{t-1,t}$ is the wage, set at the beginning of period t on the basis of all available information at t-1. This wage level applies to a fraction 1-k of the workforce and is fixed for the whole period t;

(*ii*) the workers with long-term contracts in the second period of validity (a fraction k) get in period t a wage previously set at the beginning of period t-1 on the basis of the information available at t-2:

$$w_{t-2,t} = E_{t-2}p_t \tag{1.10}$$

Given this composite wage structure, the aggregate supply curve becomes:

$$y_t = -(1-k)(w_{t-1,t} - p_t) - k(w_{t-2,t} - p_t)$$

where two different wage levels coexist. Using (1.9) and (1.10), the AS curve can be written in terms of "price surprises":

$$y_t = (1 - k) \left(p_t - E_{t-1} p_t \right) + k \left(p_t - E_{t-2} p_t \right)$$
(1.11)

Having derived the aggregate supply curve in the presence of wage stickiness, we can solve for the price level by equating (1.11) to aggregate demand (1.4):

$$p_t = \frac{1}{2} \left(m_t + v_t \right) + \frac{1-k}{2} E_{t-1} p_t + \frac{k}{2} E_{t-2} p_t \tag{1.12}$$

Taking expected values of (1.12) as of t - 2 and t - 1 we get:

$$E_{t-2}p_t = E_{t-2}m_t + E_{t-2}v_t$$

$$E_{t-1}p_t = \frac{1}{2}(E_{t-1}m_t + E_{t-1}v_t) + \frac{1-k}{2}E_{t-1}p_t + \frac{k}{2}E_{t-2}p_t$$

$$\Rightarrow E_{t-1}p_t = \frac{1}{1+k}(E_{t-1}m_t + E_{t-1}v_t) + \frac{k}{1+k}(E_{t-2}m_t + E_{t-2}v_t)$$

which, substituted into (1.12), yield the final form for the price level:

$$p_{t} = \frac{1}{1+k} \left(E_{t-1}m_{t} + E_{t-1}v_{t} \right) + \frac{k}{1+k} \left(E_{t-2}m_{t} + E_{t-2}v_{t} \right) + \frac{1}{2} \left(m_{t} - E_{t-1}m_{t} \right) + \frac{1}{2} \left(v_{t} - E_{t-1}v_{t} \right)$$
(1.13)

Then, subtracting $E_{t-1}p_t$ and $E_{t-2}p_t$ from (1.13) we obtain the (one-period and two-period) price level "surprises"

$$p_{t} - E_{t-1}p_{t} = \frac{1}{2}(m_{t} - E_{t-1}m_{t}) + \frac{1}{2}(v_{t} - E_{t-1}v_{t})$$

$$p_{t} - E_{t-2}p_{t} = \frac{1}{1+k}(E_{t-1}m_{t} - E_{t-2}m_{t}) + \frac{1}{1+k}(E_{t-1}v_{t} - E_{t-2}v_{t})$$

$$+ \frac{1}{2}(m_{t} - E_{t-1}m_{t}) + \frac{1}{2}(v_{t} - E_{t-1}v_{t})$$

Finally, using the price surprises into aggregate supply (1.11) we get the equilibrium output as:

$$y_{t} = \frac{1-k}{2(1+k)} \left[(m_{t} - E_{t-1}m_{t}) + (v_{t} - E_{t-1}v_{t}) \right] + \frac{k}{1+k} \left[(m_{t} - E_{t-2}m_{t}) + (v_{t} - E_{t-2}v_{t}) \right]$$
(1.14)

(it can be easily checked that, if all contracts last one period, i.e. k = 0, output is at the level given by (1.8) in the preceding subsection). In the presence of long-term contracts, aggregate demand and money supply shocks have real effects on output that last two periods before being entirely absorbed into price level changes. This occurs when all contracts have been renewed and for all workers the nominal wage has adjusted to the new levels of money and aggregate demand.

