The New Keynesian Model

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- Non-monetary RBC model (seen with Prof. Bagliano) and frictionless monetary model (seen with me) clearly not suitable to talk about non-neutrality and a stabilizing role for central banks
- Key source of neutrality: FULL PRICE FLEXIBILITY
  $\Longrightarrow$ inflation fully absorbs the impact of any nominal shock

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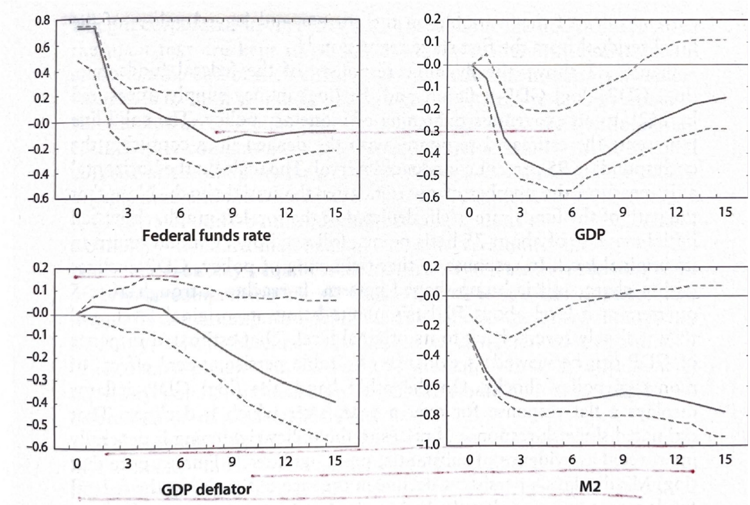
  - similar evidence for Euro Area
  - similar evidence for nominal wages (avg. duration around 1 year)

Source: Gali's Textbook



Source: Nakamura and Steinsson (Annual Rev. Econ, '13)

Table 1 Frequency of price change in consumer prices

	Median		Mean	
	Frequency (% per month)	Implied duration (months)	Frequency (% per month)	Implied duration (months)
Nakamura & Steinsson (2008)				
Regular prices (excluding substitutions 1988–1997)	11.9	7.9	18.9	10.8
Regular prices (excluding substitutions 1998–2005)	9.9	9.6	21.5	11.7
Regular prices (including substitutions 1988–1997)	13.0	7.2	20.7	9.0
Regular prices (including substitutions 1998–2005)	11.8	8.0	23.1	9.3
Posted prices (including substitutions 1998–2005)	20.5	4.4	27.7	7.7
Klenow & Kryvtsov (2008)				
Regular prices (including substitutions 1988–2005)	13.9	7.2	29.9	8.6
Posted prices (including substitutions 1988–2005)	27.3	3.7	36.2	6.8

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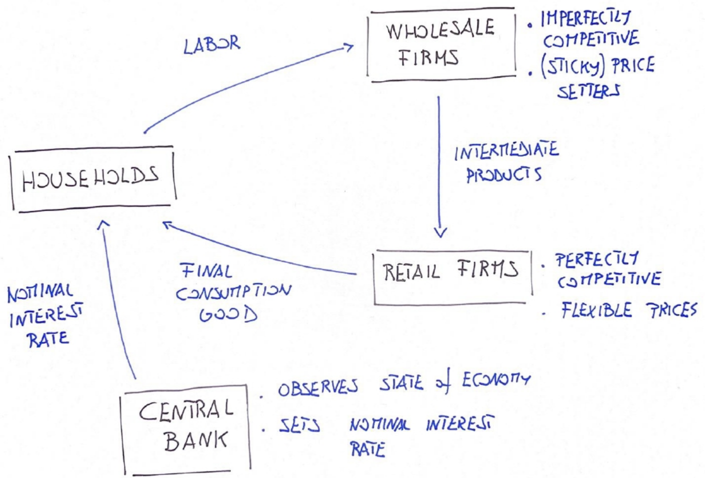
     $\Longrightarrow$ market power allows us to model them as *price makers*
  2. Though prices are set optimally, wholesale firms cannot adjust them at will due to resource costs (menu costs) or long-term contracts
     $\Longrightarrow$ aggregate price level will not fully absorb nominal shocks
     $\Longrightarrow$ some real quantities will have to adjust (non-neutral effects)

Overview

Key Elements



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  3. for $\epsilon \to \infty$, we have $Y_t = \int_0^1 Y_t(i) \, di$ (perfect substitutability)

- The firm chooses intermediate products $Y_t(i)$ to maximize profits

$$\max_{Y_t(i),\ i\in[0,1]} P_t \left[\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} - \int_0^1 P_t(i)\, Y_t(i)\, di \tag{1}$$

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- Its solutions gives **optimal demand of inputs**:

$$Y_t^d(i) \equiv Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\epsilon} Y_t \tag{2}$$

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- A zero profit condition (due to perfect competition) gives:

$$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}} \qquad (3)$$

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  - given TFP, technology will determine how much labor to hire

