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An alternative explanation of the price puzzle☆

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Abstract

This paper proposes an explanation for the frequent appearance of a price puzzle in VARs designed for monetary policy analysis. It assumes that the data are generated by a model in which output and output gap are not equivalent, while an econometrician follows the common practice of including only output in the VAR. The omission of the output gap is shown to spuriously produce a price puzzle (and several other incorrect conclusions) in a class of commonly used models. This can happen even if the model admits a triangular identification and if the forecasts produced by the misspecified VAR are optimal. When the model is tested on US data, all predictions are supported. A commodity price index is not needed to solve the puzzle.

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1. Introduction

A vast amount of literature has produced a reference framework for VAR analysis of monetary policy.[1] This reference VAR includes a commodity price index. The first VAR studies showed that omitting a commodity price and taking a short interest rate as the policy instrument produced a response of the price level to contractionary monetary policy shocks which was positive for many quarters, a finding that took the name of price puzzle. Sims (1992) proposes a rationale for the puzzle and a way to fix it. His conjecture was that the information set available to policy makers may include variables useful in forecasting future inflation that the econometrician has not considered. If the VAR forecast of inflation is not as good as the central banker's, interest rate movements which are, in fact, endogenous responses to signals of future inflation will be mistakenly identified as monetary policy shocks, hence the finding that prices increase after a contractionary monetary policy shock (henceforth *MP* shock).

Sims himself (1992) and later studies building on this suggestion have found that the puzzle disappears in the US, at least to a large extent, when the VAR is extended to include a commodity price index, a variable useful in forecasting inflation. Besides solving the price puzzle, the inclusion of a commodity price changes the overall picture of monetary policy, in that the response of output to an *MP* shock is smaller and *MP* shocks are less important in the variance decomposition of output and of the federal funds rate (the policy instrument). Based on these results, Leeper et al. (1996) warn that the exclusion of a commodity price can result in serious misspecification.

This explanation has been recently questioned by Hanson (2000) and Barth and Ramey (2001). Both papers find little correlation between a variable's ability to mitigate the price puzzle and its usefulness in forecasting inflation. Giordani (2001) includes accurate inflation forecasts in the VAR and concludes that they do not help to solve the puzzle. Barth and Ramey (2001) suggest that rising prices following a monetary contraction need not be a puzzle if monetary policy operates not only through demand effects but also through supply effects. However, if real-world monetary policy is to make any sense, the contractionary effects must eventually dominate, so the supply-side argument cannot motivate price puzzles of 3 or 4 years, as Sims (1992) finds when he excludes a commodity price from his VAR. Another limit of the supply-side story is that it does not explain why a commodity price reduces the puzzle.

This paper explores the possibility that the price puzzle may be due to something other than the omission of a variable useful in forecasting inflation (such as a commodity price). It shows that the (spurious) appearance of a price puzzle is predicted by a popular class of models, if the econometrician follows the common and seemingly innocent practice of not including a measure of output gap (or potential output) in the VAR. The intuition is that since the output gap is omitted from the inflation equation, the interest rate spuriously appears in that equation with

[1] For a summary of this literature see Christiano et al. (2000, 2001) or Leeper et al. (1996).

a positive coefficient, because the interest rate reacts positively to output gap increases and thus acts as its proxy. In this paper, we present this misspecification as a stand-alone explanation of the price puzzle. However, our argument is not per se incompatible with Sims' (poor inflation forecast), Barth and Ramey's (supply effects) or Leeper and Zha's (2001, money should belong to the policy function): whatever the reason to expect a price puzzle in a VAR that does include a good measure of the output gap, we suggest that a much bigger puzzle can be expected in a VAR that does not.

The rest of the paper proceeds as follows. Taking the model in Svensson (1997) as the data-generating process, Section 2 derives analytical results on the consequences of estimating a three variable VAR that includes output but not the output gap. Among other things, a price puzzle emerges and the variance of *MP* shocks is overestimated. The impact of an *MP* shock on output is also overestimated. The consequences of the misspecification are also shown through impulse responses, giving more color to the analytical results.

Section 3 investigates the generality of these results. Sufficient conditions are given for a model to generate a price puzzle if the econometrician does not include a measure of output gap in the VAR. Forward-looking models that satisfy these conditions include Svensson (2000) and Christiano et al. (2001). These are not necessary conditions, and simulations of two models which do not satisfy them (Fuhrer and Moore, 1995; Clarida et al., 1999) give remarkably similar outcomes.

Section 4 takes the theory to the data, using as the output gap a measure of capacity utilization produced by the Federal Reserve Board. A three variable VAR, incorporating output gap, inflation and federal funds rate (in that recursive order) is compared to a VAR including output rather than the output gap. The second VAR produces a large price puzzle, the first none. In the first VAR monetary policy is more endogenous and accounts for much less of the forecast error variance of output. These results are shown to be robust to a variety of changes. However, results based on monthly data are not as sharp as those based on quarterly data. A possible explanation relies on the presence of measurement errors. Section 5 argues that the commodity price index does not solve the price puzzle because it is useful in forecasting inflation, but rather because it is correlated with the output gap. Section 6 concludes.

2. A simple model for monetary policy analysis: Svensson (1997)

The idea that omitting a theoretically relevant variable from a VAR can render identification exercises invalid is of course not new. The contribution of this paper is to show in detail that the omission from the VAR of a measure of output gap is predicted to generate a price puzzle in a commonly used class of models. While members of this class include forward-looking models, to obtain analytical solutions we will work with a backward-looking model due to Svensson (1997). Section 3 discusses generalizations of the results.

The model is designed to capture some key features of the transmission mechanism of monetary policy. The same model is used in Rudebusch and Svensson (1999), Judd and Rudebush (1998), Romer (2000), Hansen and Sargent (2000) and, extended to a small open economy, Ball (1999). Forward-looking versions appear in Clarida et al. (1999) and in Svensson (2000). The model consists of an *IS* equation, a Phillips curve and a Taylor rule obtained from the monetary authority's optimization problem. This core three-equation structure is shared by many recent New–Keynesian models for monetary policy analysis. The model incorporates delays in the transmission of monetary policy. The interest rate can affect the output gap only with a lag; the output gap, in turn, affects inflation with a lag. Since the transmission from policy action to prices goes through output variations, monetary policy affects prices with two lags.

The *IS* relation is given by

$$y^g_{t+1} = \beta_y y^g_t - \beta_r(i_t - \pi_t) + \varepsilon^{\mathrm{AD}}_{t+1}, \tag{1}$$

where i_t is a short-term interest rate set by the monetary authority, y^g is the output gap, defined as $y^g_t = y_t - y^{\mathrm{N}}_t$, where y_t is the log of real output and y^{N}_t the log of real potential (or "natural") output. Potential output follows an exogenous AR(1) process[2]

$$y^{\mathrm{N}}_{t+1} = \rho y^{\mathrm{N}}_t + \varepsilon^{\mathrm{N}}_{t+1}. \tag{2}$$

The Phillips curve is modeled as

$$\pi_{t+1} = \pi_t + \alpha_y y^g_t + \varepsilon^{\mathrm{CP}}_{t+1}. \tag{3}$$

All shocks are *i.i.d.*[3] They are labeled aggregate demand shock, technology shock and cost-push shock, and their standard deviations are denoted by σ_{AD}, σ_{N}, σ_{CP}. The model is supplemented by a loss function for the monetary authority of the standard type

$$L_t = \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i[\lambda(y^g_{t+i})^2 + \pi^2_{t+i}],$$

where the inflation target has been normalized to zero. The solution takes the form of a Taylor rule[4]

$$i_t = \gamma_\pi \pi_t + \gamma_y y^g_t. \tag{4}$$

A monetary policy shock ($\varepsilon^{\mathrm{MP}}_t$ with std σ_{MP}) can be added by assuming that the Taylor rule is not followed deterministically.

[2] Svensson (1997) makes no assumption about potential output. We follow Svensson (2000) in assuming an AR(1) process.

[3] The assumption of *i.i.d.* shocks is not particularly restrictive, as more lags can be added to Eqs. (1)–(3) without any difficulty.

[4] See Svensson (1997) for the closed-form solution. Since the model is backward-looking the discretionary solution and the commitment solution are the same

2.1. The correct representation and identification

The model given by Eqs. (1)–(4) admits the following VAR(1) representation:

$$\begin{bmatrix} y^g_{t+1} \\ y_{t+1} \\ \pi_{t+1} \\ i_{t+1} \end{bmatrix} = \begin{bmatrix} \beta_y & 0 & \beta_r & -\beta_r \\ \beta_y - \rho & \rho & \beta_r & -\beta_r \\ \alpha_y & 0 & 1 & 0 \\ \gamma_y\beta_y + \gamma_\pi\alpha_y & 0 & \gamma_y\beta_r + \gamma_\pi & -\gamma_y\beta_r \end{bmatrix} \begin{bmatrix} y^g_t \\ y_t \\ \pi_t \\ i_t \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma_y & 0 & \gamma_\pi & 1 \end{bmatrix} \begin{bmatrix} \varepsilon^{\mathrm{AD}}_{t+1} \\ \varepsilon^{\mathrm{N}}_{t+1} \\ \varepsilon^{\mathrm{CP}}_{t+1} \\ \varepsilon^{\mathrm{MP}}_{t+1} \end{bmatrix}, \tag{5}$$

or more compactly

$$Y_{t+1} = BY_t + D\varepsilon_{t+1}. \tag{6}$$

The corresponding reduced-form VAR is written as

$$Y_{t+1} = BY_t + u_{t+1}, \qquad DD' = \mathrm{E}(u_t u_t') \equiv \Sigma. \tag{7}$$

Since D is lower triangular, all structural shocks can be recovered by a Cholesky decomposition of Σ. In particular, *MP* shocks are correctly identified by the assumption that they have no effect on the output gap, output and inflation within the period.