Now, in order to evaluate the role for stabilization monetary policy, we have to make assumptions on: (i) the stochastic process generating the demand disturbance v_t ; and (ii) how monetary policy is conducted.

First, let the aggregate demand shock follow a first-order autoregressive (AR(1)) process:

$$v_t = \rho v_{t-1} + \varepsilon_t \qquad 0 \le \rho \le 1 \tag{1.15}$$

where ε_t is a white noise process, with zero mean and variance σ_{ε}^2 . From (1.15) we directly get:

$$v_t - E_{t-1}v_t = \varepsilon_t$$

$$v_t - E_{t-2}v_t = \varepsilon_t + \rho\varepsilon_{t-1}$$

which, substituted into (1.14), give equilibrium output under the AR(1) assumption for demand disturbances:

$$y_t = \frac{1-k}{2(1+k)} \left[(m_t - E_{t-1}m_t) + \varepsilon_t \right] + \frac{k}{1+k} \left[(m_t - E_{t-2}m_t) + (\varepsilon_t + \rho\varepsilon_{t-1}) \right]$$
(1.16)

Second, let us consider two different ways in which monetary policy is conducted:

1. a *passive* policy: $m_t = \overline{m}$ for all t. In this case there are no "surprises" on the money supply path; (1.16) then becomes:

$$y_t = \frac{1}{2}\varepsilon_t + \frac{k}{1+k}\rho\varepsilon_{t-1}$$

The variance of output around its natural level $y^* = 0$ is then given by:

$$\sigma_y^2 = \left[\frac{1}{4} + \left(\frac{k}{1+k}\right)^2 \rho^2\right] \sigma_\varepsilon^2$$

Note that the higher the fraction of long-term contracts $(k \rightarrow 0.5)$, the larger the output variance;

2. a *feedback rule* designed to *stabilize* output, by minimizing the output variance around the natural level. Let the monetary rule be of the simple form:

$$m_t = \overline{m} + \delta \,\varepsilon_{t-1} \tag{1.17}$$

Monetary authorities choose the value of the policy parameter δ^* which minimizes output variance. To this aim, we obtain from (1.16) the expression for output under the feedback monetary rule (1.17), using the fact that $m_t - E_{t-1}m_t = 0 \ e \ m_t - E_{t-2}m_t = \delta \varepsilon_{t-1}$:

$$y_t = \frac{1}{2}\varepsilon_t + \frac{k}{1+k}(\rho+\delta)\varepsilon_{t-1}$$

The output variance is then:

$$\sigma_y^2 = \left[\frac{1}{4} + \left(\frac{k}{1+k}\right)^2 \left(\rho + \delta\right)^2\right] \sigma_{\varepsilon}^2$$

which is minimized for $\delta^* = -\rho$. Therefore, the optimal monetary policy rule is:

$$m_t = \overline{m} - \rho \varepsilon_{t-1}$$

1.3. Monetary policy implications

The model delivers several important implications for monetary policy:

- (i) monetary feedback rules designed to stabilize output can be effective. Monetary policy can completely offset the negative effects (in terms of a higher output variability) due to long-term contracts. However, note that this conclusion crucially depends on the persistence of the disturbances hitting the economy: in the model above, ρ must be larger than 0. A feedback monetary policy rule would be totally impotent in the face of shocks with no persistence over time ($\rho = 0$);
- (*ii*) with long-term (two-period) contracts, demand shocks (ε) have somewhat persistent effects on output. This persistence is limited, though, by the maximum length of the contracts (here, two periods only);
- (*iii*) monetary policy would lose all stabilization power only if long-term contracts were *indexed* in a way capable to exactly duplicate the effects of one-period contracts; in this case we should set $w_{t-2,t} = E_{t-1}p_t$. For all other indexation schemes, monetary policy does retain its stabilization effectiveness.³

We can now see the effectiveness of monetary policy using the standard AD - AS diagram. Start with Figure 2, where the case of all one-period contracts is depicted (k = 0). The initial aggregate demand curve, given by (1.4) with $v_t = 0$, is AD_0 . In the absence of shocks, output is at its natural level y^* (set to zero in the model), and the aggregate supply curve is vertical. When unexpected money supply or aggregate demand shocks occur, for a given expected price level $E_{t-1}p_t$, the supply function is positively sloped. Let the demand disturbance v follow the stochastic process in (1.15) with $\rho = 1$ (this means that any shock ε has a permanent nature), and suppose that at t a positive realization of the demand functions shifts permanently from AD_0 to AD_1 .