- Substituting $Y_t\left(i\right)=\left[\frac{P_t(i)}{P_t}\right]^{-\epsilon}Y_t$ and $H_t\left(i\right)=\frac{Y_t(i)}{Z_t}=\left[\frac{P_t(i)}{P_t}\right]^{-\epsilon}\frac{Y_t}{Z_t}$ in profits (5), and taking FOC with respect to $P_t\left(i\right)$, gives

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- If we stopped here, monetary policy would remain neutral since firms are still able to move prices freely

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Price Stickiness

Calvo Pricing

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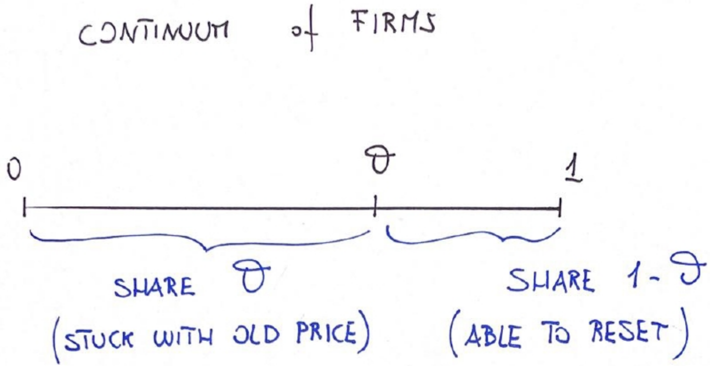
- REMARKS
  1. probability of being (or not being) able to reset the price is *history independent*
  2. newly set price $\tilde{P}_t(i)$ likely not aligned with optimal price $P_t^*(i)$



CONTINUUM of FIRMS
0
ϑ
1
SHARE ϑ
(STUCK WITH OLD PRICE)
SHARE 1-ϑ
(ABLE TO RESET)

- Recall the aggregate price (a.k.a. Consumer Price Index, CPI)

$$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon} \, di \right]^{\frac{1}{1-\epsilon}} \implies P_t^{1-\epsilon} = \int_0^1 P_t(i)^{1-\epsilon} \, di \qquad (8)$$

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- Skipping some technical details, **CPI** evolves as

$$P_t^{1-\epsilon} = (1-\theta)\,\tilde{P}_t^{1-\epsilon} + \theta P_{t-1}^{1-\epsilon} \tag{10}$$

- **CPI** motion is approximately (with $\hat{x}_t = \frac{X_t - \bar{X}}{\bar{X}} \approx \ln \frac{X_t}{\bar{X}}$):

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- Let gross inflation be $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ and assume $\bar{\Pi} = 1$ (zero steady state *net* inflation), we have that

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- Then

$$\begin{aligned} \hat{\pi}_t &\equiv \pi_t - \bar{\pi} = \underbrace{(p_t - \bar{p})}_{\hat{p}_t} - \underbrace{(p_{t-1} - \bar{p})}_{\hat{p}_{t-1}} \\ &\underset{\text{eq. (11)}}{=} (1-\theta)\left(\widehat{\tilde{p}}_t - \hat{p}_{t-1}\right) \end{aligned} \tag{12}$$

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Approximate Price Index and Inflation

- **CPI** motion is approximately (with $\hat{x}_t = \frac{X_t - \bar{X}}{\bar{X}} \approx \ln \frac{X_t}{\bar{X}}$):

$$\hat{p}_t = \theta \hat{p}_{t-1} + (1-\theta)\,\widehat{\tilde{p}}_t \tag{11}$$

- Let gross inflation be $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ and assume $\bar{\Pi} = 1$ (zero steady state *net* inflation), we have that

$$\pi_t \equiv \ln \Pi_t = p_t - p_{t-1}, \qquad \bar{\pi} \equiv \ln \bar{\Pi} = 0$$

- Then

$$\begin{aligned} \hat{\pi}_t \quad &\equiv \quad \pi_t - \bar{\pi} = \underbrace{(p_t - \bar{p})}_{\hat{p}_t} - \underbrace{(p_{t-1} - \bar{p})}_{\hat{p}_{t-1}} \\ &\underset{\text{eq. (11)}}{=} (1-\theta)\left(\widehat{\tilde{p}}_t - \hat{p}_{t-1}\right) \end{aligned} \tag{12}$$

- Hence, inflation occurs when the newly set price $\widehat{\tilde{p}}_t$ is above the average price of the previous period, $\hat{p}_{t-1}$

- Calvo's original set-up: firm i chooses the **optimal price** $\widehat{\tilde{p}}_t(i)$ to solve

$$\min_{\widehat{\tilde{p}}_t(i)} \frac{1}{2} E_t \sum_{k=0}^{\infty} (\theta\beta)^k \left[\widehat{\tilde{p}}_t(i) - \hat{p}^*_{t+k}(i)\right]^2 \tag{13}$$

that is, it seeks to minimize the discrepancy with the ideal price $\hat{p}^*_{t+k}(i) \equiv \ln P^*_{t+k}(i)$ (defined in eq. (6)). Note:

