Monetary Policy in DSGE Models

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Course Overview

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- **Main references**
  Monetary Policy, Inflation, and the Business Cycle (J. Gali)
  Interests and Prices (M. Woodford)
  Monetary Theory and Policy (C. Walsh)

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- With money, we have to distinguish between *real* and *nominal* variables, such as real vs. nominal wage, real vs. nominal interest rates, etc.

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- Alternative ways of doing it. Two most common are:

$$\underbrace{M_t \geq \alpha P_t C_t}_{\text{cash-in-advance (CIA)}} \qquad \text{or} \qquad \underbrace{U_t = U\left(C_t, H_t, \frac{M_t}{P_t}\right)}_{\text{money-in-utility (MIU)}}$$

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- Others are *transaction-cost* (cash is not subject to fees) and *shopping-time* (cash saves you time in shopping) models.

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- Shocks follow a *1st order autoregressive* (AR1) process

$$x_t = \underbrace{\rho x_{t-1}}_{\text{lagged term}} + \underbrace{\varepsilon_t}_{\text{noise}}, \quad \varepsilon_t \sim \text{iid} N\left(0, \sigma_\varepsilon^2\right), \quad \underbrace{|\rho| < 1}_{\text{persistence}} \tag{1}$$

$\Longrightarrow$ **k-period ahead forecast**: $E_t x_{t+k} = \rho^k x_t$

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- We start by analyzing money and monetary policy in a frictionless RBC-like model.

Baseline Model

Households

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$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, H_t, \frac{M_t}{P_t}\right) \tag{2}$$

subject to a budget constraint holding in every $t \geq 0$

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- Resources are spent to buy consumption $P_t C_t$, get cash new M_t, and buy new bonds B_t

- The household's problem is

$$\max L = E_0 \sum_{t=0}^{\infty} \beta^t \left[U\left(C_t, H_t, \frac{M_t}{P_t} \right) + \lambda_t (M_{t-1} + R_{t-1} B_{t-1} + W_t H_t + T_t - P_t C_t - M_t - B_t) \right] \quad (4)$$

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- To get transparent results (without loss of generality), let's consider the following utility specification (common in the literature)

$$U\left(C_t, H_t, \frac{M_t}{P_t} \right) = \frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{H_t^{1+\chi}}{1+\chi} + \nu \frac{\left(\frac{M_t}{P_t} \right)^{1-\gamma}}{1-\gamma}$$

NOTE: instead of putting *leisure* as a source of utility, I have put *hours worked* H_t as a source of dis-utility. Leisure would be given by $1 - H_t$

- First order conditions give

$$
\begin{aligned}
\text{FOC}(C_t) &: \quad C_t^{-\sigma} - \lambda_t P_t = 0 && (5)\\
\text{FOC}(H_t) &: \quad -\psi H_t^{\chi} + \lambda_t W_t = 0 && (6)\\
\text{FOC}(B_t) &: \quad -\lambda_t + \beta R_t E_t \lambda_{t+1} = 0 && (7)\\
\text{FOC}(M_t) &: \quad \frac{\nu \left(\frac{M_t}{P_t}\right)^{-\gamma}}{P_t} - \lambda_t + \beta E_t \lambda_{t+1} = 0 && (8)
\end{aligned}
$$

- Combining (5)-(6) gives the optimal **trade-off between working and taking leisure** (MB = marginal benefit, MC = marginal cost)

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- Combining (5)-(7) gives optimal **consuming vs saving trade-off**

$$\underbrace{\lambda_t}_{C_t^{-\sigma}/P_t} = \beta R_t \underbrace{E_t \lambda_{t+1}}_{C_{t+1}^{-\sigma}/P_{t+1}} \Longrightarrow \underbrace{C_t^{-\sigma}}_{\text{MB of consuming today}} = \underbrace{\beta R_t E_t \left[\frac{C_{t+1}^{-\sigma}}{\Pi_{t+1}} \right]}_{\text{MB of consuming tomorrow}} \tag{10}$$

where $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$ is gross inflation

- From the FOC of money (8):

$$\frac{\nu\left(\frac{M_t}{P_t}\right)^{-\gamma}}{P_t} = \lambda_t - \beta E_t\lambda_{t+1} = \lambda_t\left(1-\beta\frac{E_t\lambda_{t+1}}{\lambda_t}\right)$$
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- Hence

$$\nu\left(\frac{M_t}{P_t}\right)^{-\gamma} = \lambda_t P_t\left(\frac{R_t-1}{R_t}\right) \underset{\text{FOC}(C)}{=} C_t^{-\sigma}\left(\frac{R_t-1}{R_t}\right)$$

