Macroeconomic Analysis

Real Business Cycles
(F. Kydland- E. Prescott)

Main theoretical features of the RBC view:

- *equilibrium approach* to business cycle fluctuations, defined as a set of properties concerning *comovements* and *persistence* of main macroeconomic quantities
- unifying theory of *growth* and *fluctuations*
- focus on the *propagation mechanisms* (especially over time) of shock based on *intertemporal substitution* effects
- *technological shocks* as main driving force of business cycle fluctuations

Analytical features:

- reference model:

  neoclassical model of growth (Solow) with *uncertainty* in the rate of technological progress

  $\Rightarrow$ fluctuations are the aggregate result of the behavioral rules of rationally optimizing agents, subject to resource constraints in a stochastic environment

- empirical methodology:

  calibration, instead of traditional econometric testing

Basic RBC model

Structure of the economy:

- *preferences*: large number of infinitely-lived agents maximizing an expected utility function

$$U_t = E_t \left[\sum_{j=0}^{\infty} \beta^j u(\underset{\text{consumption}}{c_{t+j}}, \underset{\text{leisure}}{l_{t+j}}) \right] \qquad 0 < \beta < 1 \tag{U}$$

  $u(c, l) \Rightarrow$ preference for smooth paths of c and l (and intertemporal substitutability of c and l in the face of changes in the real wage and in the real interest rate)

- time *endowment*: amount of time normalized to 1 to divide between work n and leisure l

$$n_{t+j} + l_{t+j} = 1 \tag{*}$$

- *technology* and *capital accumulation*: constant-return-to-scale production function

$$y_{t+j} = z_{t+j}\, f(k_{t+j}, n_{t+j}) \tag{F}$$

  z_{t+j}: total productivity shock;

$$k_{t+j+1} = i_{t+j} + (1 - \delta)\, k_{t+j} \tag{K}$$

  i_{t+j}: investment; δ: rate of physical capital depreciation;

- total *resource constraint* (no government spending, closed economy):

$$y_{t+j} = c_{t+j} + i_{t+j} \tag{Y}$$

- combining (F), (K) e (Y)

$$c_{t+j} + k_{t+j+1} = z_{t+j}\, f(k_{t+j}, n_{t+j}) + (1 - \delta) k_{t+j} \tag{**}$$

Problem:

> find the sequences $\{c_{t+j}\}_0^\infty$, $\{l_{t+j}\}_0^\infty$, $\{n_{t+j}\}_0^\infty$ e $\{k_{t+j}\}_0^\infty$ that maximize expected utility given the time endowment and resource constraint (*) e (**) and the stochastic process generating technological shocks

$$
\begin{aligned}
\max \quad L \quad = \quad & \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) + \sum_{j=0}^{\infty} \beta^j \, \omega_{t+j}(1 - n_{t+j} - l_{t+j}) \\
& + \sum_{j=0}^{\infty} \beta^j \, \lambda_{t+j} \, [z_{t+j} \, f(k_{t+j}, n_{t+j}) + (1-\delta)k_{t+j} - c_{t+j} - k_{t+j+1}]
\end{aligned}
$$

with $\{\omega_{t+j}\}_0^\infty$ and $\{\lambda_{t+j}\}_0^\infty$ sequences of Lagrange multipliers associated with the costraints (and interpreted as shadow prices of one additional unit of time and capital, respectively)

Solution:
system of first-order conditions for c_t, l_t, n_t and k_{t+1} with constraints (*) and (**)

$$
\begin{gathered}
u_c(c_t, l_t) = \lambda_t \\
u_l(c_t, l_t) = \omega_t \\
\lambda_t \, z_t \, f_n(k_t, n_t) = \omega_t \\
\beta \, E_t \left(\lambda_{t+1} \, [z_{t+1} \, f_k(k_{t+1}, n_{t+1}) + 1 - \delta]\right) = \lambda_t
\end{gathered}
$$

Special case (with closed-form solution):

$$\begin{aligned} u(c,l) &= \theta \log c + (1-\theta) \log l \\ y_t &= z_t\, k_t^{1-\alpha} n_t^{\alpha} \\ \delta &= 1 \end{aligned}$$

$\Rightarrow$ **solution**:

$$\frac{\theta}{c_t} = \lambda_t$$

$$\frac{1-\theta}{l_t} = \omega_t$$

$$\alpha\, \lambda_t\, z_t\, k_t^{1-\alpha}\, n_t^{\alpha-1} = \omega_t$$

$$\beta\,(1-\alpha)\, E_t \left(\lambda_{t+1}\, z_{t+1}\, n_{t+1}^{\alpha}\, k_{t+1}^{-\alpha}\right) = \lambda_t$$

to be solved for:

$$\begin{aligned} c_t &= c(k_t, z_t) \\ n_t &= n(k_t, z_t) \\ k_{t+1} &= k(k_t, z_t) \end{aligned}$$