Technology shocks have no effect on the output gap, inflation or the interest rate at any lag. We therefore often focus on a smaller VAR ('VARgap') which eliminates output from (5)

$$\begin{bmatrix} y^g_{t+1} \\ \pi_{t+1} \\ i_{t+1} \end{bmatrix} = \begin{bmatrix} \beta_y & \beta_r & -\beta_r \\ \alpha_y & 1 & 0 \\ \gamma_\pi\alpha_y + \beta_y\gamma_y & \gamma_\pi + \beta_r\gamma_y & -\beta_r\gamma_y \end{bmatrix} \begin{bmatrix} y^g_t \\ \pi_t \\ i_t \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \gamma_y & \gamma_\pi & 1 \end{bmatrix} \begin{bmatrix} \varepsilon^{\mathrm{AD}}_{t+1} \\ \varepsilon^{\mathrm{CP}}_{t+1} \\ \varepsilon^{\mathrm{MP}}_{t+1} \end{bmatrix}. \tag{8}$$

The smaller VAR in (8) allows recovery of *AD*, *CP* and *MP* shocks by a recursive ordering. However, most applied research has not estimated (5) or (8), but rather VARs devoid of explicit measures of the output gap.[5]

[5] Some exceptions are Rotemberg and Woodford (1997) and Rudebusch and Svensson (1999).

2.2. From the VAR implied by theory to the empirical VAR, taking a false step

Let us assume that the Svensson (1997) model (Eqs. (1)–(4)) is the DGP, and that an econometrician follows the common practice of estimating a VAR including output, inflation and the interest rate,[6] but not the output gap or potential output, and then attempts to recover *MP* shocks with a recursive identification.[7] It is apparent that the econometrician's decision can result in misspecification, since she is omitting a variable (the output gap). Our aim is to uncover a detailed picture of the consequences of this omission. Starting from (8), use $y_t^g \equiv y_t - y_t^{\mathrm{N}}$, expand out the y_{t+1}^{N} and y_t^{N} terms and use $y_{t+1}^{\mathrm{N}} = \rho y_t^{\mathrm{N}} + \varepsilon_{t+1}^{\mathrm{N}}$. Then eliminate y_t^{N} rearranging the Taylor rule. The result is

$$\begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ i_{t+1} \end{bmatrix} = \begin{bmatrix} \rho & (\rho-\beta_y)\frac{\gamma_\pi}{\gamma_y}+\beta_r & -[(\rho-\beta_y)\frac{1}{\gamma_y}+\beta_r] \\ 0 & 1-\alpha_y\frac{\gamma_\pi}{\gamma_y} & \frac{\alpha_y}{\gamma_y} \\ 0 & \beta_r\gamma_y-\beta_y\gamma_\pi+\gamma_\pi-\frac{\alpha_y}{\gamma_y}\gamma_\pi^2 & -\beta_r\gamma_y+\beta_y-\frac{\alpha_y}{\gamma_y}\gamma_\pi \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix}$$

$$+ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \gamma_y & \gamma_\pi & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{t+1}^{\mathrm{N}} \\ \varepsilon_{t+1}^{\mathrm{AD}} \\ \varepsilon_{t+1}^{\mathrm{CP}} \\ \varepsilon_{t+1}^{\mathrm{MP}} \end{bmatrix} + \frac{1}{\gamma_y} \begin{bmatrix} \rho-\beta_y \\ -\alpha_y \\ -\gamma_y\beta_y-\gamma_\pi\alpha_y \end{bmatrix} \varepsilon_t^{\mathrm{MP}}. \tag{9}$$

Abstracting from the last term for the moment, notice that the three reduced-form shocks available to the econometrician are linear combinations of four structural shocks. Correct identification of all shocks is therefore impossible unless the structural covariance matrix Σ (defined in (7)) is singular. And even if Σ is singular, recursive identifying restrictions will deliver inconsistent estimates of the true impulse responses in all but a few special cases. In particular, the rest of this section shows that one such inconsistent estimate delivers the price puzzle.

If the standard deviation of all shocks is strictly positive, this system does not admit a VAR(1) representation. It is easily proved (see Giordani, 2001) that it admits a VARMA(2,1) representation. This implies that the econometrician who is selecting a lag length for the VAR is likely to need more than one lag, and will produce sub-optimal fit and forecasts (even is she estimates a VARMA(2,1)) for all variables, including inflation. However, the origin of the price puzzle lies elsewhere, namely in

[6] Throughout the paper I refer to the VAR in output, inflation and interest rate as "misspecified VAR".

[7] This group of three variables, with the same ordering, plus a commodity price index ordered after prices, is at the core of the framework VAR model for monetary policy analysis (see, for example, Bagliano and Favero, 1998).

the fact that the coefficient of i_t in the inflation equation is positive. In order to focus on this point, let us assume that the Taylor rule is deterministic ($\sigma_{\text{MP}} = 0$). The system in (9) then simplifies to a VAR(1)

$$\begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ i_{t+1} \end{bmatrix} = \begin{bmatrix} \rho & (\rho - \beta_y)\frac{\gamma_\pi}{\gamma_y} + \beta_r & -[(\rho - \beta_y)\frac{1}{\gamma_y} + \beta_r] \\ 0 & 1 - \alpha_y \frac{\gamma_\pi}{\gamma_y} & \frac{\alpha_y}{\gamma_y} \\ 0 & \beta_r\gamma_y - \beta_y\gamma_\pi + \gamma_\pi - \frac{\alpha_y}{\gamma_y}\gamma_\pi^2 & -\beta_r\gamma_y + \beta_y - \frac{\alpha_y}{\gamma_y}\gamma_\pi \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & \gamma_y & \gamma_\pi \end{bmatrix} \begin{bmatrix} \varepsilon_{t+1}^{\text{N}} \\ \varepsilon_{t+1}^{\text{AD}} \\ \varepsilon_{t+1}^{\text{CP}} \end{bmatrix}. \tag{10}$$

Call Σ^* the reduced-form covariance matrix of the misspecified VAR in (10). Denote with $[\Sigma]_2^4$ the 3×3 matrix obtained deleting the first row and the first column of Σ (defined in (7)). Because $\sigma_{\text{MP}} = 0$, the covariance matrix Σ of the correctly specified VAR in (5) is singular, and $[\Sigma]_2^4 = \Sigma^*$ (the deterministic form of the Taylor rule allows the retrieval of y_t^g with no error as a linear combination of i_t and π_t). That is, the exclusion of the output gap produces no loss of fit in any equation of the misspecified VAR. Therefore, the misspecified VAR gives optimal forecasts of inflation at all time horizons, implying that the standard explanation for the price puzzle is not relevant in this setting.

The econometrician identifies the system by assuming a recursive structure, with the interest rate ordered last. If she attempts a complete identification, she needs labels for three structural shocks, say AD^*, CP^* and MP^*. Denote with D^* the Cholesky decomposition of Σ^*, and with D the Cholesky decomposition of Σ. The econometrician will then interpret D^* as

$$D^* = \begin{bmatrix} \sigma_{\text{AD}}^* & 0 & 0 \\ b_{21} & \sigma_{\text{CP}}^* & 0 \\ \gamma_y^*\sigma_{\text{AD}}^* & \gamma_\pi^*\sigma_{\text{CP}}^* & \sigma_{\text{MP}}^* \end{bmatrix}. \tag{11}$$

Write $[\Sigma]_2^4 = \Sigma^*$ as[8]

$$D^*D^{*'} = [DD']_2^4,$$

[8]To avoid mixing problems of misspecification and of parameter uncertainty due to small sample size, the sample size is assumed infinite.

where $[DD']_2^4$ is the 3×3 matrix obtained deleting the first row and the first column of DD'. In expanded form

$$[DD']_2^4 = \begin{bmatrix} \sigma_{\mathrm{N}}^2 + \sigma_{\mathrm{AD}}^2 & 0 & \gamma_y \sigma_{\mathrm{AD}}^2 \\ & \sigma_{\mathrm{CP}}^2 & \gamma_\pi \sigma_{\mathrm{CP}}^2 \\ & & \gamma_y^2 \sigma_{\mathrm{AD}}^2 + \gamma_\pi^2 \sigma_{\mathrm{CP}}^2 \end{bmatrix} = D^* D^{*'}$$
$$= \begin{bmatrix} \sigma_{\mathrm{AD}}^{*2} & \sigma_{\mathrm{AD}}^{*} b_{21} & \gamma_y^* \sigma_{\mathrm{AD}}^{*2} \\ & \sigma_{\mathrm{CP}}^{*2} + b_{21}^2 & b_{21} \gamma_y^* \sigma_{\mathrm{AD}}^* + \gamma_\pi^* \sigma_{\mathrm{CP}}^{*2} \\ & & \gamma_y^{*2} \sigma_{\mathrm{AD}}^{*2} + \gamma_\pi^{*2} \sigma_{\mathrm{CP}}^{*2} + \sigma_{\mathrm{MP}}^{*2} \end{bmatrix}. \tag{12}$$