³In particular, Fischer shows that a plausible indexation scheme such as:

$$w_{t-2,t} = w_{t-2,t-1} + (p_{t-1} - p_{t-2})$$

= $E_{t-2}p_{t-1} + (p_{t-1} - p_{t-2})$

according to which the wage set for the second period of the contract (t) is equal to the wage set for the first period (t-1) adjusted for the observed inflation rate $(p_{t-1} - p_{t-2})$, allows monetary policy to be effective in stabilizing output.

In period t both output and the price level are affected by the demand shock. Output reacts because nominal wages are fixed by contracts and do not adjust within the period to the changed demand conditions. In the subsequent period t+1 all contracts are renewed and the new level of the nominal wage can incorporate the (revised) expectations on the price level. Output goes back to the initial level and the increase in demand is entirely reflected in the price level: the economy follows the path $A \to B \to C$ in Figure 2. Monetary policy rules such as (1.17) cannot have any real stabilization effect: before money supply can react to the demand shock, all contracts are renewed and output is back at its natural level.

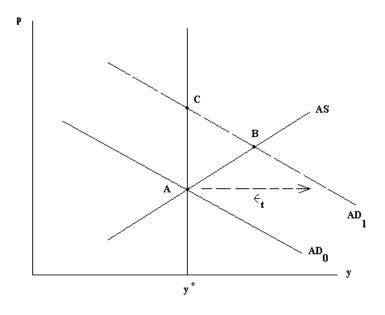


Figure 2

Different conclusions are reached when also two-period contracts are available (k > 0). In Figure 3, which refers to this case, the AD_0 and AD_1 schedules and the aggregate supply in the absence of shocks are the same as in Figure 2. However, in the long-term contracts case, we have two curves showing the adjustment of supply to unanticipated changes in the price level. In period t, when aggregate demand shifts from AD_0 to AD_1 , nominal wages are fixed for all workers: the effect of $\varepsilon_t > 0$ on y and p is the same as in the case of short-term contracts and the equilibrium is reached at point B. In the subsequent period t + 1, differently from

the previous case, not all contracts are renewed: half of the existing two-period contracts are in their second period and will be renewed only at the end of period t + 1. Therefore, aggregate supply does not adjust completely: the curve shifts from AS to AS', yielding a new temporary equilibrium at point C. In period t+2 also those long-term contracts will have incorporated the revised expectations on the level of demand and the supply adjustment will be completed: the increase in demand will be reflected only in the price level, with output back at its initial equilibrium level (point D). With a "passive" monetary policy (i.e. without any reaction of the money supply to the demand shock) the economy follows the path $A \to B \to C \to D$ in Figure 3.

A monetary stabilization policy conducted according to the rule (1.17) with the value of δ set optimally ($\delta^* = -1$) can make the economy go back to the initial equilibrium output (point A) already in period t + 1, when not all contracts are renewed yet. The path of the economy is now modified, being $A \to B \to A$. This effect is achieved by means of a reduction of money supply which takes aggregate demand back from AD_1 to AD_0 in t + 1. Output deviates from the natural rate for only one period (instead of two periods, as in the long-term contracts case): monetary policy is then effective in stabilizing output fluctuations.