NB: with log utility, labor supply does not change as wage w changes:

$$\max\ \theta\ \log c + (1-\theta)\ \log l$$

$$\text{sub} \quad c = w\, n = w\,(1-l)$$

$$\text{f.o.c.:} \quad -\frac{\theta\, w}{w(1-l)} + \frac{1-\theta}{l} = 0 \quad \Rightarrow \quad l = 1-\theta$$

labor supply independent of w

$\Rightarrow$ *conjectures* (guesses) on the analytical form of the solutions:

$$\begin{aligned} n_t &= \bar{n} \\ c_t &= \pi_C\, z_t\, k_t^{1-\alpha} \\ k_{t+1} &= \pi_K\, z_t\, k_t^{1-\alpha} \end{aligned}$$

with π_C and π_K undetermined coefficients, related by the total resources constraint (with $n_t = \bar{n}$):

$$\underbrace{z_t\, \bar{n}^\alpha\, k_t^{1-\alpha}}_{y_t} = \underbrace{\pi_C\, z_t\, k_t^{1-\alpha}}_{c_t} + \underbrace{\pi_K\, z_t\, k_t^{1-\alpha}}_{k_{t+1}}$$

$$\Rightarrow \pi_C + \pi_K = \bar{n}^\alpha$$

Combining the f.o.c. for c and k and using the undetermined form of the solutions for n_t and c_t:

$$\begin{aligned} \frac{\theta}{\pi_C\, z_t\, k_t^{1-\alpha}} &= \beta\,(1-\alpha)\, E_t \left(\frac{\theta}{\pi_C\, z_{t+1}\, k_{t+1}^{1-\alpha}}\, z_{t+1}\, \bar{n}^\alpha\, k_{t+1}^{-\alpha} \right) \\ \Rightarrow \frac{\theta}{\pi_C\, z_t\, k_t^{1-\alpha}} &= \beta\,(1-\alpha)\, \bar{n}^\alpha\, E_t \left(\frac{\theta}{\pi_C\, k_{t+1}} \right) \end{aligned}$$

using the solution for k_{t+1}:

$$\frac{\theta}{\pi_C\, z_t\, k_t^{1-\alpha}} = \beta\,(1-\alpha)\, \bar{n}^\alpha\, E_t \left(\frac{\theta}{\pi_C\, \left(\pi_K\, z_t\, k_t^{1-\alpha}\right)} \right)$$

$$\Rightarrow \quad \pi_K = \beta\,(1-\alpha)\, \bar{n}^\alpha$$

$$\Rightarrow \quad \pi_C = [1 - \beta\,(1-\alpha)]\; \bar{n}^\alpha$$

$\Rightarrow$ solutions for c_t and k_{t+1}:

$$\begin{aligned} c_t &= [1 - \beta\,(1-\alpha)]\; z_t\, \bar{n}^\alpha\, k_t^{1-\alpha} \\ k_{t+1} &= \beta\,(1-\alpha)\, z_t\, \bar{n}^\alpha\, k_t^{1-\alpha} \end{aligned}$$

To check that the conjecture $n_t = \bar{n}$ is correct, by combining the f.o.c. for l and n and using the solution for c_t:

$$\frac{1-\theta}{1-\bar{n}} = \alpha\, \frac{\theta}{c_t}\, z_t\, k_t^{1-\alpha}\, \bar{n}^{\alpha-1}$$

$$\Rightarrow \bar{n} = \frac{\alpha\,\theta}{\alpha\,\theta + (1-\theta)\, [1 - \beta\,(1-\alpha)]} \quad \text{constant}$$

Dynamic properties of k and c:

$$\begin{aligned}\log\, k_{t+1} &= \phi_0 + (1-\alpha)\,\log\, k_t + \log\, z_t \\ \log\, c_t &= \phi_1 + (1-\alpha)\,\log\, k_t + \log\, z_t\end{aligned}$$

assumption of $AR(1)$ stochastic process for z:

$$\log\, z_t = \rho\,\log\, z_{t-1} + \varepsilon_t \qquad 0 < \rho < 1$$

From:

$$\begin{aligned}\log\, k_{t+1} &= \phi_0 + (1-\alpha)\,\log\, k_t + \log\, z_t \\ \rho\,\log\, k_t &= \rho\,\phi_0 + \rho\,(1-\alpha)\,\log\, k_{t-1} + \rho\,\log\, z_{t-1}\end{aligned}$$

taking $\log\, k_{t+1} - \rho \log\, k_t$, we get the $AR(2)$ processes for k:

$$\log\, k_{t+1} = (1-\rho)\phi_0 + (1-\alpha+\rho)\,\log\, k_t - \rho\,(1-\alpha)\,\log\, k_{t-1} + \varepsilon_t$$