The relations between the actual and the estimated shocks are straightforwardly obtained from the equalities in (12). b_{21} is correctly set to zero, since $\sigma_{\mathrm{AD}}^* b_{21} = 0$. It follows that:

1. $\sigma_{\mathrm{AD}}^{*2} = \sigma_{\mathrm{N}}^2 + \sigma_{\mathrm{AD}}^2$. The variance of the labeled *AD* shock is the sum of the variances of the *AD* shock and of the technology shock.
2. $\sigma_{\mathrm{AD}}^{*2} = \sigma_{\mathrm{AD}}^2 + \sigma_{\mathrm{N}}^2$ and $\gamma_y \sigma_{\mathrm{AD}}^2 = \gamma_y^* \sigma_{\mathrm{AD}}^{*2}$ imply $\frac{\gamma_y}{\gamma_y^*} = (\sigma_{\mathrm{AD}}^2 + \sigma_{\mathrm{N}}^2)/\sigma_{\mathrm{AD}}^2 > 1$: the output gap coefficient in the Taylor rule is underestimated. Simple algebra gives $\gamma_y \sigma_{\mathrm{AD}} - \gamma_y^* \sigma_{\mathrm{AD}}^* = \gamma_y \sigma_{\mathrm{AD}} (1 - \sigma_{\mathrm{AD}} / \sqrt{(\sigma_{\mathrm{AD}}^2 + \sigma_{\mathrm{N}}^2)}) > 0$. This means that the intensity of the response of the monetary authority to a one std *AD* shock is underestimated even though the std of *AD* shocks is overestimated. The intuition is that some unforecasted movements in output are due to technology shocks, to which monetary policy does not respond. Since the misspecified VAR registers a small average reaction of the interest rate to unforecasted output movements, the coefficient γ_y in the Taylor rule is underestimated. The underestimation grows with σ_{N}.
3. $\sigma_{\mathrm{CP}}^2 = \sigma_{\mathrm{CP}}^{*2}$, following from the fact that $b_{21} = 0$.
4. Finally, the variance of *MP* shocks is overestimated. To derive the result, start from (12), which sets $\gamma_y^2 \sigma_{\mathrm{AD}}^2 + \gamma_\pi^2 \sigma_{\mathrm{CP}}^2 = \gamma_y^{*2} \sigma_{\mathrm{AD}}^{*2} + \gamma_\pi^{*2} \sigma_{\mathrm{CP}}^{*2} + \sigma_{\mathrm{MP}}^{*2}$. Use the results obtained so far, namely

(a) $\gamma_\pi^2 \sigma_{\mathrm{CP}}^2 = \gamma_\pi^{*2} \sigma_{\mathrm{CP}}^{*2}$,
(b) $\sigma_{\mathrm{AD}}^{*2} = \sigma_{\mathrm{N}}^2 + \sigma_{\mathrm{AD}}^2$,
(c) $\gamma_y / \gamma_y^* = (\sigma_{\mathrm{AD}}^2 + \sigma_{\mathrm{N}}^2)/\sigma_{\mathrm{AD}}^2$,

to obtain

$$\sigma_{\mathrm{MP}}^{*2} = \gamma_y^2 \frac{\sigma_{\mathrm{AD}}^2 \sigma_{\mathrm{N}}^2}{\sigma_{\mathrm{AD}}^2 + \sigma_{\mathrm{N}}^2} > 0. \tag{13}$$

Even though the Taylor rule is deterministic, the VAR finds that the variance of the labeled *MP* shock is strictly positive. The intuition is that since the interest rate does not react in the same way to technology and *AD* shocks, when a movement in output (of a given amount) is observed the VAR sometimes registers a certain change in the interest rate (when the movement is caused by an *AD* shock) and sometimes a different change (when caused by a technology shock) and is tricked into interpreting this as random behavior of the monetary authority.[9] If $\sigma_{\text{MP}} > 0$, the right-hand side of Eq. (13) gives a lower bound for $\sigma^{*2}_{\text{MP}} - \sigma^2_{\text{MP}}$. Notice that if either σ_{N} or σ_{AD} are zero, σ^{*2}_{MP} is also correctly estimated to be zero.[10] However, since $\sigma^*_{\text{MP}} > 0$ in general, the occurrence of a price puzzle in the misspecified system follows immediately from the coefficients of the inflation equation in the misspecified VAR, which from (10) is

$$\pi_{t+1} = \left[1 - \alpha_y \frac{\gamma_\pi}{\gamma_y}\right]\pi_t + \frac{\alpha_y}{\gamma_y} i_t + \varepsilon^{\text{CP}}_{t+1}. \tag{14}$$

Eq. (14) is both the inflation equation in the VAR and the structural equation of the recursive system, since $b_{21} = 0$ (see (12)) implies that y_{t+1} has a zero coefficient. If $\sigma_{\text{AD}} = 0$, (14) simplifies to

$$\pi_{t+1} = \left[1 - \alpha_y \frac{\gamma_\pi}{\gamma_y} + \frac{\alpha_y}{\gamma_y}\gamma_\pi\right]\pi_t + \varepsilon^{\text{CP}}_{t+1}. \tag{15}$$

Since the interest rate has a zero coefficient in the inflation equation, there is no price puzzle (moreover, $\sigma^*_{\text{MP}} = 0$ in this case). If $\sigma_{\text{AD}} > 0$ and $\sigma_{\text{N}} = 0$, (14) is equivalent to

$$\pi_{t+1} = \pi_t + \alpha_y y_t + \varepsilon^{\text{CP}}_{t+1}. \tag{16}$$

y_t, π_t and i_t are then perfectly collinear and choosing between (14) and (16) is a matter of taste. On the other hand, if σ_{AD} and σ_{N} are positive, no other autoregressive representation fits as well as (14). Therefore, OLS will retrieve (14). The reason why i_t appears with a positive coefficient in (14) is that movements in the interest rate help retrieve movements in the output gap, which is omitted. If $\alpha_y/\gamma_y > 0$ the impact of the retrieved *MP** shock (which causes i_t to be higher than forecasted) on inflation is estimated to be zero contemporaneously and positive at one lag. In other words, a positive response of inflation to a contractionary *MP* shock (a price puzzle) at lag one is guaranteed as long as the variance of the retrieved *MP* shocks is estimated to be strictly positive, which will be the case if $\sigma_{\text{N}} > 0$ (see Eq. (13)). The magnitude of the puzzle at lag one is given by $(\alpha_y/\gamma_y)\sigma^*_{\text{MP}}$, so it grows with the variance of technology shocks (see Eq. (13)). If the econometrician is ordering the interest rate first she will obtain an even more pronounced price puzzle, since σ^*_{MP} is obviously larger and the inflation equation will show a positive coefficient on the contemporaneous value of the interest rate, so the impulse response to a contractionary *MP* shock will show inflation raising immediately.

[9] In this model the interest rate does not react at all to technology shocks, but the intuition applies more generally, as long as technology and *AD* shocks do not have the same impact on output and the interest rate.

[10] σ^{*2}_{MP} is also zero when γ_y is zero. This case will be taken up shortly.

The derivation of Eq. (14) assumes that $\gamma_y > 0$. In the Svensson model, the optimal γ_y is positive even if inflation is the only argument in the loss function. However, $\gamma_y > 0$ is not a necessary condition for the system to be stable, and if γ_y is zero the interest rate looses its role as a proxy for the omitted output gap, and the misspecified VAR correctly retrieves the *MP* shocks and their effects. In what follows we assume $\gamma_y > 0$.

To gain further understanding of the puzzle, it is useful to show that the misspecified *MP** shocks are positively correlated with the true *AD* shocks and negatively correlated with the true technology shocks.

In the DGP, the one-step-ahead forecast error of i is given by

$$i_{t+1} - \mathrm{E}_t i_{t+1} = \gamma_y \varepsilon_{t+1}^{\mathrm{AD}} + \gamma_\pi \varepsilon_{t+1}^{\mathrm{CP}}, \tag{17}$$

while the one-step-ahead forecast error in the misspecified model is given by

$$i_{t+1} - \mathrm{E}_t^* i_{t+1} = \gamma_y^* \varepsilon_{t+1}^{*\mathrm{AD}} + \gamma_\pi^* \varepsilon_{t+1}^{*\mathrm{CP}} + \varepsilon_{t+1}^{*\mathrm{MP}}. \tag{18}$$

Asterisks denote the shocks obtained from the misspecified VAR. Since the assumption that $\sigma_{\mathrm{MP}} = 0$ implies $\mathrm{E}_t i_{t+1} = \mathrm{E}_t^* i_{t+1}$, we can equate the right-hand sides and use the results just obtained, namely $\gamma_\pi \varepsilon_{t+1}^{\mathrm{CP}} = \gamma_\pi^* \varepsilon_{t+1}^{*\mathrm{CP}}$, $\varepsilon_{t+1}^{*\mathrm{AD}} = \varepsilon_{t+1}^{\mathrm{AD}} + \varepsilon_{t+1}^{\mathrm{N}}$ to obtain as expression for $\varepsilon^{*\mathrm{MP}}$,

$$\varepsilon_{t+1}^{*\mathrm{MP}} = (\gamma_y - \gamma_y^*)\varepsilon_{t+1}^{\mathrm{AD}} - \gamma_y^* \varepsilon_{t+1}^{\mathrm{N}}. \tag{19}$$

