Optimal Monetary Policy

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Overview

Taylor Rule vs Optimal Monetary Policy

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- QUESTION: does a Taylor rule represents an *optimal monetary policy*?
  $\Longrightarrow$ we must first to take a stand on what *optimal* means
  $\Longrightarrow$ we must identify a policy criterion the central banker is trying to optimize

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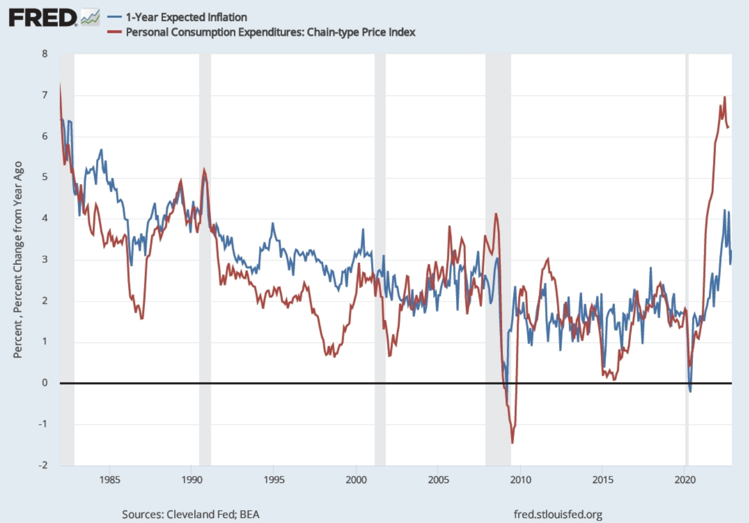
- It can also shield the economy from **belief-driven shocks** to inflation itself
  Ex: agents' pessimistic belief that inflation will move above target
- A Taylor rule with $\phi_\pi > 1$ can rule out this possibility by "convincing" economic agents (firms or households) that the initial pessimistic belief is unjustified

Inflation Expectations and the Taylor Principle

Anchored and Un-Anchored Inflation Expectations



- Suppose agents come to believe that inflation will be *persistently* above target (steady state)

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- From Euler equation for consumption, with $\hat{r}_t = \phi_\pi \hat{\pi}_t$

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- For given $E_t \hat{c}_{t+1}$,

$$\hat{c}_t \begin{cases} > 0 \text{ (higher consumption)} & \text{if} \quad \phi_\pi < 1 \text{ (dovish Fed)} \\ < 0 \text{ (lower consumption)} & \text{if} \quad \phi_\pi > 1 \text{ (hawkish Fed)} \end{cases}$$

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A Dovish Fed

- If Fed is **dovish**

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- **Result.** Inflation indeed increases, $\hat{\pi}_t > 0$: the initial belief of higher inflation is *self-fulfilled*

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  - the *stabilizing* effect of a hawkish Taylor rule is called **the Taylor Principle**

Central Bank's Objectives

Fed's Dual Mandate

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- Unfortunately, this is hard to achieve as the economy is subject to an **inflation vs the output gap trade-off** captured by the Phillips curve

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa_x \hat{x}_t + \hat{u}_t$$

  That is: for given $E_t \hat{\pi}_{t+1}$, lowering inflation ($\hat{\pi}_t \downarrow$) leads to lower output gap ($\hat{x}_t \downarrow$), possibly negative (recession)

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  - for $\alpha_x = \alpha_r = 0$, the Fed is a *strict inflation targeter*

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2. A loss function can be completely arbitrary
    1. What should be the optimal value of α_x? Should it be zero (strict inflation targeting)?
    2. Why not trying to stabilize also other variables? For instance, should the Fed care about *financial stability*?

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  - Fed chooses $\hat{r}_t$, anticipating its impact on AD and AS

Optimal Monetary Policy

A Welfare-Based Loss Function

- It can be shown that, under suitable assumptions

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, H_t\right) \approx \min \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\hat{\pi}_t^2 + \alpha_x \hat{x}_t^2\right)$$

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- NOTE: if utility included money, $U\left(C_t, H_t, \frac{M_t}{P_t}\right)$, the loss would include a term $\alpha_r \hat{r}_t^2$. However, for standard calibrations, $\alpha_r \approx 0$.