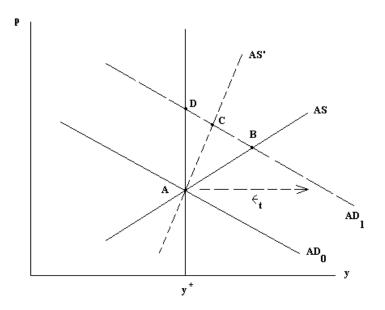


Figure 3

2. A model with *fixed* nominal wages (Taylor 1979)

In order to generate persistent real effects of the shocks hitting the economy, we now change our assumptions on nominal wage setting. As in Taylor (1979, 1980), we adopt the following hypotheses on the contract structure:

- (i) all workers have two-period contracts;
- (ii) contracts fix the nominal wage at the same level in the two periods of validity (whereas, under Fischer's scheme, the wage could be predetermined at two different levels);
- (*iii*) contracts are "*staggered*": in each period half of the workforce must renew the contract for the current and the following period.

In contracting the wage, two elements are taken into account by workers: (i) the *relative wage* level, and (ii) the expected *future output* level. The following equation, determining the nominal wage set at the beginning of period t by half of the workforce to be kept fixed for t and t + 1, incorporates both elements:

$$w_t = \frac{1}{2}w_{t-1} + \frac{1}{2}E_{t-1}w_{t+1} + \frac{\gamma}{2}\left(E_{t-1}y_t + E_{t-1}y_{t+1}\right) + \eta_t$$
(2.1)

where η_t is a shock to the process determining nominal wages, and γ is a positive parameter. On the basis of information available at t-1, half of the workers set the wage as an average of the current wage of the other half of the workforce, set in the previous period, w_{t-1} , and the value of the wage expected to be set in the next period, when the contract will be renewed, $E_{t-1}w_{t+1}$. Moreover, the wage depends also on the expected level of output over the two periods of the contract (as a proxy for the expected state of the economy).

To close the model, the price level in t is specified as a mark-up on the average wage prevailing in the period:

$$p_t = \frac{1}{2}(w_t + w_{t-1}) \tag{2.2}$$

(therefore, unlike in Fischer's model where it is countercyclical, here the real wage is constant over the cycle).

Aggregate demand has the simple form:

$$y_t = (m_t - p_t) + v_t (2.3)$$

with v_t a mean zero white noise process. Finally, monetary authorities set nominal money supply according to the rule:

$$m_t = \delta p_t \tag{2.4}$$

with $0 \leq \delta \leq 1$.

2.1. Model solution

We start by deriving an expression for the nominal wage after substituting out the terms involving expected output. Using (2.2) and (2.3) we have

$$E_{t-1}y_t = E_{t-1}m_t - \frac{1}{2}E_{t-1}w_t - \frac{1}{2}w_{t-1}$$
$$E_{t-1}y_{t+1} = E_{t-1}m_{t+1} - \frac{1}{2}E_{t-1}w_{t+1} - \frac{1}{2}E_{t-1}w_t$$

Substituting the equations above into (2.1) and using (2.4) we can express the nominal wage in period t as a function only of its own past and expected future values:

$$w_{t} = \left[\frac{1}{2} - \frac{\gamma(1-\delta)}{4}\right] w_{t-1} - \frac{\gamma(1-\delta)}{2} E_{t-1} w_{t} + \left[\frac{1}{2} - \frac{\gamma(1-\delta)}{4}\right] E_{t-1} w_{t+1} + \eta_{t} \quad (2.5)$$

This can be further simplified by computing the wage at t expected as of t - 1:

$$E_{t-1}w_t = \theta \left(w_{t-1} + E_{t-1}w_{t+1} \right) \tag{2.6}$$

where θ is given by

$$\theta = \frac{2 - \gamma(1 - \delta)}{4 + 2\gamma(1 - \delta)}$$

Finally, substituting (2.6) into (2.5) we obtain:

$$w_t = \theta w_{t-1} + \theta E_{t-1} w_{t+1} + \eta_t \tag{2.7}$$

The degree of dependence of the current wage level on its past and expected future levels is captured by the coefficient θ , which depends non only upon the structural parameter γ , but also on the policy parameter δ , that characterizes the monetary policy rule chosen by the policymaker.