Since $cov(\varepsilon_{t+1}^{\mathrm{AD}}, \varepsilon_{t+1}^{\mathrm{N}}) = 0$ by assumption, using $\gamma_y/\gamma_y^* = (\sigma_{\mathrm{AD}}^2 + \sigma_{\mathrm{N}}^2)/\sigma_{\mathrm{AD}}^2 > 0$ gives

$$cov(\varepsilon_{t+1}^{*\mathrm{MP}}, \varepsilon_{t+1}^{\mathrm{AD}}) = (\gamma_y - \gamma_y^*)\sigma_{\mathrm{AD}}^2 > 0, \tag{20}$$

$$cov(\varepsilon_{t+1}^{*\mathrm{MP}}, \varepsilon_{t+1}^{\mathrm{N}}) = -\gamma_y^* \sigma_{\mathrm{N}}^2 < 0. \tag{21}$$

These results provide further insight into the origin of the price puzzle: the misspecified monetary policy shocks are positively correlated with the true aggregate demand shocks, which in turn raise inflation with a lag. Since at lag one the true monetary policy shocks cannot affect inflation, only the spurious part is active at lag one, so we are certain to find a price puzzle. Moreover, the misspecified monetary policy shocks are correlated with technology shocks. This means that the misspecified impulse response of output to a contractionary monetary policy shock is contaminated by the response of output to a negative technology shock. If potential output is more persistent than the output gap, the response of output to a monetary policy shock will be longer lived than the true one.

There are more potentially erroneous conclusions that can be derived from the misspecified system. The reduced form equation for output in the misspecified system is

$$y_{t+1} = \rho y_t + \left[(\rho - \beta_y)\frac{\gamma_\pi}{\gamma_y} + \beta_r\right]\pi_t - \left[(\rho - \beta_y)\frac{1}{\gamma_y} + \beta_r\right] i_t + \varepsilon_{t+1}^{\mathrm{AD}} + \varepsilon_{t+1}^{\mathrm{N}}. \tag{22}$$

If $\rho > \beta_y$, as plausible, the effect of a given interest rate shock on output one step ahead is overestimated, since the coefficient attached to i_t is larger than β_r.

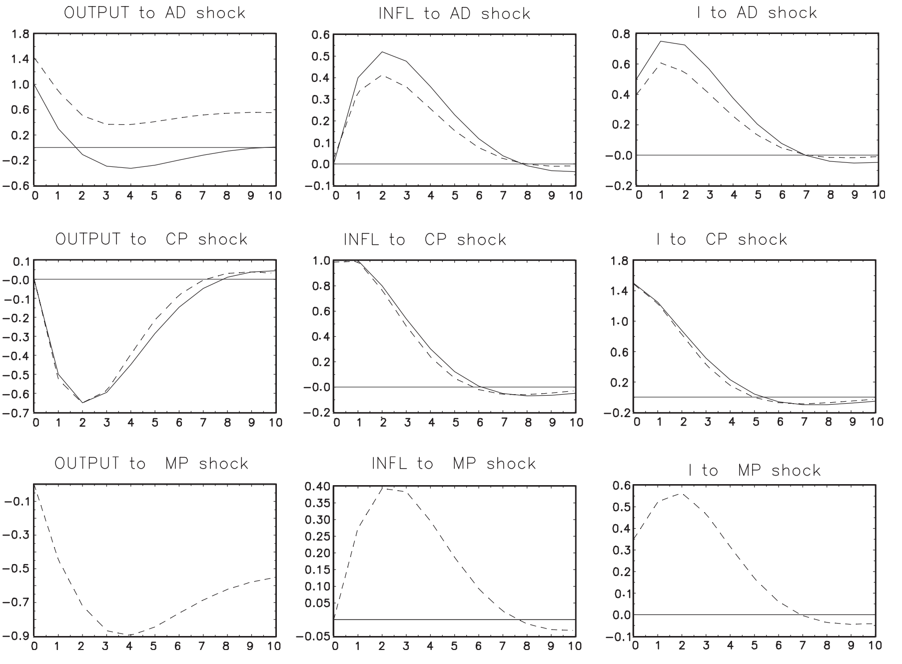

Fig. 1. True impulse responses (solid line) and impulse responses from misspecified VAR (dashed line) in the Svensson model. $\sigma_{MP} = 0$.

2.3. Impulse responses

A more complete picture of the consequences of the misspecification can be gained from looking at impulse responses. The experiment is as follows. Each graph plots the response of output or inflation or interest rate to a shock in the theoretical economy together with the response to the same shock in the misspecified three variable VAR (output, inflation, interest rate).

The model parameters are set as in Ball (1999): $\alpha_y = 0.4$, $\beta_y = 0.8$, $\beta_r = 1$. Reflecting the idea that potential output is a highly persistent process, $\rho = 0.98$. The standard deviations are $\sigma_{AD} = \sigma_{CP} = \sigma_N = 1$. For ease of comparison, the parameters in the Taylor function are not set to the optimal value in each case, but are kept constant at $\gamma_y = 0.5$, $\gamma_\pi = 1.5$.[11] The response of output to a labeled *AD* shock is higher than the true one upon impact and more persistent thereafter, while the response of the interest rate to an *AD* shock is underestimated. In the low-right corner of Fig. 1, notice that the estimated std of an *MP* shock, which is zero in the *DGP*, is a substantial 0.35. Therefore, in the variance decomposition of all variables

[11]The optimal value of the parameters of the Taylor function has a closed-form solution given in Svensson (1997).

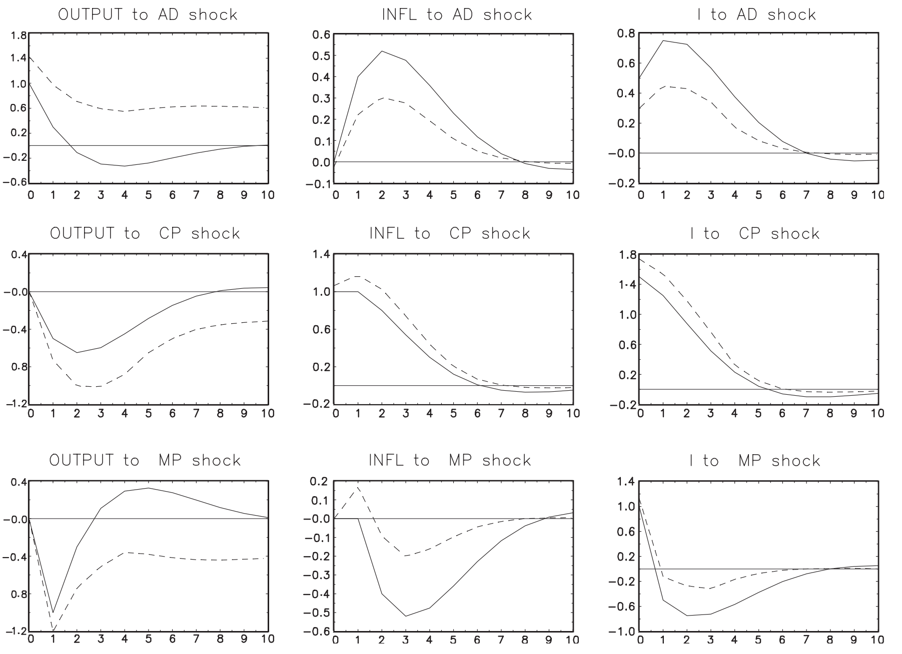

Fig. 2. True impulse responses (solid line) and impulse responses from misspecified VAR (dashed line) in the Svensson model. $\sigma_{\mathrm{MP}} = 1$.

the role of *MP* shocks, which is truly zero, is estimated to be positive. The price puzzle has warning proportions. The response of output to a misspecified *MP* shock is highly persistent, reflecting the fact that the misspecified *MP* shocks are negatively correlated with the true technology shocks.

In the second experiment a stochastic element is added to the behavior of the monetary authorities by setting $\sigma_{\mathrm{MP}} = 1$. The results are displayed in Fig. 2. The misspecified system is no longer VAR(1) when $\sigma_{\mathrm{MP}} > 0$. If a VAR is estimated on simulated data,[12] the Portmanteau test of residual correlation is first passed (at 5%) at four lags, so a VAR(4) is fitted to the misspecified system. If less than four lags are chosen, the misspecifications maintain the same qualitative pattern but become larger. The response of the interest rate to an *AD* shock is underestimated, again as expected. Responses to *MP* shocks are once again those that display the most obvious misspecifications. On the low-right corner, the std of the *MP* shock is overestimated. The price puzzle is substantial and can now be confronted with the true behavior of inflation in response to an *MP* shock. As for the response of output to an *MP* shock, both its size and its persistence are overestimated.

[12]The misspecified impulse responses are obtained by fitting a VAR on data generated by the *DGP*. The first 500 observations are not used in estimation. The VAR is estimated on 10 000 observations to eliminate parameter uncertainty.