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- FOC of (13) with respect to $\widehat{\tilde{p}}_t(i)$ gives

$$E_t \sum_{k=0}^{\infty} (\theta\beta)^k \left[\widehat{\tilde{p}}_t(i) - \hat{p}^*_{t+k}(i)\right] = 0 \tag{14}$$

- Working out the summation

$$\widehat{\tilde{p}}_t(i) - \hat{p}_t^*(i) + \theta\beta\left[\widehat{\tilde{p}}_t(i) - E_t\hat{p}_{t+1}^*(i)\right] + + (\theta\beta)^2\left[\widehat{\tilde{p}}_t(i) - E_t\hat{p}_{t+2}^*(i)\right] + \ldots = 0 \qquad (15)$$

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- $\Longrightarrow$

$$\begin{aligned}&\widehat{\tilde{p}}_t(i)\underbrace{\left[1 + \theta\beta + (\theta\beta)^2 + ..\right]}_{1/(1-\theta\beta)\text{ since }|\theta\beta|<1} \\ &= E_t\left[\hat{p}^*_t(i) + \theta\beta\hat{p}^*_{t+1}(i) + (\theta\beta)^2\hat{p}^*_{t+2}(i) + ..\right] \qquad (16)\end{aligned}$$

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- $\Longrightarrow$ if able to reset, firm's optimal price chosen at t is

$$\widehat{\tilde{p}}_t(i) = (1 - \theta\beta) E_t \sum_{k=0}^{\infty} (\theta\beta)^k \hat{p}_{t+k}^*(i) \quad (17)$$

- Recall that the optimal (flex) price was $P_t^*(i) = P_t^* = \mu MC_t$ (see equation (6))
  $\Longrightarrow$ in a generic period $t+k$

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- Hence, the **optimal (sticky) price** is proportional to the expected PDV of future nominal marginal costs

$$\widehat{\tilde{p}}_t(i) = \widehat{\tilde{p}}_t = (1-\theta\beta)\, E_t \sum_{k=0}^{\infty} (\theta\beta)^k\, \widehat{mc}_{t+k} \tag{18}$$

NOTE: if $\theta = 0$, **optimal (flex) price** would be $\widehat{\tilde{p}}_t = \hat{p}_t^* = \widehat{mc}_t$.

New Keynesian Phillips Curve

- We start by writing the optimal pricing condition (18) recursively:

$$
\begin{aligned}
\widehat{\tilde{p}}_t &= (1-\theta\beta)\left[\widehat{mc}_t + \theta\beta E_t \widehat{mc}_{t+1} + (\theta\beta)^2 E_t \widehat{mc}_{t+2} + ..\right] \\
&= (1-\theta\beta)\,\widehat{mc}_t + \theta\beta\,(1-\theta\beta)\left[E_t \widehat{mc}_{t+1} + \theta\beta E_t \widehat{mc}_{t+2} + ..\right] \\
&= (1-\theta\beta)\,\widehat{mc}_t + \theta\beta E_t \underbrace{\left[(1-\theta\beta)\, E_{t+1} \sum_{k=0}^{\infty} (\theta\beta)^k\, \widehat{mc}_{t+1+k}\right]}_{\widehat{\tilde{p}}_{t+1}}
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\end{aligned}$$

- In summary:

$$\widehat{\tilde{p}}_t = (1-\theta\beta)\,\widehat{mc}_t + \theta\beta E_t \widehat{\tilde{p}}_{t+1} \tag{19}$$

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  1. Define **real marginal costs**

$$MC_t^r = \frac{MC_t}{P_t} \underset{\text{usual steps}}{\Longrightarrow} \widehat{mc}_t^r = \widehat{mc}_t - \hat{p}_t \Longrightarrow \widehat{mc}_t = \widehat{mc}_t^r + \hat{p}_t \quad (20)$$

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- Plugging (20)-(21) into (19), simple algebra yields the **NKPC**

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \underbrace{\frac{(1-\theta)(1-\theta\beta)}{\theta}}_{\kappa} \widehat{mc}_t^r \quad (22)$$

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1. the coefficient on marginal costs, κ, is strictly decreasing in price stickiness θ: $\frac{\partial \kappa}{\partial \theta} < 0$
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$\Longrightarrow$ as prices get stickier (longer expected duration), firms respond less to current marginal costs, putting (relatively) more emphasis on expected future inflation
2. iterating forward (23),

$$\hat{\pi}_t = \kappa E_t \sum_{k=0}^{\infty} \beta^k \widehat{mc}^r_{t+k}$$

$\Longrightarrow$ it is enough to expect marginal cost to increase at some point in the future (even if very far) for inflation to move today

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Households

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- Letting $W_t^r \equiv \frac{W_t}{P_t}$ be the real wage, his optimal behavior is summarized by the following two relationships