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  - **PRO**: if credible, it helps central bank affect market's expectations about key macro variables (inflation, interest rates)
    Ex: Fed commits to *do whatever it takes* to keep inflation at 2% target
    If markets trusts the Fed $\Longrightarrow$ inflation expectations are anchored at 2%

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  EX: Fed commits to keep rates high until end of '23
- Is *commitment* **desirable**?
  - **PRO**: if credible, it helps central bank affect market's expectations about key macro variables (inflation, interest rates)
    Ex: Fed commits to *do whatever it takes* to keep inflation at 2% target
    If markets trusts the Fed $\Longrightarrow$ inflation expectations are anchored at 2%
  - **CON**: unforeseen events may call for policy deviations from announcement, which the public could interpret as opportunistic behavior
    Ex: since early '22, U.S. inflation has been way above 2% target. Bad luck (COVID + gas crisis) or broken promise?

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  2. *Commitment is time inconsistent*. An *ex ante* optimal announced policy is *ex post* suboptimal

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Optimal Discretionary Policy

A Discretionary Central Bank

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- Market participants anticipate that when forming expectations about future variables
- No incentive for the central bank to make announcements since they will not be credible
  $\Longrightarrow$ central bank unable to manipulate market expectations ($E_t\hat{\pi}_{t+t}$ and $E_t\hat{x}_{t+1}$ in our model), so it will take them as given

- The central bank solves

$$\min \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\hat{\pi}_t^2 + \alpha_x \hat{x}_t^2 \right) \tag{4}$$

subject to the NKPC

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- Treating $E_t \hat{\pi}_{t+1}$ like a constant, from the policy-marker's perspective, the dynamic problem in (4)-(5) is a sequences of disconnected static problems:

$$\min_{\hat{x}_t} L_t = \frac{1}{2} \left[\left(\kappa_x \hat{x}_t + \hat{u}_t + \underbrace{\beta E_t \hat{\pi}_{t+1}}_{\text{given}} \right)^2 + \alpha_x \hat{x}_t^2 \right] \tag{6}$$

- Taking FOCs with respect to $\hat{x}_t$,

$$\underbrace{\left(\kappa_x \hat{x}_t + \hat{u}_t + \beta \underbrace{E_t \hat{\pi}_{t+1}}_{\text{given}}\right)}_{\hat{\pi}_t} \kappa_x + \alpha_x \hat{x}_t = 0 \implies \hat{\pi}_t = -\frac{\alpha_x}{\kappa_x} \hat{x}_t \tag{7}$$

Optimal Discretionary Policy

Optimal Targeting Rule

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  - but agents *rationally* anticipate (7) will be used

Optimal Discretionary Policy

REE Solution

- We combine $\hat{\pi}_t = -\frac{\alpha_x}{\kappa_x}\hat{x}_t$ with the NKPC

$$\underbrace{\hat{\pi}_t}_{-\frac{\alpha_x}{\kappa_x}\hat{x}_t} = \beta \underbrace{E_t\hat{\pi}_{t+1}}_{-\frac{\alpha_x}{\kappa_x}E_t\hat{x}_{t+1}} + \kappa_x\hat{x}_t + \hat{u}_t$$

$$\Longrightarrow \hat{x}_t\left(1+\frac{\kappa_x^2}{\alpha_x}\right) = \beta E_t\hat{x}_{t+1} - \frac{\kappa_x}{\alpha_x}\hat{u}_t$$

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\end{aligned} \tag{8}$$

- This is a standard linear difference equation. Conjecture the solution:

$$\hat{x}_t = \eta_{x,u}\hat{u}_t$$

and use the method of undetermined coefficients to find $\eta_{x,u}$. Recall that

$$\hat{u}_t = \rho_u\hat{u}_t + \hat{\varepsilon}_t^u, \qquad \hat{\varepsilon}_t^u \sim \text{iid}N\left(0,\sigma_u^2\right), \; 0 \le \rho_u < 1 \tag{9}$$

- Substitute $E_t \hat{x}_{t+1} = \eta_{x,u} \rho_u \hat{u}_t$ into (8)

$$\hat{x}_t = a\eta_{x,u}\rho_u \hat{u}_t - b\hat{u}_t = \left(a\eta_{x,u}\rho_u - b\right) \hat{u}_t$$

$\Longrightarrow$ solve $a\eta_{x,u}\rho_u - b = \eta_{x,u}$:

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- Use optimal targeting rule to find inflation

$$\hat{\pi}_t = -\frac{\alpha_x}{\kappa_x}\hat{x}_t = -\frac{\alpha_x}{\kappa_x}\eta_{x,u}\hat{u}_t = \underbrace{\frac{\alpha_x}{\alpha_x\left(1 - \beta\rho_u\right) + \kappa_x^2}}_{\eta_{\pi,u} \geq 0}\hat{u}_t \tag{11}$$

- We have found: $\hat{x}_t = \eta_{x,u}\hat{u}_t$ and $\hat{\pi}_t = \eta_{\pi,u}\hat{u}_t$:

$$\eta_{x,u} = -\frac{\kappa_x}{\alpha_x\left(1-\beta\rho_u\right)+\kappa_x^2} < 0, \quad \eta_{\pi,u} = \frac{\alpha_x}{\alpha_x\left(1-\beta\rho_u\right)+\kappa_x^2} > 0$$

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Optimal Discretionary Policy

Summary

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2. however, under baseline calibration ($\theta = 2/3$, $\beta = 0.99$, $\sigma = \chi = 1$ and $\epsilon = 8$)

$$\alpha_x = \frac{\kappa_x}{\epsilon} = \frac{(1-\theta)(1-\theta\beta)}{\theta} \frac{\sigma + \chi}{\epsilon} \approx 0.04$$

Price stability should be the key objective!

Optimal Discretionary Policy

Summary

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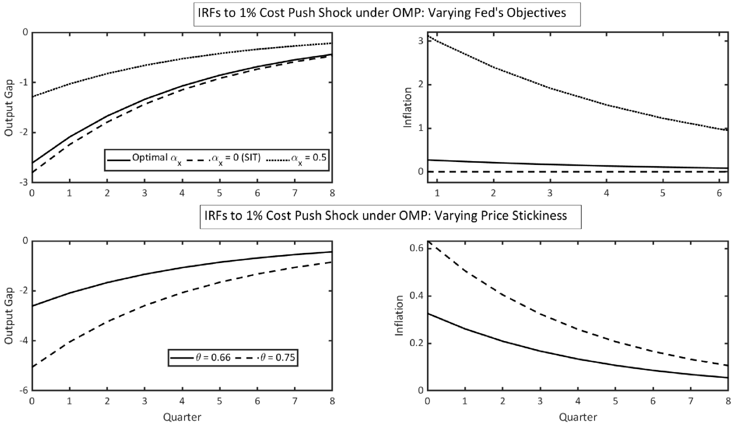
  4. From targeting rule $\hat{\pi}_t = -\frac{\alpha_x}{\kappa_x}\hat{x}_t$, the **optimal relative volatility** of inflation to the output gap is

$$ORV \equiv \frac{Std.Dev(\hat{\pi}_t)}{Std.Dev(\hat{x}_t)} = \frac{\alpha_x}{\kappa_x} = \frac{\kappa_x/\epsilon}{\kappa_x} = \frac{1}{\epsilon}$$

Since $\epsilon > 1$, inflation is always *less volatile* than the output gap

Optimal Discretionary Policy

Impulse Responses to 1% Cost Push Shock



Optimal Discretionary Policy

Interest Rates

- What about the nominal and real interest rate? From the AD curve

$$\underbrace{\hat{x}_t}_{\eta_{x,u}\hat{u}_t} = \underbrace{E_t\hat{x}_{t+1}}_{\eta_{x,u}\rho_u\hat{u}_t} - \delta\left(\hat{r}_t - \underbrace{E_t\hat{\pi}_{t+1}}_{\eta_{\pi,u}\rho_u\hat{u}_t} - \widehat{rr}_t^n\right)$$

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  - real interest rate

$$\hat{r}_t - E_t\hat{\pi}_{t+1} = \underbrace{\frac{\sigma\kappa_x\left(1-\rho_u\right)}{\alpha_x\left(1-\beta\rho_u\right)+\kappa_x^2}}_{\eta_{rr,u}>0}\hat{u}_t + \underset{\text{TFP driven}}{\widehat{rr}_t^n} \qquad (12)$$

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  - nominal interest rate

$$\hat{r}_t = \underbrace{\frac{\alpha_x\rho_u + \sigma\kappa_x\left(1-\rho_u\right)}{\alpha_x\left(1-\beta\rho_u\right)+\kappa_x^2}}_{\eta_{r,u}>0}\hat{u}_t + \underset{\text{TFP driven}}{\widehat{rr}_t^n} \tag{13}$$