Eq. (9) shows that the misspecified system takes a VAR(1) form if $\sigma_{\mathrm{MP}} = 0$. When $\sigma_{\mathrm{MP}} > 0$, driving the standard deviation of any other shock to zero does not produce this effect: the system remains a VARMA(2,1). Simulations show (not reported) that the effects of *MP* shocks are correctly retrieved if $\sigma_{\mathrm{AD}} = 0$, but only if the VAR has a generous lag structure. Otherwise, we still observe a price puzzle. Setting σ_{CP} to zero does not eliminate the puzzle.

3. Robustness of the main results and forward-looking models

The adoption of a simple backward-looking model has allowed us to derive analytical solutions. This section shows that the conclusions extend to a broader class of models, which include microfounded and forward-looking models. We assume that potential output is stochastic and exogenous, although it need not be a Gaussian AR process. For example, potential output (or its first differences) may be a constant which is subject to occasional breaks.

We will distinguish between two situations. In the first, the structure of the model is such that the occurrence of a price puzzle in a VAR omitting the output gap can be proven. These models possess certain properties, mainly that monetary policy affects output with a lag and inflation with a longer lag. In the second, these properties do not hold: they may be just rough approximations or not even that, so we cannot be certain to find a price puzzle, and must resort to numerical simulations.

3.1. Sufficient conditions for a price puzzle

The overestimation of *MP* shocks and the occurrence of a price puzzle when the output gap is omitted from the VAR can be established analytically (see Giordani (2001) for the proof) in a class of New–Keynesian models whose reduced form resembles the Svensson model in certain key features: (i) inflation responds with a lag and positively to the output gap, (ii) the monetary policy authority can affect output with a lag and inflation with a longer lag, (iii) the output gap appears with a positive coefficient in the monetary policy function and (iv) y_t^g cannot be reduced to a linear combination of a constant, y_t, π_t, and variables dated t–1 or earlier. The overestimation of *MP* shocks follow from the fact that the output gap appears in the true policy function. The price puzzle generates from the fact that the policy instrument appears in the misspecified inflation equation to pick up the role of the omitted output gap.

The key assumption that *MP* shocks affect output with a lag and inflation with a longer lag, although not uncontroversial, is commonly incorporated in models for monetary policy analysis (like the MPS macromodel of the US) and in both theoretical and applied work. For example, Christiano et al. (2000) include this feature of the transmission mechanism among the 'stylized facts' produced by the VAR literature, and Ball (1999) and Svensson (2000) suggests that it is common central-bank experience. Microfounded, forward-looking models that either impose or estimate the timing of monetary policy implied by Svensson (1997) include Clarida et al. (1999, Section 6), Svensson (2000) and Christiano et al. (2001).

3.2. Price puzzles in other models

The conditions for a price puzzle stated above are not necessary. For example, many economists believe that the peak responses of output and inflation following an *MP* shock occur after several months, but do not believe that there is no reaction at all within a month or a quarter. In other words, one's prior may be that conditions (i)–(iv) hold only as a rough approximation. Intuition then suggests that the main results should go through as long as the initial movement in inflation is sufficiently small.

To test this conjecture, we repeat the experiment summarized in Figs. 1 and 2 on a different model: a somewhat simplified version of Fuhrer and Moore (1995) estimated in Söderlind (1999). The model has an *IS* relation similar to (1), except that the output gap depends on the expected real yield on a 40-period bond, computed with RE using the term structure hypothesis (on a short interest rate, the policy instrument). The price level is determined by overlapping contracts for nominal wages. A positive output gap and expected future inflation both exercise upward pressure on the currently renegotiated wages. The model is forward looking, and inflation can react to both output and interest rate movements within the period, so conditions (i)–(ii) do not hold. However, inflation has a degree of inertia (an inverse function of the share of contracts renegotiated in each period).

To understand if a price puzzle is likely to show up in this model, we repeat the experiment of Section 2.2: the model is solved (which can no longer be done analytically) and used to generate a large sample of data; a VAR(4) is then estimated on the generated data, including output, inflation and the short interest rate, but excluding the output gap; shocks are identified with a Cholesky decomposition (order: output, inflation, interest rate) and the VAR responses are compared with the true responses. The model's parameters are those estimated by Söderlind (1999), except for the monetary policy rule, which is set to a deterministic Taylor rule with coefficients 0.5 and 1.5 on output gap and inflation.[13] Potential output is assumed to be an AR(1) process with coefficient 0.98 and $\sigma_{\mathrm{N}} = 0.5$.

An obvious difference with Section 2 is that in this model the VAR with a Cholesky identification cannot retrieve the exact responses even if it includes the output gap, since the DGP no longer has a triangular structure. However, the impulse responses of a VAR that also includes the output gap come to match the true responses rather closely (not reported). On the other hand, Fig. 3 shows that the misspecified VAR displays exactly the same problems found in the Svensson model: a sizable *MP* shock, which produces a persistent price puzzle and a very persistent negative response of output; following an *AD* shock, the response of output is too persistent, while inflation and the interest rate move too little.

[13] Söderlind (1999) estimates the model assuming a commitment solution, so the policy function also depends on Lagrange multipliers.

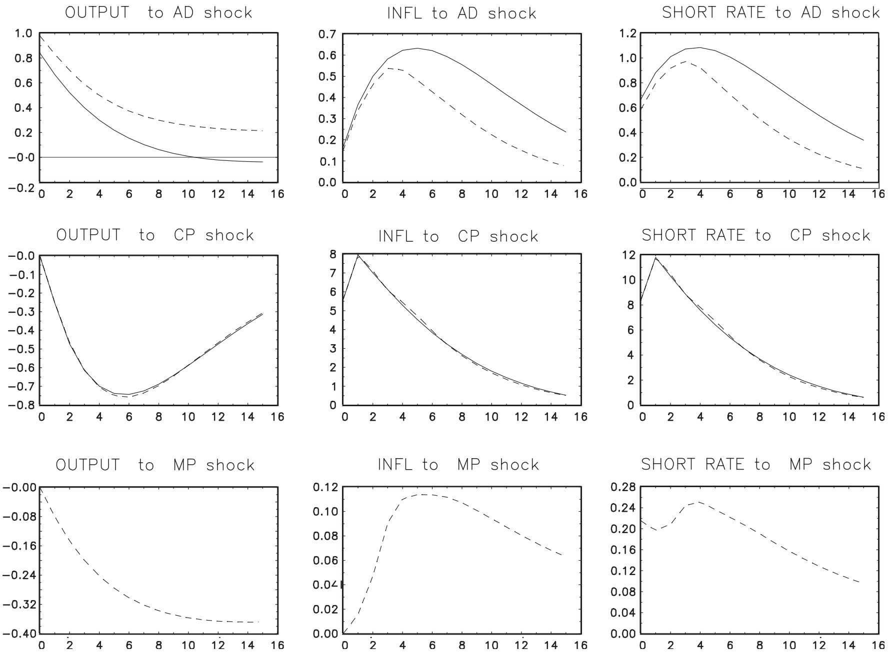

Fig. 3. True impulse responses (solid line) and impulse responses from misspecified VAR (dashed line) in the Fuhrer–Moore model.

It is harder to guess what could happen in a model with pervasive simultaneity and little inertia. We consider a simple New–Keynesian model (Euler equation for consumption/income, Calvo-style Phillips curve and Taylor rule) supplemented with a process for potential output

$$y_t = \mathrm{E}_t y_{t+1} - \frac{1}{\sigma}(i_t - \mathrm{E}_t \pi_{t+1}) + \varepsilon_t^{\mathrm{AD}}, \tag{23}$$

$$y_{t+1}^{\mathrm{N}} = \rho y_t^{\mathrm{N}} + \varepsilon_{t+1}^{\mathrm{N}}, \tag{24}$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \alpha_y y_t^g + \varepsilon_t^{\mathrm{CP}}, \tag{25}$$

$$i_t = \gamma_\pi \pi_t + \gamma_y y_t^g. \tag{26}$$

The parameters are set to the following values: $\sigma = 2$, $\beta = \rho = 0.98$, $\alpha_y = 0.5$, $\gamma_\pi = 1.5$, $\gamma_y = 0.5$. $\varepsilon_t^{\mathrm{AD}}, \varepsilon_t^{\mathrm{N}}, \varepsilon_t^{\mathrm{CP}}$ are all $nid(0, 1)$. Notice that the *IS* equation is now expressed in terms of output rather than of the output gap. There is a lot of simultaneity in this economy: every variable responds within the period to all shocks. Therefore, it is not surprising that the VAR with Cholesky identification gives a