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\begin{aligned}
\psi H_t^{\chi} &= W_t^r C_t^{-\sigma} \\
C_t^{-\sigma} &= \beta R_t E_t \left[\frac{C_{t+1}^{-\sigma}}{\Pi_{t+1}} \right]
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- Their approximation gives

$$\textbf{Labor Supply} : \hat{w}_t^r = \chi \hat{h}_t + \sigma \hat{c}_t \tag{24}$$

$$\textbf{Euler Equation} : \hat{c}_t = E_t \hat{c}_{t+1} - \sigma^{-1} \left(\hat{r}_t - E_t \hat{\pi}_{t+1} \right) \tag{25}$$

- Recall from firm's problem (see eq. (6)) that

$$MC_t^r = \frac{W_t^r}{Z_t} \underset{\text{usual steps}}{\Longrightarrow} \widehat{mc}_t^r = \hat{w}_t^r - \hat{z}_t \tag{26}$$

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$$C_t = Y_t \underset{\text{usual steps}}{\Longrightarrow} \hat{c}_t = \hat{y}_t, \quad \text{and} \quad Y_t = Z_t H_t \underset{\text{usual steps}}{\Longrightarrow} \hat{h}_t = \hat{y}_t - \hat{z}_t$$

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- We plug the latter back into the NKPC (23)

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa\, (\chi + \sigma)\, \hat{y}_t - \kappa\, (1 + \chi)\, \hat{z}_t \tag{27}$$

- If we use $\hat{c}_t = \hat{y}_t$ also in the Euler equation (25), we have the **equilibrium system** describing the dynamics of our economy around the steady state

$$\textbf{AD Curve} \quad : \quad \hat{y}_t = E_t \hat{y}_{t+1} - \sigma^{-1} \left(\hat{r}_t - E_t \hat{\pi}_{t+1} \right) \tag{28}$$

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- Let's look at them more closely.
- For now, let's take expectations $E_t \hat{y}_{t+1}$ and $E_t \hat{\pi}_{t+1}$ as given (of course, they are both endogenous...we'll deal with it later)

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- REMARK: in the baseline model changes in the *real interest rate* do not have direct impact on the NKPC

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- Define the output gap:

$$\hat{x}_t \equiv \hat{y}_t - \hat{y}_t^F \qquad \Longrightarrow \qquad \hat{y}_t = \hat{x}_t + \hat{y}_t^F \tag{34}$$

- Plug this into the NKPC

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa(\chi+\sigma)\underbrace{\left(\hat{x}_t + \hat{y}_t^F\right)}_{\hat{y}_t} - \kappa(1+\chi)\hat{z}_t$$

$$= \beta E_t \hat{\pi}_{t+1} + \kappa(\chi+\sigma)\hat{x}_t + \kappa(\chi+\sigma)\frac{1+\chi}{\sigma+\chi}\hat{z} - \kappa(1+\chi)\hat{z}_t$$

$$= \beta E_t \hat{\pi}_{t+1} + \underbrace{\kappa(\chi+\sigma)}_{\kappa_x}\hat{x}_t \qquad (35)$$

- Plug this into the NKPC

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- This is a dynamic version (because of $\beta E_t \hat{\pi}_{t+1}$ term) of the original equation Peter C. Phillips estimated on U.S. data to show inverse relationship between inflation and the rate of unemployment

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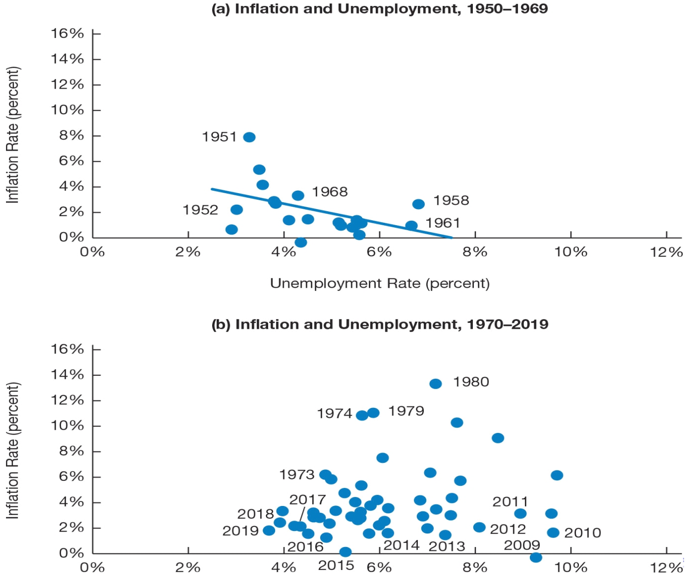
- This is a dynamic version (because of $\beta E_t \hat{\pi}_{t+1}$ term) of the original equation Peter C. Phillips estimated on U.S. data to show inverse relationship between inflation and the rate of unemployment
- Here the relationship is with the output gap, which is *negatively* related to unemployment (in the data, NOT here since there is no unemployment in the baseline NK model)