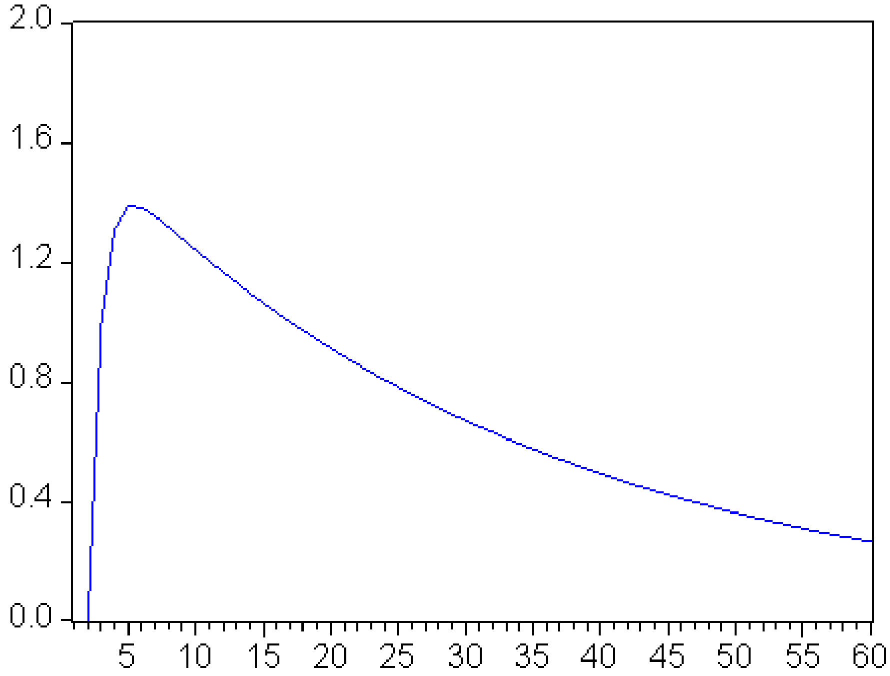
and c:

$$\log\, c_t = [\alpha(1-\rho)\phi_1 + (1-\alpha)(1-\rho)\phi_0] + (1-\alpha+\rho)\,\log\, c_{t-1} - \rho\,(1-\alpha)\,\log\, c_{t-2} + \varepsilon_t$$

Example of dynamic response (*impulse response*) of log k to a unit realization of ε with:

$$\log k_{t+1} = (1 - \alpha + \rho) \log k_t - \rho (1 - \alpha) \log k_{t-1} + \varepsilon_t$$

for $\alpha = 0.66$, $\rho = 0.98$ (constant omitted)



Main results from calibration of a "standard" RBC model

(R. King-S. Rebelo, "Resuscitating business cycles", *Handbook of Macroeconomics*, 2000)

Main parameter values used in calibration: $\alpha = 0.66$, $\delta = 0.025$ (quarterly), $\rho = 0.98$.

Table 3
Business Cycle Statistics for Basic RBC Model[35]

	Standard Deviation	Relative Standard Deviation	First Order Auto-correlation	Contemporaneous Correlation with Output
Y	1.39	1.00	0.72	1.00
C	0.61	0.44	0.79	0.94
I	4.09	2.95	0.71	0.99
N	0.67	0.48	0.71	0.97
Y/N	0.75	0.54	0.76	0.98
w	0.75	0.54	0.76	0.98
r	0.05	0.04	0.71	0.95
A	0.94	0.68	0.72	1.00

Note: All variables have been logged (with the exception of the real interest rate)and detrended with the HP filter.

Table 1
Business Cycle Statistics for the U.S. Economy

	Standard Deviation	Relative Standard Deviation	First Order Auto-correlation	Contemporaneous Correlation with Output
Y	1.81	1.00	0.84	1.00
C	1.35	0.74	0.80	0.88
I	5.30	2.93	0.87	0.80
N	1.79	0.99	0.88	0.88
Y/N	1.02	0.56	0.74	0.55
w	0.68	0.38	0.66	0.12
r	0.30	0.16	0.60	-0.35
A	0.98	0.54	0.74	0.78

Note: All variables are in logarithms (with the exception of the real interest rate) and have been detrended with the HP filter. Data sources are described in Stock and Watson [1998], who created the real rate using VAR inflation expectations. Our notation in this table corresponds to that in the text, so that Y is per capita output, C is per capita consumption, I is per capita investment, N is per capita hours, w is the real wage (compensation per hour), r is the real interest rate, and A is total factor productivity.

Problems (some results are not consistent with the observed properties of macroeconomic time series, especially concerning the labor market):

- "*employment variability puzzle*":

data: employment is as much volatile as output and is strongly procyclical, whereas real wage is less volatile and only weakly procyclical

RBC: the observed pattern is obtained only by assuming a very large wage elasticity of labor supply (which is not supported by empirical microeconomic evidence)

but: by introducing *indivisibility* (non convexity) in labor supply decisions (i.e. workers can choose *whether* to work or not to work, but not *how many hours* to work per week), it is possible to reconcile a high volatile employment with a low microeconomic labor supply elasticity

- "*productivity puzzle*":

data: labor productivity and employment are not highly correlated

RBC: large correlation between productivity and employment (due to the technological nature of the shocks)

but: (1) the productivity-employment correlation can be reduced by the introduction of shocks to labor supply (e.g. due to monetary disturbances in the presence of nominal rigidities)
(2) in the presence of *labor hoarding* behavior by firms, the correlation between *effective* labor input and productivity could be higher than that measured using data on hours worked; in this case the measure of productivity shocks based on the Solow residual (SR):

$$\log SR_t = \log Y_t - (1 - \alpha) \log K_t - \alpha \log N_t$$

would overestimate the actual changes in total factor productivity.