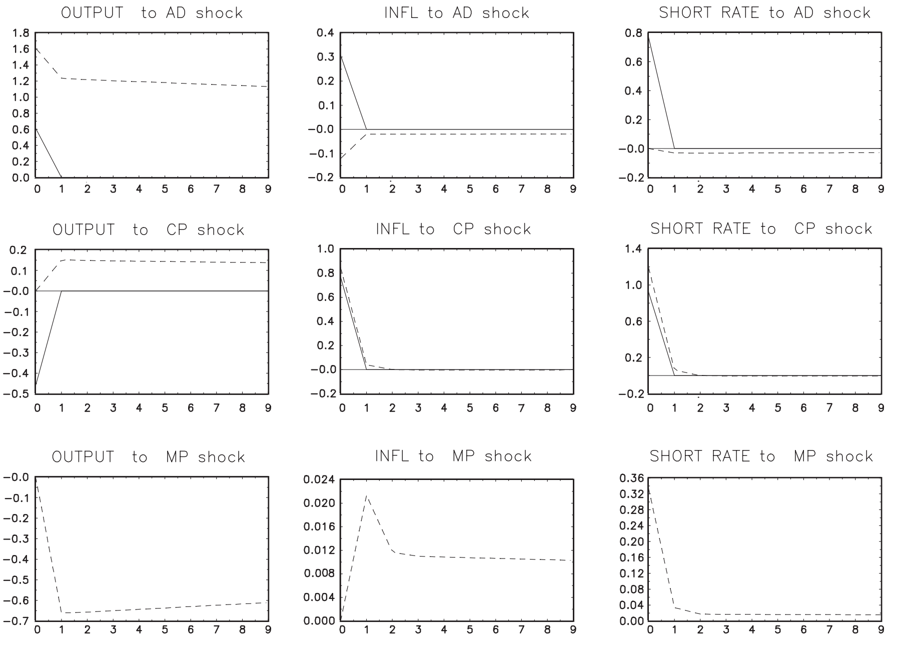

Fig. 4. True impulse responses (solid line) and impulse responses from misspecified VAR (dashed line) in the model of Clarida et al. (1999).

misleading picture of the economy.[14] However, as long as the interest rate is ordered last, the VAR does not find an *MP* shock if $\sigma_N = 0$ (not reported). When σ_N is set to one, the VAR recovers a sizable *MP* shock which produces a small but very persistent price puzzle and persistently lowers output (Fig. 4).

4. Solving the puzzle on US data

The strategy we adopt to test the hypothesis presented so far is:

1. Estimate a three variable VAR (the misspecified VAR) including: the log of real GDP, CPI inflation and the federal funds rate (same identification ordering).
2. Estimate the same VAR but with a measure of output gap rather than output ('VARgap').
3. Compare the impulse responses of the two VARs and check whether they behave as predicted by the analysis of Sections 2 and 3.

[14]Canova and Pina (1999) have an example of misspecification arising when the econometrician imposes short-run restrictions, while the DGP does not have enough restrictions on contemporaneous responses to identify any shock.

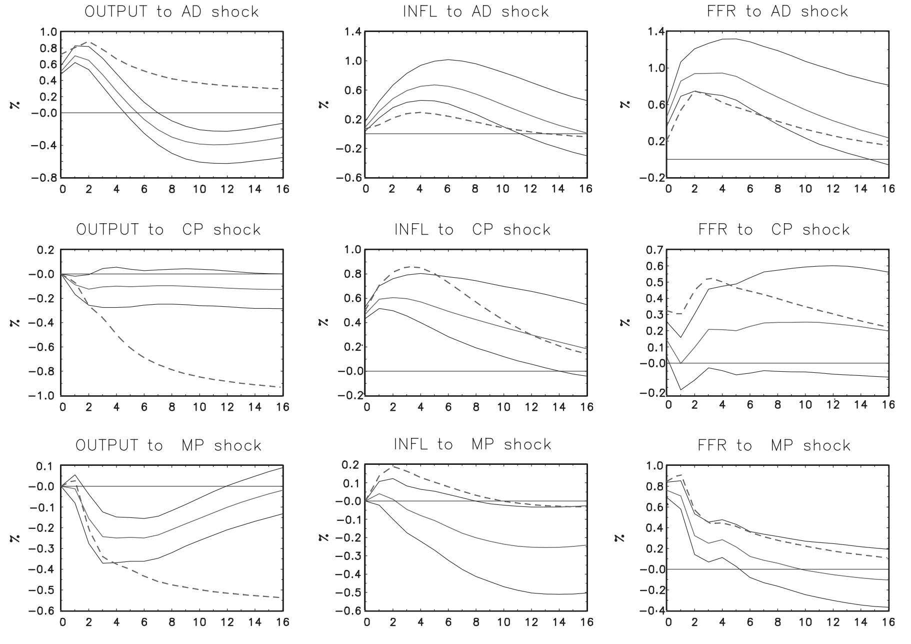

Fig. 5. VAR (real GDP, inflation and federal funds rate, dashed line) compared with those from VARgap (capacity utilization rather than real GDP, solid line). Ninety percent error bands for VARgap (computed as in Doan, 1992). Sample 1966Q1 2001Q3.

As a measure of the output gap we use the series of capacity utilization in the manufacturing sector built by the Federal Reserve Board.[15] A VAR(3) was estimated in all cases.[16] Fig. 5 plots impulse responses for the two VARs estimated on the sample 1966Q1–2001Q3 (unless otherwise stated all figures are produced with a VAR(3) on the same sample). Ninety per cent error bands for VARgap are included. Forecast error variance decomposition for VARgap and for the misspecified VAR are presented in Tables 1 and 2, respectively.

The reader is invited to compare Figs. 2 and 5: in all nine cases the theoretical model correctly predicts whether the impulse response of VARgap lies above or

[15]All series are from FRED. All series except the federal funds rate are s.a. Capacity utilization in the manufacturing sector and GDP at constant 1995 prices (logged) are available quarterly. The federal funds rate and CPI are aggregated by averaging monthly data. Inflation is the annual percentual increase in the CPI. Since capacity utilization is expressed as a percentage of full capacity for the manufacturing sector, a scale adjustment was used to account for the fact that industrial production (manufacturing) is more volatile than GDP. Therefore, the series used in estimation is capacity $\times 0.5$. The number 0.5 is the result of the following computations. We assume that the output gap for GDP (in logs) is a multiple of the output gap in manufacturing. That is $y_t^g = \alpha y_t^{g,m}$, implying that $\alpha = std(y_t^g)/std(y_t^{g,m})$. Using data on s.a. industrial production and computing the std of deviations from linear trends, we estimate $\alpha = 0.5$. A very similar result is obtained by comparing the standard deviations of the growth rates.

[16]The Schwarz and HQ criteria both select two lags for VARgap and, respectively, two and four lags for the misspecified VAR.

Table 1
Forecast error variance decomposition at selected horizons for VAR gap

Period	VD of capacity u.			VD of inflation			VD of FFR		
	AD	CP	MP	AD	CP	MP	AD	CP	MP
0	100	0	0	3.5 (3.1)	96.4 (3.1)	0	26.5 (6.2)	2.4 (2.2)	71.0 (6.3)
2	96.1 (2.8)	1.9 (2.2)	2 (1.9)	26.4 (8.2)	73.4 (8.2)	0.1 (0.7)	60.1 (7.5)	1.0 (1.5)	38.9 (7.2)
4	88.6 (7.2)	2.7 (3.9)	8.7 (4.9)	40.5 (10.9)	59.1 (10.9)	0.4 (1.5)	60.1 (7.5)	1.0 (1.5)	38.9 (7.2)
8	78.2 (7.2)	4.1 (5.3)	17.7 (6.8)	50.0 (14.1)	47.8 (13.6)	2.1 (3.1)	78.0 (8.4)	4.2 (5.8)	17.8 (6.3)
16	80.4 (6.7)	5.8 (5.4)	13.8 (5.8)	45.9 (16.1)	45.7 (14.5)	8.4 (7.5)	77.5 (10.3)	7.8 (9.1)	14.6 (6.1)

Standard errors within parentheses (obtained by Monte Carlo, as described in Doan, 1992).

Table 2
Forecast error variance decomposition at selected horizons for the misspecified VAR

Period	VD of real GDP			VD of inflation			VD of FFR		
	AD	CP	MP	AD	CP	MP	AD	CP	MP
0	100	0	0	1.8 (3.5)	98.2 (2.3)	0	5.0 (3.6)	12.0 (4.8)	82.9 (5.6)
2	94.3 (3.0)	3.5 (2.5)	2.1 (1.7)	4.3 (4.3)	92.1 (4.8)	3.5 (2.5)	28.2 (8.3)	12.4 (5.6)	59.4 (8.4)
4	80.0 (8.0)	11.9 (6.6)	8.0 (4.7)	7.1 (6.4)	89.8 (6.8)	3.1 (2.8)	35.6 (9.9)	18.5 (8.1)	45.8 (9.2)
8	52.9 (12.7)	32.7 (12.3)	14.4 (7.7)	8.0 (8.0)	89.7 (8.6)	2.2 (3.1)	38.4 (11.4)	23.2 (11.2)	38.3 (10.0)
16	29.0 (13.7)	51.8 (15.6)	19.1 (11.7)	7.4 (8.1)	90.6 (9.3)	2.0 (3.9)	38.0 (12.1)	27.4 (14.3)	34.6 (11.0)

Standard errors in parenthesis (obtained by Monte Carlo, as described in Doan 1992).

below the impulse response of the misspecified VAR. The following results are all consistent with the analysis of Section 2:

1. There is a huge price puzzle in the misspecified VAR: error bands (not reported) show that the response of inflation is significantly positive for several quarters. On the other hand, the response of inflation in VARgap is never significantly positive and is significantly negative for several quarters.
2. The response of the federal funds rate to an *AD* shock is higher in VARgap, even though the *AD* shock has a lower standard deviation.
3. The responses of output (gap) to all shocks are shorter lived in VARgap.
4. The std of *MP* shocks is 12% lower in VARgap, and the residual standard deviations of the inflation equation and of the federal funds rate equation (in reduced form) are 7% and 11% lower. VARgap has a dramatically higher ability to explain movements in inflation (refer to the variance decomposition of inflation).
5. Monetary policy looks much more endogenous as the percentage of the federal funds rate forecast error variance due to *MP* shocks is substantially reduced in VARgap.