- Rearranging terms in previous equation gives the expression for **real money demand**

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- NOTE: if we had $\nu = 0$ (no utility from holding cash), then $\frac{M_t}{P_t} = 0$ (no demand for money)

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- NOTE: no capital accumulation, hence no physical investments
- Profit maximization:

$$\max_{H_t^f} P_t Z_t H_t^f - W_t H_t^f \underset{\text{FOC}(H_t^f)}{\Longrightarrow} P_t Z_t = W_t \Longrightarrow W_t^r \equiv \frac{W_t}{P_t} = Z_t \tag{13}$$

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- Assume (for simplicity) the central bank can control *real* money supply. Money market equilibrium requires

$$\underbrace{\frac{M_t{}^s}{P_t}}_{\text{money supply}} = \underbrace{(\nu C_t^{\sigma})^{\frac{1}{\gamma}} \left(\frac{R_t}{R_t - 1} \right)^{\frac{1}{\gamma}}}_{\text{money demand}} \tag{14}$$

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- choose R_t, and then let eq. (14) determine money supply $\frac{M_t}{P_t}^s$

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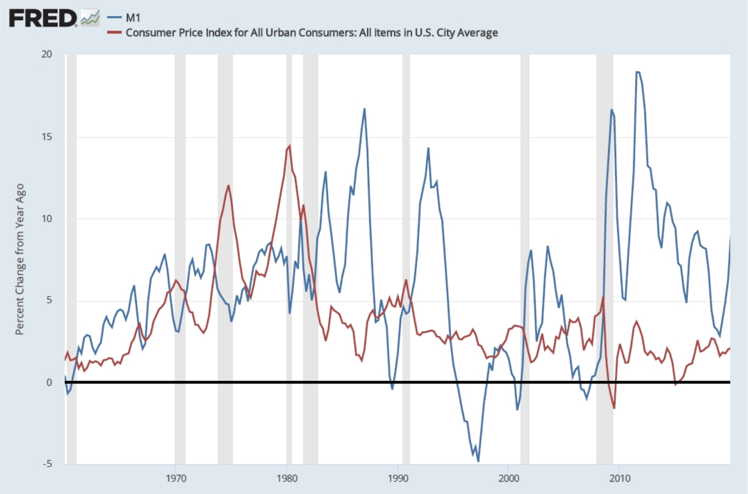
Avg. Money Growth: 1960-1980 = 5.1%; 1981-2019 = 6.1%

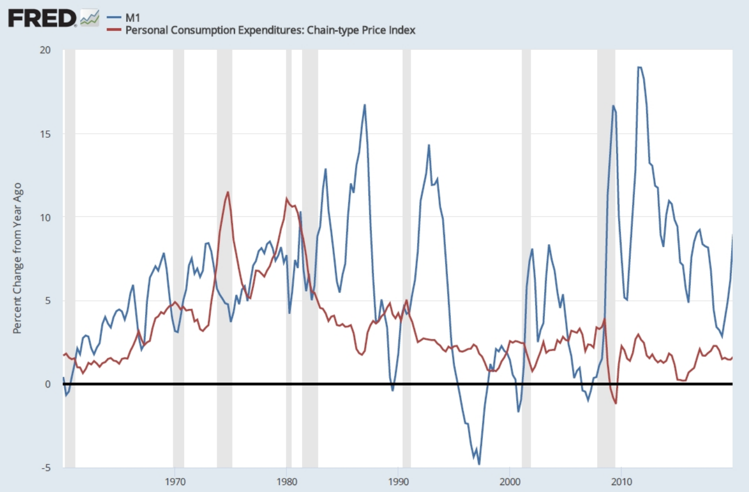
Avg. Inflation: 1960-1980 = 5.1%; 1981-2019 = 3%