Equilibrium System

Visual Fit of the Phillips Curve



Equilibrium System

The AD Curve and the Output Gap

- We can re-write also the AD curve in output gap terms

$$\underbrace{\hat{x}_t + \hat{y}_t^F}_{\hat{y}_t} = E_t \underbrace{\left(\hat{x}_{t+1} + \hat{y}_{t+1}^F\right)}_{\hat{y}_{t+1}} - \delta\left(\hat{r}_t - E_t \hat{\pi}_{t+1}\right)$$

$$\Longrightarrow \quad \hat{x}_t = E_t \hat{x}_{t+1} - \delta\left(\hat{r}_t - E_t \hat{\pi}_{t+1}\right) + E_t \hat{y}_{t+1}^F - \hat{y}_t^F \qquad (36)$$

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- Since $\hat{y}_t^F = \eta_{y,z}^F \hat{z}_t$ in (32) and $E_t \hat{z}_{t+1} = \rho_z \hat{z}_t$:

$$E_t \hat{y}_{t+1}^F - \hat{y}_t^F = \eta_{y,z} \left(\rho_z - 1\right) \hat{z}_t \qquad (37)$$

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$$E_t \hat{y}^F_{t+1} - \hat{y}^F_t = \eta_{y,z}\left(\rho_z - 1\right)\hat{z}_t \tag{37}$$

- Then (recall $\delta = \sigma^{-1}$)

$$\hat{x}_t = E_t \hat{x}_{t+1} - \delta\left[\hat{r}_t - E_t \hat{\pi}_{t+1} - \sigma \eta_{y,z}\left(\rho_z - 1\right)\hat{z}_t\right] \tag{38}$$

- Define

$$\widehat{rr}_t^n \equiv \sigma\eta_{y,z}\left(\rho_z - 1\right)\hat{z}_t \tag{39}$$

This is the so-called **natural real interest rate** we found in the frictionless (flexible price) model
NOTE: if there was zero output gap in every period ($\hat{x}_t = E_t\hat{x}_{t+1} = 0$) the real interest rate would be equal to this

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- Then, the **final AD curve** is

$$\hat{x}_t = E_t\hat{x}_{t+1} - \delta\left(\hat{r}_t - E_t\hat{\pi}_{t+1} - \widehat{rr}_t^n\right) \tag{40}$$

- Let's summarize what we have

$$\hat{x}_t = E_t \hat{x}_{t+1} - \delta \left(\hat{r}_t - E_t \hat{\pi}_{t+1} - \widehat{rr}_t^n \right) \quad (41)$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa_x \hat{x}_t + \hat{u}_t \quad (42)$$

Equilibrium System

Cost-Push Shock and Need of Monetary Policy

- Let's summarize what we have

$$
\begin{aligned}
\hat{x}_t &= E_t\hat{x}_{t+1} - \delta\left(\hat{r}_t - E_t\hat{\pi}_{t+1} - \widehat{rr}^n_t\right) && (41)\\
\hat{\pi}_t &= \beta E_t\hat{\pi}_{t+1} + \kappa_x\hat{x}_t + \hat{u}_t && (42)
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$$

- NOTE: I have added an exogenous **cost-push shock** $\hat{u}_t$ to the AS curve

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  - $\hat{u}_t$ allows to capture pure supply side shocks. Ex: oil price/energy shocks (relevant today!), mark-up shocks
  - assume (as for other shocks) that

$$\hat{u}_t = \rho_u \hat{u}_t + \hat{\varepsilon}^u_t, \qquad \hat{\varepsilon}^u_t \sim \text{iid}N\left(0, \sigma^2_u\right), \; 0 \leq \rho_u < 1 \quad (43)$$

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- The system (41)-(42) includes 2 EXOGENOUS ($\widehat{rr}_t^n$ and $\hat{u}_t$) and 3 ENDOGENOUS variables
  $\Longrightarrow$ we need a 3rd equation for monetary policy

Equilibrium System

Solving the Model with an Instrumental Taylor Rule

- Assume the Fed adopts a Taylor rule

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$$\text{Taylor Rule I} : \hat{r}_t = \widehat{rr}_t^n + \phi_\pi \hat{\pi}_t + \phi_x \hat{x}_t + \hat{v}_t \quad (44)$$

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$$\text{Taylor Rule II}: \hat{r}_t = \phi_\pi \hat{\pi}_t + +\phi_x \hat{x}_t + \hat{v}_t \tag{45}$$

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- In both cases, we assume $\phi_\pi > 1$ and $\phi_x \geq 0$, with shock $\hat{v}_t$

$$\hat{v}_t = \rho_v \hat{v}_{t-1} + \hat{\varepsilon}_t^v, \qquad \hat{\varepsilon}_t^v \sim \text{iid} N\left(0, \sigma_v^2\right), \; 0 \leq \rho_v < 1 \tag{46}$$

capturing either Fed's discretionary decisions (independent from state of the economy) or, simply, policy mistakes

Equilibrium System

Solving the Model under Taylor Rule I (TR1)

- Fed adopts

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- Plugging the policy rule (47) into the system:

$$\hat{x}_t = E_t \hat{x}_{t+1} - \delta \left(\phi_\pi \hat{\pi}_t + \phi_x \hat{x}_t + \hat{\upsilon}_t - E_t \hat{\pi}_{t+1} \right) \qquad (48)$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa_x \hat{x}_t + \hat{u}_t \qquad (49)$$

REMARK: as $\widehat{rr}_t^n$ drops out of the system, this rule fully neutralizes the effects of TFP!