To solve for w_t we apply the method of undetermined coefficients, and guess a simple linear solution for w_t of the form:

$$w_t = \pi_1 w_{t-1} + \eta_t \tag{2.8}$$

The only past variable affecting the current wage is last period wage; moreover, there is a role for the stochastic disturbance in the wage setting process (on which we already impose a unitary coefficient in (2.8)). From (2.8) we get:

$$E_{t-1}w_{t+1} = \pi_1^2 w_{t-1} \tag{2.9}$$

Equating (2.8) to (2.7), with (2.9) substituted in, we get:

$$\pi_1 w_{t-1} + \eta_t = \theta w_{t-1} + \theta \pi_1^2 w_{t-1} + \eta_t$$

from which π_1 can be obtained as the solution of the equation

$$\pi_1 = \theta(1 + \pi_1^2)$$

Choosing the solution value $\pi_1 < 1$ (to ensure a stable process for the wage), we finally obtain the following solution for the wage w_t :

$$w_t = \frac{1 - \sqrt{1 - 4\theta^2}}{2\theta} w_{t-1} + \eta_t \tag{2.10}$$

This is an autoregressive process: the effect of a shock to the nominal wage η_t lasts for several periods, and the degree of persistence is captured by the magnitude of the coefficient π_1 .

It is now possible to derive the price level and output:

$$p_t = \frac{1 - \sqrt{1 - 4\theta^2}}{2\theta} p_{t-1} + \frac{1}{2}(\eta_t + \eta_{t-1})$$
(2.11)

$$y_t = \frac{1 - \sqrt{1 - 4\theta^2}}{2\theta} y_{t-1} - \frac{1 - \delta}{2} (\eta_t + \eta_{t-1}) + v_t - \frac{1 - \sqrt{1 - 4\theta^2}}{2\theta} v_{t-1} \qquad (2.12)$$

In this model with staggered contracts, output is given by a (first-order) autoregressive process (displaying persistence of the effects of shocks), and two moving average components involving the two stochastic disturbances (the wage shock and the aggregate demand disturbance).

2.2. Implications

The main implications of the model are:

- (i) Shocks can affect output for a long time, even beyond the length of the contracts that fix the nominal wage. This is the case (in the version of the model presented above) of the wage setting disturbance η_t , but, in an extended model, the same would occur for a money supply shock. Persistence is due here to the gradual adjustment of wages and, given mark-up pricing, of prices to shocks. On the contrary, in Fischer's model with predetermined wages, when the longest contracts have been renewed, any effect of the shocks has been entirely incorporated into the new levels of wages and prices, and no further effect on output occurs.
- (*ii*) If monetary authorities aim at minimizing the variance of output around its natural level (here normalized to zero), a monetary policy implemented according to the rule (2.4) is effective, since the parameter δ can be set optimally. In fact, from (2.12), output variance is given by:

$$\sigma_y^2 = \frac{(1-\delta)^2}{2(1-\pi_1^2)} \sigma_\eta^2 + \sigma_v^2$$

and it is minimized for $\delta \to 1$: optimal monetary policy entails full adjustment of money supply to the current level of prices (and then wages).

(*iii*) However, if monetary policy is successful in reducing output variability (σ_y^2) , it loses control of the variability of the price level. Indeed, from (2.11) the variance of p_t is:

$$\sigma_p^2 = \frac{1}{2(1 - \pi_1^2)} \sigma_\eta^2$$

When $\delta \to 1$, $\sigma_y^2 \to \sigma_v^2$ but at the same time $\sigma_p^2 \to \infty$, since $\pi_1 \to 1$. Therefore, a *trade-off* between the *variability* of output and prices arises (and not between the levels of real and nominal variables, as depicted by the traditional Phillips curve).

References

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