6. The share of *MP* shocks in the variance decomposition of output gap in VARgap is less (one-half after 16 periods) than in the decomposition of output in the misspecified VAR. The share of *MP* shocks in the variance decomposition of output in the misspecified VAR grows with the forecast horizon, as predicted (the reason being that the labeled *MP* shocks are correlated with technology shocks). In contrast, *MP* shocks in VARgap display no such behavior (the result does not change at forecast horizons longer than 4 years).
7. The inflation equation in the two VARs take the form suggested by Section 2 (refer to Eqs. (3) and (14)). In VARgap, the lagged values of capacity utilization are highly significant, while the lags of the federal funds rate are insignificant (the F-test for their exclusion has p-value 0.39). In the misspecified VAR, the lagged values of output are insignificant (p-value 0.38) and lags of the federal funds rate figure prominently (p-value 0.02). The signs of significant variables are those suggested by theory in all cases.

4.1. Robustness of the results

This section discusses the robustness of the results, focusing on the response of inflation to a monetary policy shock.

Future information: The algorithm employed by the Fed to construct capacity utilization is quite complex.[17] In synthesis: a measure of capacity (note: not capacity utilization) is available once a year for the fourth quarter. Quarterly and monthly data on capacity are obtained by interpolation, and quarterly and monthly data on capacity utilization are obtained by dividing industrial production indexes (available monthly) by the interpolated data. This procedure gives rise to two concerns, which we discuss in turn.

The first concern relates to the fact that capacity utilization for the first three quarters of each year contains information about capacity in the fourth quarter, because of the interpolation procedure. It is then possible that capacity utilization proxies for future output and thus for future inflation, helping to solve the puzzle because of the reasons exposed in Sims (1992). To examine this possibility, we added the four-quarter lead of real GDP to the misspecified VAR, so that in fact three lags and four leads of output are available in the VAR, but this did not mitigate the puzzle.

The second concern is that we expect the interpolated data to contain rather large measurement errors, particularly at monthly frequencies. Measurement errors, however, act to increase the price puzzle, as we discuss next.

How much robustness should we expect?: Let us assume that the DGP is given by the Svensson model (Eqs. (1)–(4)). There is a large number of potential empirical counterparts for both inflation and the output gap, and the econometrician will have to make a choice concerning these variables. Can we expect the results to be relatively insensitive to this choice? Unfortunately not. We consider inflation and the output gap in turn.

[17] Detailed information is available at www.federalreserve.gov/releases/G17/cap_notes.htm.

Let us say that the measure of inflation used by the econometrician, denoted π_t^*, is not the same as the measure of inflation π_t which belongs to the DGP. Specifically, assume that

$$\pi_t^* = \pi_t - u_t, \quad u_t \sim nid(0, \sigma_u^2). \tag{27}$$

The error term u_t could represent a measurement error (so π_t is the true inflation level). Alternatively, π_t^* could be an accurate measure of inflation, and in the model given by Eqs. (1)–(4), π_t would then be an underlying, 'core' inflation level, devoid of the highest frequency components of π_t^*. It is then easy to show that the VAR estimated using π_t^* will display a price puzzle as long as $\sigma_u^2 > 0$, and that the size of the puzzle is increasing in σ_u^2. Simply rewrite Eq. (3) in terms of π_t^*

$$\pi_{t+1}^* = \pi_t^* + \alpha_y y_t^g + u_t + (\varepsilon_{t+1}^{\mathrm{CP}} - u_{t+1}), \tag{28}$$

and (using the Taylor rule) notice that $\mathrm{E}[(i_t - \mathrm{E}(i_t|\pi_t^*, y_t^g)), u_t] = \gamma_\pi \sigma_u^2$, so in a regression of π_{t+1}^* on π_t^*, y_t^g and i_t, i_t will appear with a positive coefficient and thus generate a puzzle. Whereas in Section 2 the interest rate appears in the inflation equation because it contains additional information on the omitted output gap, here it appears because it contains information on the omitted core inflation π_t.

Turning to the output gap now, we can expect measurement errors to result in a price puzzle for similar reasons: if the output gap is measured imprecisely, the interest rate will contain additional information on the true level of the output gap and will therefore appear in the inflation equation. Formally, let

$$y_t^{g*} = y_t^g - u_t, \quad u_t \sim nid(0, \sigma_u^2), \tag{29}$$

where y_t^g is the true output gap and y_t^{g*} is the measure used by the econometrician. The inflation equation can be rewritten as

$$\pi_{t+1} = \pi_t + \alpha_y y_t^{g*} + \alpha_y u_t + \varepsilon_{t+1}^{\mathrm{CP}}, \tag{30}$$

and $\mathrm{E}[(i_t - \mathrm{E}(i_t|\pi_t^*, y_t^g)), u_t] = \gamma_\pi \sigma_u^2$, implying once again a price puzzle.

Simulations on the model (not reported) show that we can in fact generate a large and prolonged puzzle by adding a measurement error to either inflation or the output gap, for reasonable (below one) values of σ_u^2. Similarly, the quarterly VARgap estimated on real data also generates a sizable puzzle if random shocks are added to inflation or capacity utilization prior to estimation (not reported).

This warrants a few remarks. If the analysis of Section 2 is correct, we are entitled to expect that any VAR which includes a reasonable proxy for the output gap will reduce the price puzzle, but we cannot demand that all such proxies perform similarly. For similar reasons, among VARs that do not consider an explicit measure of the output gap, we expect those that include highly cyclical variables (unemployment, investments, asset prices and so on) to show a smaller puzzle than those that do not, but not necessarily to eliminate it. Finally, notice that a puzzle due to *i.i.d.* measurement errors is reduced by time aggregation (by the law of large numbers, the time-averaged π^* and y^{g*} converge to the time-averaged π and y^g), so we can expect the choice of which inflation and output gap measure to use to be more important at monthly than at quarterly frequencies.

Lags, identification, and inclusion of other variables: We checked the robustness of the results to different lag lengths (ranges 1–8), using the log of the CPI (in levels), and using GDP deflator inflation instead of CPI inflation. Switching capacity and inflation in the identification ordering also has little effect on both impulse responses and variance decompositions, as predicted by the model. Adding money (M2, logged) also changes little (ordered either before or after the federal funds rate). Of course, there is no reason why output should be excluded from the VAR as long as a measure of output gap is also included. All results are remarkably consistent if real GDP is added to VARgap. The responses of capacity and of real GDP to *AD*, *CP* and *MP* shocks are nearly identical.[18] The same is true using industrial production rather than GDP.

Sample: The results are also robust to starting the sample in 1980Q1. Several researchers[19] have noticed that a commodity price index does not solve the price puzzle on the pre-1979 sample. VARgap is not as robust on the 1966–1979 sample, in the sense that some specifications do produce a sizable (though generally not significant) price puzzle. Including both output and the output gap reduces the puzzle, possibly because capacity utilization is a less efficient proxy of the output gap in this subsample. For purposes of comparison with previous literature, we estimate a VAR(4) including capacity, real GDP (in logs), the GDP deflator (in logs) and the federal funds rate on the sample 1966Q1–1979Q1. This specification produces 'well-behaved' responses (refer to Fig. 6).

Alternative measures of the output gap: The following alternative measures of output gap have been considered: log deviations from a linear and from a quadratic trend, the CBO measure of output gap, unemployment and the cycle component of HP filtered log output ($\lambda = 1600$).[20] Notice that not all these measures are compatible with the assumption of stochastic potential output made in the theoretical analysis, but they have been tried because they are commonly used. The results are sensitive to the choice of filter, and some combinations of lag length and filter do not eliminate the price puzzle. However, all measures of output gap improve the VAR performance, in the sense that the price puzzle and the other allegedly spurious results are always sizably reduced. Capacity utilization gives the best fit in both the inflation and the interest rate equation.

Monthly data: Capacity utilization in the industrial sector is available monthly from 1967, so a monthly VARgap can be estimated and compared with a misspecified VAR which includes industrial production rather than GDP. This VAR

[18]Section 2 showed that, in the theoretical framework, the *MP** shocks retrieved from the misspecified VAR are positively correlated with the true *AD* shocks and negatively correlated with the true technology shocks. Using technology shocks (from GDP, ordered second, as suggested by the model) and *AD* shocks taken from the four variable VAR and *MP** shocks from the misspecified VAR, this prediction can be tested and is in fact correct: $corr(\varepsilon_{\rm AD}, \varepsilon^*_{\rm MP}) = 0.35$, $corr(\varepsilon_{\rm N}, \varepsilon^*_{\rm MP}) = -0.32$.

[19]See, for instance, Hanson (2000) and Barth and Ramey (2001).

[20]The HP filter is two sided, therefore the filtered data should not be used in regression analysis, since they will lead to inconsistent estimates. However, the same is true, strictly speaking, of linear and quadratic detrending, which are commonly used, and some researchers may nonetheless be curious about the results given by both filters.