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REMARK: as $\widehat{rr}_t^n$ drops out of the system, this rule fully neutralizes the effects of TFP!

- **PROPOSITION**: if $\phi_\pi > 1$ and $\phi_x \geq 0$, the system has a unique Rational Expectations Equilibrium where

$$\hat{\pi}_t = \eta_{\pi,u} \hat{u}_t + \eta_{\pi,\upsilon} \hat{\upsilon}_t \tag{50}$$

$$\hat{x}_t = \eta_{x,u} \hat{u}_t + \eta_{x,\upsilon} \hat{\upsilon}_t \tag{51}$$

Equilibrium Solution

Finding the MSV by Method of Undetermined Coefficients (MUC)

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- The MUC is a "guess and verify" process
    1. Given initial guess $\left(\eta_{\pi,u}, \eta_{\pi,v}, \eta_{x,u}, \eta_{x,v}\right)$ we compute expectations

$$\begin{aligned}
\underbrace{E_t \hat{\pi}_{t+1}}_{\text{entering AD\&AS}} &= \eta_{\pi,u} \underbrace{E_t \hat{u}_{t+1}}_{\rho_u \hat{u}_t} + \eta_{\pi,v} \underbrace{E_t \hat{v}_{t+1}}_{\rho_v \hat{v}_t} \\
\underbrace{E_t \hat{x}_{t+1}}_{\text{entering AD}} &= \eta_{x,u} \underbrace{E_t \hat{u}_{t+1}}_{\rho_u \hat{u}_t} + \eta_{x,v} \underbrace{E_t \hat{v}_{t+1}}_{\rho_v \hat{v}_t}
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2. Plug them back into system (41)-(42)

3. Solve system for $\hat{x}_t$ and $\hat{\pi}_t$: both will be linear functions of $\hat{u}_t$ and $\hat{v}_t$

$$\hat{\pi}_t = N_{\pi,u}\hat{u}_t + N_{\pi,v}\hat{v}_t \quad (52)$$
$$\hat{x}_t = N_{x,u}\hat{u}_t + N_{x,v}\hat{v}_t \quad (53)$$

with the N coefficients depending on both structural parameters of the model $(\beta, \sigma, \chi, \kappa, \rho_u, \rho_v)$, policy parameters (ϕ_π, ϕ_x) and "guesses" $\left(\eta_{\pi,u}, \eta_{\pi,v}, \eta_{x,u}, \eta_{x,v}\right)$

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4. A REE is found by matching coefficients (initial guesses are confirmed)

$$N_{\pi,u} = \eta_{\pi,u}, \qquad N_{\pi,v} = \eta_{\pi,v}$$

$$N_{x,u} = \eta_{x,u}, \qquad N_{x,v} = \eta_{x,v}$$

- Once we have solved for $\hat{\pi}_t$ and $\hat{x}_t$, we can find all remaining quantities using (linear) equilibrium conditions

$$
\begin{aligned}
\text{Expected Output Gap} &: \quad E_t\hat{x}_{t+1} = \eta_{x,u}\rho_u\hat{u}_t + \eta_{x,v}\rho_v\hat{v}_t \\
\text{Expected Inflation} &: \quad E_t\hat{\pi}_{t+1} = \eta_{\pi,u}\rho_u\hat{u}_t + \eta_{\pi,v}\rho_v\hat{v}_t \\
\text{Output and Consumption} &: \quad \hat{y}_t = \hat{c}_t = \hat{x}_t + \hat{y}_t^F, \\
\text{Employment} &: \quad \hat{h}_t = \hat{y}_t - \hat{z}_t \\
\text{Nominal Rate} &: \quad \hat{r}_t = \widehat{rr}_t^n + \phi_\pi\hat{\pi}_t + \phi_x\hat{x}_t + \hat{v}_t \\
\text{Real Rate} &: \quad \hat{r}_t - E_t\hat{\pi}_{t+1}
\end{aligned}
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- The provided Excel file allows to study how the solution changes when we change the parameterization of the model, e.g. changes in price stickiness θ, IES δ, labor elasticity parameter χ, etc. More on parameterization below.