Monetary Policy
Annual Money (M1) Growth and PCE Inflation

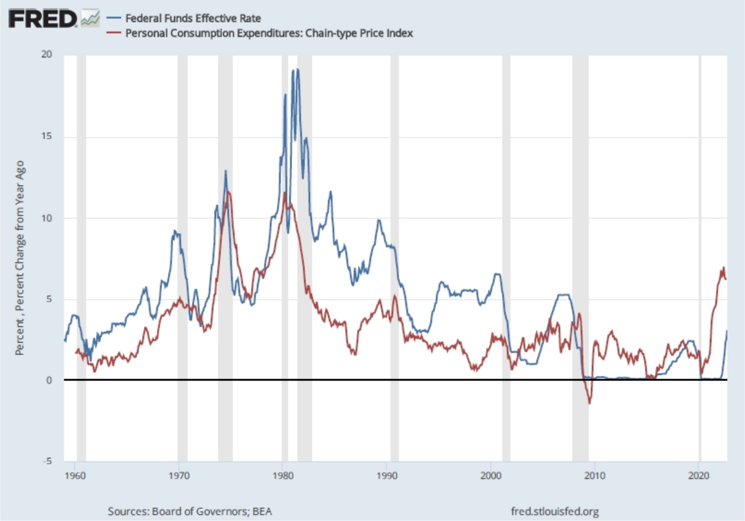
Avg. Money Growth: 1960-1980 = 5.1%; 1981-2019 = 6.1%

Avg. Inflation: 1960-1980 = 4.6%; 1981-2019 = 2.6%



Monetary Policy

Fed Funds Rate and PCE Inflation



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  - **Post-Volcker**: *elasticity* of $\frac{R_t}{R^*}$ to $\frac{\Pi_t}{\Pi^*}$ LARGER than 1(**hawkish Fed**) $\implies$ a 1% inflation increase (above target) triggers a *more than* 1% FFR increase (above target)

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Real Variables

- In equilibrium, all markets clear

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- **All variables driven by TFP!**

- Let $\bar{X}$ denote the steady state of any variable X_t.

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Fluctuations around the Steady State

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- Taking logs of both (16) and (18), with $y = \ln Y$ and $z = \ln Z$:

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- Subtract the latter from the former

$$y_t - \bar{y} = \frac{1+\chi}{\sigma+\chi} \left(z_t - \bar{z}\right) \tag{19}$$

- Let $\hat{x}_t$ be the percent deviation of any variable X_t from its steady state $\bar{X}$:

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- REMARK: since we require $\hat{x}_t$ to be *small*, this approach is valid for *small* fluctuations around the steady state
$\Longrightarrow$ model cannot handle large shocks (ex: financial crisis, covid, large rare events, etc.)

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$$\hat{y}_t = \underbrace{\frac{1+\chi}{\sigma+\chi}}_{\eta_{y,z}} \hat{z}_t \tag{21}$$

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- From equilibrium conditions (16) and (17) it then follows that

$$\hat{c}_t = \eta_{y,z}\hat{z}_t, \qquad \hat{h}_t = \underbrace{\frac{1-\sigma}{\sigma+\chi}}_{\eta_{h,z}} \hat{z}_t \tag{22}$$

NOTE: sign of $\eta_{h,z}$ depend on $\sigma \gtreqless 1$. So labor can be pro (counter) cyclical for σ less (more) than 1.

- Back to the Euler equation (10):

$$C_t^{-\sigma} = \beta E_t \left[\frac{R_t}{\Pi_{t+1}} C_{t+1}^{-\sigma} \right] \tag{23}$$

REMARK: similar to the Euler eq. of RBC model, with $\frac{R_t}{\Pi_{t+1}}$ (the *ex ante* real interest rate) replacing the real return from capital investment $R_{t+1}^k = Z_{t+1} F' \left(K_{t+1} \right) + 1 - \delta$