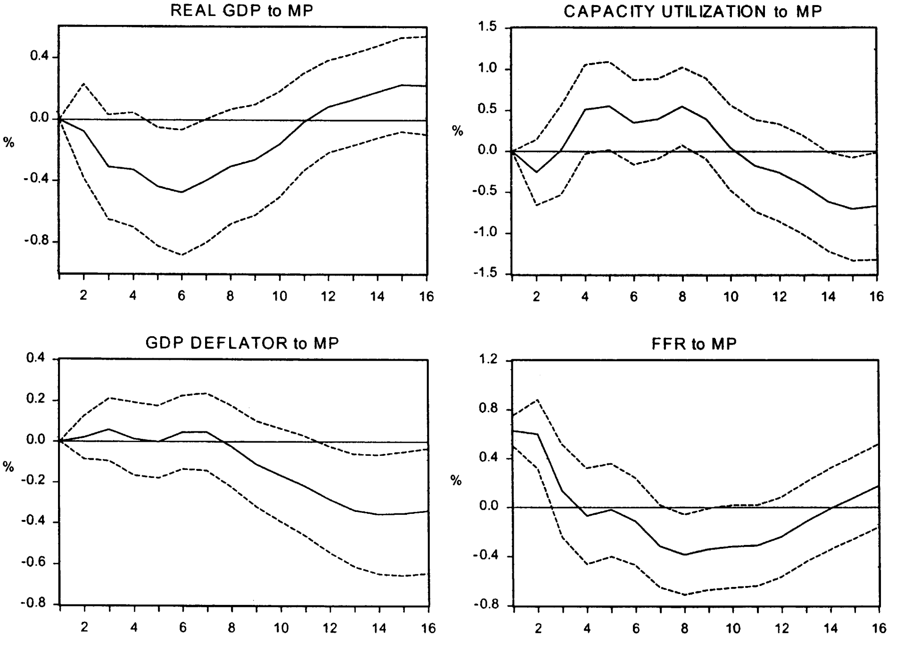

Fig. 6. Responses to an *MP* shock, with ±2 std error bands (analytical). VAR(4) including: capacity, log real GDP, log GDP deflator, federal funds rate. Sample 1966Q1–1979Q1.

(estimated with 6 lags on the sample 1967–2001) shows a price puzzle (see Fig. 7a): inflation is positive for the first year. If instead of annual inflation we use the CPI in log levels, the puzzle is even larger (not reported), although the misspecified VAR always fares substantially worse.

Measurement errors are the only explanation we can offer which rationalizes the difference in results between monthly and quarterly data,[21] although no doubt there are others, less favorable to the thesis of this paper. In order to provide some backing for this interpretation, we consider a measure of core inflation, namely annual inflation built from CPI less food and energy.[22] We first notice that this results in a substantial improvement in the fit of the federal funds rate equation. A likelihood ratio test performed on this equation of the VAR, augmented with six lags of core inflation, suggests that the standard measure of inflation is redundant (p-value of the F-statistic 0.26), while core inflation is not (p-value 0.003).

When core inflation is added to VARgap (same sample and lag structure), the price puzzle is smaller and shorter lived (see Fig. 7b). On the other hand, core inflation does little to solve the price puzzle in either quarterly or monthly VARs if

[21]Refer to the heading 'How much robustness should we expect?', earlier in this section.
[22]Source: FRED II. Series ID: CPILFESL.

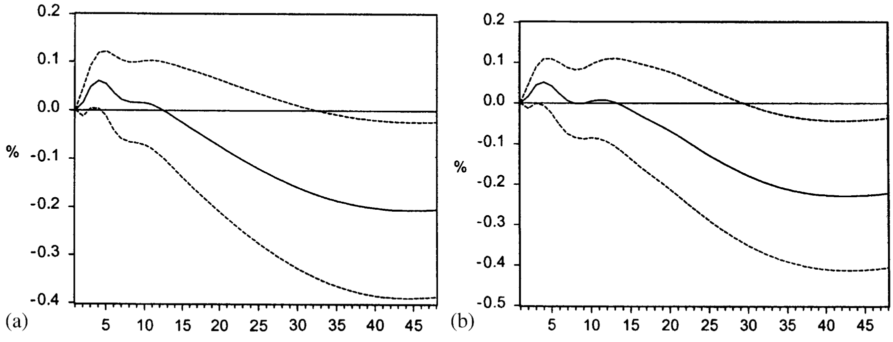

Fig. 7. Monthly data. (a) Response of annual inflation to an *MP* shock in VARgap. (b) Response of annual inflation to an *MP* shock in a four variable VAR (VARgap plus core inflation, federal funds rate ordered last).

capacity utilization is omitted (and real GDP or industrial production alone are used).

To summarize, introducing capacity utilization and core inflation into a monthly VAR reduce the price puzzle substantially, but does not eliminate it. The discrepancies between results based on monthly and quarterly data may be due to errors in measuring core inflation and the output gap, but it seems worthwhile to think of alternative explanations.

5. Why does a commodity price index reduce the price puzzle?

This section argues that the commodity price index mitigates the price puzzle mainly because it contains useful information about the output gap, not because it is a good predictor of future inflation (which it is). PcomCEE[23] and capacity utilization do tend to move together, as shown in Fig. 8 (correlation 0.58 on the sample 1970Q1–1998Q4).[24] PcomCEE also Granger causes both capacity utilization and real GDP: the *F*-tests (not reported) reject very strongly. Therefore it is clear that PcomCEE carries information on the state of the business cycle. Of course, it is also a predictor of inflation. But does it help solve the puzzle because it predicts inflation or because it proxies for the output gap?

The standard explanation implies that the price puzzle should disappear when a good leading indicator of inflation is included in the VAR. Since commodity prices have added value in predicting inflation, commodity prices should solve the price

[23] I call PcomCEE the index used by Christiano et al. (2000, 2001). This index does not display a trend (so it is meaningful to talk of its correlation with other stationary variables).

[24] The publication of the commodity price index used in Christiano et al. (2000, 2001) was discontinued around 1996. The series was used by the Department of Commerce (DOC) as a leading indicator. Data up to 1998Q4 have been kindly provided by Charles Evans, who constructed the last few data points following the procedure used by DOC.

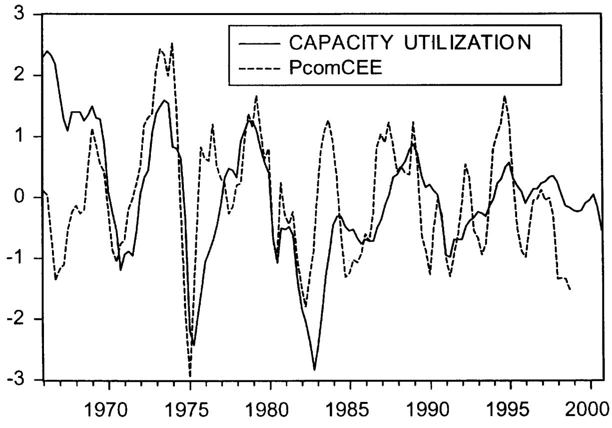

Fig. 8. Standardized capacity utilization and PcomCEE, quarterly.

puzzle. Since capacity utilization is a powerful predictor of inflation, the standard explanation can also rationalize its ability to solve the puzzle.[25] But other useful leading indicators of inflation should also go at least some way in solving the puzzle, if this theory is correct. However, Hanson (2000, Abstract) can "*find little correlation between an ability to forecast inflation and an ability to resolve the price puzzle*". Similar results are reported by Barth and Ramey (2001). Giordani (2001) explicitly includes accurate inflation forecasts in the VAR and concludes that they do not help to solve the puzzle. It should also be noted that not all variables that have been used as 'commodity prices' work as well. In fact, the price of intermediate goods or industrial prices (from FRED) do not perform nearly as well as the leading indicator used by Christiano et al. (2000, 2001).

As a further attempt to show that PcomCEE proxies for the output gap, we perform some statistical tests (the statements are for the null): (1) Once capacity is included in the Fed reaction function, PcomCEE is redundant. (2) Once PcomCEE in included in the Fed reaction function, capacity is redundant.

The testing procedures start with a model that nests both: the federal funds rate regressed on a constant, three lags of itself, contemporaneous and lagged (three lags) values of inflation, output, PcomCEE, capacity utilization (sample 1966:1–1998:4). The p-value for the F-statistic that commodity prices are redundant is 0.41. On the other hand, the hypothesis that capacity is redundant is clearly rejected (p-value 0.0000). However, if capacity is not included, PcomCEE shows up prominently in the equation (p-value 0.0055). Therefore, we conclude that PcomCEE reduces the price puzzle because it is a cyclical variable, not for its inflation forecasting ability.

[25] Stock and Watson (1999) find that the capacity utilization rate is the leading example of a stable predictor of inflation.

6. Conclusions

This paper argues that the finding of a positive response of inflation to a contractionary *MP* shock (price puzzle) in VARs designed for monetary policy analysis may not be due (or not only) to monetary authorities having better forecasts than those produced by the VAR. Rather, it may be due to the omission of an accurate measure of output gap in the VAR. This omission is shown to spuriously produce a price puzzle (and several other incorrect conclusions) in a common class of models. When the implications of the theoretical analysis are tested on quarterly US data, all the main predictions are confirmed. While other elements may be at play in generating the price puzzle, there is solid evidence to suggest that trying to capture the output gap pays good dividends in improving the performance of structural VARs.

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