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- The provided Excel file allows to study how the solution changes when we change the parameterization of the model, e.g. changes in price stickiness θ, IES δ, labor elasticity parameter χ, etc. More on parameterization below.
- Usually, we perturb the model with one shock at a time
  Ex: we feed in a cost-push shock $\hat{u}_t$, but shut down the policy shock $\hat{v}_t = 0$ (and viceversa)

- Recall that $\hat{\pi}_t = \eta_{\pi,u}\hat{u}_t, \hat{x}_t = \eta_{x,u}\hat{u}_t$. Simple algebra yields

$$\eta_{\pi,u} = \frac{1-\rho_u}{(1-\rho_u)(1-\beta\rho_u)+\kappa_x\delta(\phi_\pi-\rho_u)} > 0$$

$$\eta_{x,u} = -\frac{\delta(\phi_\pi-\rho_u)}{(1-\rho_u)(1-\beta\rho_u)+\kappa_x\delta(\phi_\pi-\rho_u)} < 0$$

Cost Push Shock

Analytical Solution

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- Key takeaways (related to ongoing real world events)

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- Key takeaways (related to ongoing real world events)
    1. a positive cost push shock **raises inflation** but **lowers the output gap**
       Ex: gas price shock can generate stagflation (inflation + stagnation)
       INTUITION: as inflation increases, the CB hikes the interest rate (by Taylor rule)
       $\Longrightarrow$ a higher interest rate has a negative impact on real activity

Cost Push Shock

Analytical Solution

2. Should the central bank be **"more hawkish", i.e. larger** ϕ_π? Harsh trade-off!

$$\frac{\partial \left|\eta_{\pi,u}\right|}{\partial \phi_\pi} < 0, \qquad \text{and} \qquad \frac{\partial \left|\eta_{x,u}\right|}{\partial \phi_\pi} > 0$$

Raising the nominal rate more aggressively tames the pressure on inflation, but leads to a worse recession

INTUITION: for given increase in inflation, a larger ϕ_π means a more contractionary MP

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- as real activity declines, so does demand faced by firms, and hence their demand for workers
- this policy-driven decline in wages counteracts the initial cost push shock via AS curve: inflation increases by less!

3. Higher price stickiness makes both $\hat{\pi}_t$ and $\hat{x}_t$ respond more to the shock

$$\frac{\partial \left|\eta_{\pi,u}\right|}{\partial\theta} = \underbrace{\frac{\partial \left|\eta_{\pi,u}\right|}{\partial\kappa_x}}_{-} \underbrace{\frac{\partial\kappa_x}{\partial\theta}}_{-} > 0, \qquad \textit{and} \qquad \frac{\partial \left|\eta_{x,u}\right|}{\partial\theta} = \underbrace{\frac{\partial \left|\eta_{x,u}\right|}{\partial\kappa_x}}_{-} \underbrace{\frac{\partial\kappa_x}{\partial\theta}}_{-} > 0$$

NOTE: $\lim_{\theta\to 0}\eta_{\pi,u} = \lim_{\theta\to 0}\eta_{x,u} = 0$

$\Longrightarrow$Under flexible prices all that matters is TFP!

- A *quantitative* assessment requires numerical values for structural parameters

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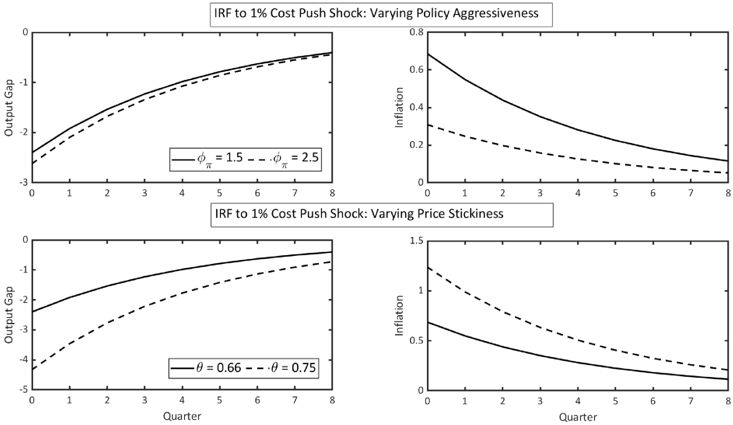
  - we want the model to fit perfectly long-run averages/trends
  - we want to assess how much it can explain of empirical fluctuations around those averages/trends (at quarterly frequency)
  - no econometric estimation!

- These are the key parameters and baseline values used in literature

$$\begin{aligned}
\beta &= 0.99 \Longrightarrow \quad \text{steady state real interest rate } \approx 4\% \\
\chi &= 1 \quad \Longrightarrow \quad \text{labor elasticity to wage } 1/\chi = 1 \\
\theta &= 2/3 \Longrightarrow \quad \text{avg. price duration } 1/\left(1-\theta\right) = 3 \text{ qrts} \\
\epsilon &= 8 \quad \Longrightarrow \quad \text{price markup } \mu = \frac{\epsilon}{\epsilon - 1} = 1.14 \\
\sigma &= 1 \quad \Longrightarrow \quad \text{risk aversion} = 1 \\
\rho_z &= 0.9 \qquad \rho_v = 0.5 \qquad \rho_u = 0.8
\end{aligned}$$



IRF to 1% Cost Push Shock: Varying Policy Aggressiveness
Output Gap
Inflation
ϕ_π = 1.5
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IRF to 1% Cost Push Shock: Varying Price Stickiness
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- Recall that $\hat{\pi}_t = \eta_{\pi,\upsilon}\hat{\upsilon}_t$ and $\hat{x}_t = \eta_{x,\upsilon}\hat{\upsilon}_t$. Simple algebra yields