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- Take logs of both side of (23) (using lower case notation) and its SS counterpart

$$\begin{aligned} -\sigma c_t &= \ln \beta + r_t - \sigma E_t c_{t+1} - E_t \pi_{t+1} \\ -\sigma \bar{c} &= \ln \beta + \bar{r} - \sigma \bar{c} - \bar{\pi} \end{aligned}$$

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- Take the difference, recalling that $x_t - \bar{x} = \hat{x}_t$:

$$\hat{c}_t = E_t \hat{c}_{t+1} - \sigma^{-1} \left(\hat{r}_t - E_t \hat{\pi}_{t+1} \right) \tag{24}$$

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- We can re-write the latter as:

$$\hat{r}_t = E_t\hat{\pi}_{t+1} + \eta_{y,z}\sigma E_t\left(\hat{z}_{t+1} - \hat{z}_t\right) \tag{25}$$

- Assume TFP $\hat{z}_t$ follows a simple AR(1) process:

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Euler Equation and the Real Interest Rate

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- By eq. (25), the real interest rate just depends on TFP

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- REMARK: up to here, I have not introduced monetary policy!

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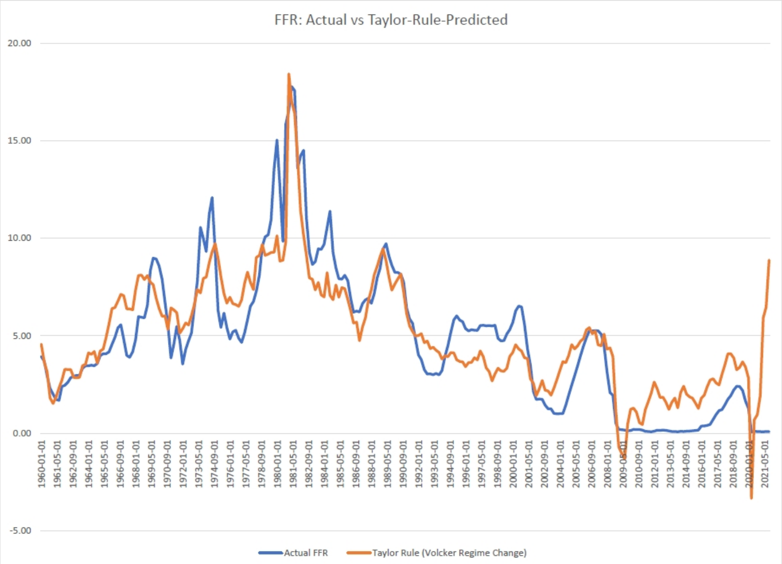
  2. $\hat{\upsilon}_t$ is a monetary policy shock capturing either policy mistakes or unsystematic (discretionary) monetary policy decision

Equilibrium

Actual vs Taylor Rule Predicted Interest Rate in the U.S.



- Plug Taylor rule (29) into (25):

$$\underbrace{\phi_\pi \hat{\pi}_t + \hat{v}_t}_{\hat{r}_t} = E_t \hat{\pi}_{t+1} + \eta_{y,z} \sigma \left(\rho_z - 1 \right) \hat{z}_t \tag{31}$$

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- Then, write it as a stochastic difference equation in inflation

$$\hat{\pi}_t = \underbrace{\frac{1}{\phi_\pi}}_{a} E_t \hat{\pi}_{t+1} + \underbrace{\frac{\eta_{y,z}\sigma\left(\rho_z - 1\right)}{\phi_\pi}}_{b<0} \hat{z}_t \underbrace{- \frac{1}{\phi_\pi}}_{d<0} \hat{\upsilon}_t \tag{32}$$

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- We can solve (33) by forward iteration or by method of undetermined coefficients

  Same result as long as $|a| < 1$ (we need $\phi_\pi > 1$).