Optimal Discretionary Policy

Targeting Rule vs Taylor Rule

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Optimal Discretionary Policy

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$$\eta^{TR}_{\pi,u} = \frac{1-\rho_u}{(1-\rho_u)(1-\beta\rho_u)+\kappa_x\delta(\phi_\pi-\rho_u)} > 0 \qquad (14)$$

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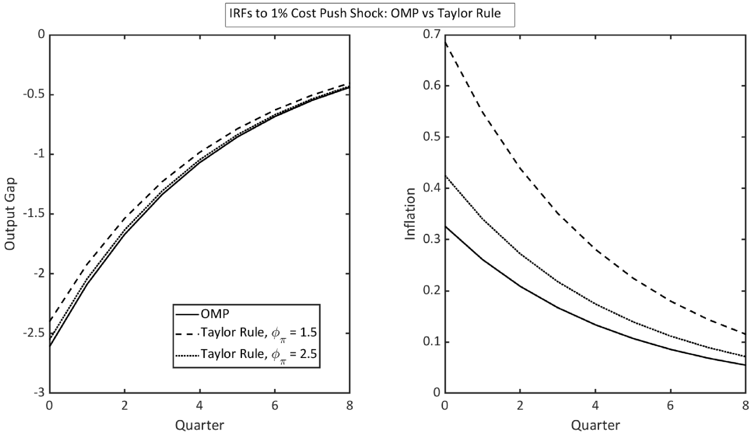
Optimal Discretionary Policy

Targeting Rule vs Taylor Rule



- For a Taylor rule to replicate the optimal policy we need:

$$\begin{aligned} \eta_{\pi,u}^{TR} &= \eta_{\pi,u}^{OMP} \\ \eta_{x,u}^{TR} &= \eta_{x,u}^{OMP} \end{aligned} \quad \Longrightarrow \quad \phi_\pi = \phi_\pi^* \equiv \rho_u + \frac{\kappa_x \sigma (1 - \rho_u)}{\alpha_x} \tag{18}$$

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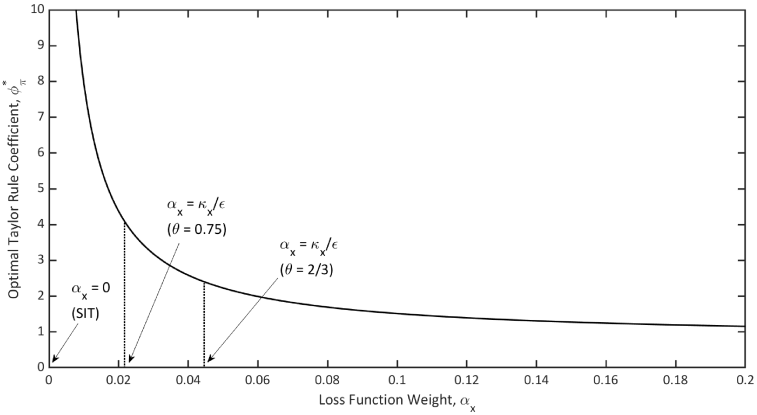
- Notice that
  1. the optimal coefficient ϕ_π^* is strictly decreasing in α_x $\implies$ the higher the Fed's concern for output stability (lower relative concern for price stability), the less responsive to inflation the *optimal Taylor rule* should be
  2. if the Fed was a *strict inflation targeter*, $\phi_\pi^* \to \infty$: no realistic Taylor rule will attain the optimum
  3. if the Fed was "benevolent", $\alpha_x = \kappa_x / \epsilon$, then $\phi_\pi^* \equiv \rho_u + \sigma\epsilon \left(1 - \rho_u\right)$



Optimal Taylor Rule Coefficient, ϕ_π^*
Loss Function Weight, α_x
$\alpha_x = 0$
(SIT)
$\alpha_x = \kappa_x/\epsilon$
(θ = 0.75)
$\alpha_x = \kappa_x/\epsilon$
(θ = 2/3)
0 0.02 0.04 0.06 0.08 0.1 0.12 0.14 0.16 0.18 0.2
0 1 2 3 4 5 6 7 8 9 10