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- Key takeaways

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- Key takeaways
  1. a positive interest rate shock (contractionary MP shock) **lowers both inflation and the output gap**
     INTUITION: a contractionary MP, $\hat{\upsilon}_t > 0$, affects negatively real activity via AD curve
     $\Longrightarrow$ Lower activity brings down goods demand by consumers, and then labor demand by firms
     $\Longrightarrow$ This drags down wages, which, in turn lead to lower inflation via AS curve

2. Higher price stickiness has opposite effects on $\eta_{\pi,v}$ and $\eta_{x,v}$

$$\frac{\partial\left|\eta_{\pi,v}\right|}{\partial\theta} = \underbrace{\frac{\partial\left|\eta_{\pi,v}\right|}{\partial\kappa_x}}_{+}\underbrace{\frac{\partial\kappa_x}{\partial\theta}}_{-} < 0, \qquad \text{and} \qquad \frac{\partial\left|\eta_{x,v}\right|}{\partial\theta} = \underbrace{\frac{\partial\left|\eta_{x,v}\right|}{\partial\kappa_x}}_{-}\underbrace{\frac{\partial\kappa_x}{\partial\theta}}_{-} > 0$$

INTUITION: contractionary MP, $\hat{v}_t > 0$, makes households less willing to buy goods from firms
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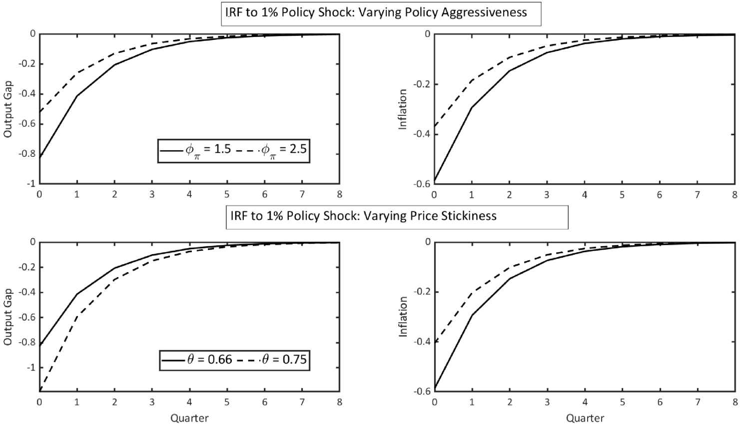
3. Response of output $\hat{y}_t$ is identical to output gap (since latter just driven by TFP)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- To assess the transmission of shocks to TFP, $\hat{z}_t$, we assume the Fed adopts TR2

$$\hat{r}_t = \phi_\pi \hat{\pi}_t + \hat{\upsilon}_t \tag{54}$$

TFP Shock

Solving the Model under Taylor Rule II (TR2)

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- We need to find $\left(\eta_{\pi,z}, \eta_{\pi,z} \right)$

- Following similar logic of cost-push and policy shock, we find

$$\eta_{\pi,z} = -\frac{\kappa_x (1-\rho_z)}{(1-\rho_z)(1-\beta\rho_z)+\kappa_x\delta(\phi_\pi-\rho_z)}\frac{1+\chi}{\sigma+\chi} < 0$$

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  Inflation: higher TFP $\Longrightarrow$ lower marginal costs $\Longrightarrow$ firms cut prices

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- Both inflation and the output gap respond negatively to a TFP shock
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- If we let $\kappa_x \to \infty$ (flex prices, RBC), we will get same coefficients found in frictionless model

$$\eta_{\pi,z} \to \frac{1-\rho_z}{\delta(\phi_\pi-\rho_z)}\frac{1+\chi}{\sigma+\chi} \quad \text{and} \quad \eta_{x,z} \to 0$$

- Since TFP is the main driver of fluctuations in a frictionless RBC model, it is interesting to look at output and hours worked

$$\hat{y}_t = \underbrace{\hat{x}_t}_{\eta_{x,z}\hat{z}_t} + \underbrace{\hat{y}_t^F}_{\eta_{y,z}^F\hat{z}_t} = (\underset{-}{\eta_{x,z}} + \underset{+}{\eta_{y,z}^F})\hat{z}_t$$

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- Weaker response to TFP compared to frictionless model: $\eta_{y,z}$ is strictly increasing in κ_x
- A positive TFP increases $\hat{y}_t^F$ more than $\hat{y}_t$, so the output gap drops!

TFP Shock

Comparrison with RBC Model

- For what concerns hours

$$\hat{h}_t = \hat{y}_t - \hat{z}_t = \eta_{y,z}\hat{z}_t - \hat{z}_t = \left(\eta_{y,z} - 1\right) = \left(\underset{-}{\eta_{x,z}} + \underbrace{\eta^F_{y,z} - 1}_{\frac{1-\sigma}{\sigma+\chi}}\right)\hat{z}_t$$

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- Empirical evidence: hours respond *negatively* to TFP
  $\Longrightarrow$ since $\eta_{x,z} < 0$, NK model can fit that!

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