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- Plug the latter back into (33):

$$\hat{\pi}_t = a\left(\eta_{\pi,z}\rho_z\hat{z}_t + \eta_{\pi,v}\rho_v\hat{v}_t\right) + b\hat{z}_t + d\hat{v}_t \tag{35}$$

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- Initial guesses $\eta_{\pi,z}$ and $\eta_{\pi,v}$ are correct if

$$a\eta_{\pi,z}\rho_z + b = \eta_{\pi,z} \implies \eta_{\pi,z} = b/\left(1 - a\rho_z\right)$$

$$a\eta_{\pi,v}\rho_v + d = \eta_{\pi,v} \implies \eta_{\pi,v} = d/\left(1 - a\rho_v\right)$$

- Forward (33) by one period and take expectations E_t (where $E_t E_{t+1}\hat{\pi}_{t+2} = E_t\hat{\pi}_{t+2}$ by law of iterated expectations):

$$E_t\hat{\pi}_{t+1} = aE_t\hat{\pi}_{t+2} + bE_t\hat{z}_{t+1} + dE_t\hat{\upsilon}_{t+1} \tag{37}$$

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- Repeat this procedure for $E_t \hat{\pi}_{t+j}$ (for $j = 2, 3, ...$) and assume that $\lim_{j \to \infty} a^j E_t \hat{\pi}_{t+j} = 0$ (indeed $\lim_{j \to \infty} a^j = 0$ since $a = \frac{1}{\phi_\pi} < 1$), we find

$$\hat{\pi}_t = b \sum_{j=0}^{\infty} a^j E_t \hat{z}_{t+j} + d E_t \sum_{j=0}^{\infty} a^j E_t \hat{v}_{t+j}$$

Equilibrium Solution under RE

Inflation under a Hawkish Taylor Rule

- Recognizing that $E_t \hat{z}_{t+j} = \rho_z^j \hat{z}_t$ and $E_t \hat{\upsilon}_{t+j} = \rho_\upsilon^j \hat{\upsilon}_t$, we can write

$$
\begin{aligned}
\hat{\pi}_t \quad &= \quad b \sum_{j=0}^{\infty} \left(a\rho_z\right)^j \hat{z}_t + d \sum_{j=0}^{\infty} \left(a\rho_\upsilon\right)^j \hat{\upsilon}_t \\
\Longrightarrow \quad & \hat{\pi}_t = \frac{b}{1 - a\rho_z} \hat{z}_t + \frac{d}{1 - a\rho_\upsilon} \hat{\upsilon}_t
\end{aligned}
$$

- Both methods give

$$\hat{\pi}_t = \frac{b}{1 - a\rho_z}\hat{z}_t + \frac{d}{1 - a\rho_v}\hat{v}_t$$

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- Using definitions of a, b and d in (32), inflation is

$$\hat{\pi}_t = -\frac{(1-\rho_z)}{\phi_\pi - \rho_z}\frac{\sigma(1+\chi)}{\sigma+\chi}\hat{z}_t - \frac{1}{\phi_\pi - \rho_z}\hat{v}_t \tag{39}$$

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  1. inflation responds *negatively* both to TFP, $\hat{z}_t$, and to the policy rate shock, $\hat{\upsilon}_t$;

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  3. a dovish Fed ($\phi_\pi \to \rho_z$) makes inflation extremely volatile

Equilibrium Solution under RE

Nominal Interest Rate under a Hawkish Taylor Rule

- Recall that the nominal interest rate is $\hat{r}_t = \phi_\pi \hat{\pi}_t + \hat{v}_t$. Plugging in the solution (39) for inflation

$$\hat{r}_t = -\phi_\pi \frac{(1-\rho_z)}{\phi_\pi - \rho_z} \frac{\sigma(1+\chi)}{\sigma+\chi} \hat{z}_t - \frac{\rho_v}{\phi_\pi - \rho_v} \hat{v}_t \quad (40)$$

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  Hence, as long as $\rho_v > 0$, a positive interest rate shock ends up lowering the interest rate as well (!!).

- It seems counter-intuitive, but there are two channels at work

$$\hat{r}_t = \underbrace{\phi_\pi \hat{\pi}_t}_{\text{systematic policy} \downarrow} + \underbrace{\hat{v}_t}_{\text{random policy} \uparrow} \tag{41}$$

Conclusions

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2. Only the nominal interest rate and inflation respond to monetary policy shocks
3. This is in contrast with VAR evidence showing real effects of MP
4. We need to introduce some nominal friction to create non-neutrality!
   $\Longrightarrow$ New Keynesian Model of Nominal Rigidities