# Macroeconomic Analysis

# Lecture notes (1) on:
# *The "natural" rate of unemployment and the long-run Phillips curve*

(F. Bagliano, 2022)

The first serious attack to the theoretical propositions and policy implications put forward by the "neoclassical synthesis" is due to Milton Friedman (1968) and Edmund Phelps (1967). At the end of the 1960s, they independently developed similar ideas aimed at explaining the positive correlation between output fluctuations and inflation in the *short-run*, preserving the neoclassical property of *long run* money neutrality. In the following notes a simple formalization of the original Friedman's idea (1968, 1977) is presented and briefly discussed.

## 1. Friedman's model (1968, 1977)

The simple model presented here has the following main features:

- the goods and labor markets operate under conditions of perfect competition and *market-clearing* (i.e. the equilibrium between demand and supply always occurs)[1]
- wages and prices are perfectly *flexible* (nominal rigidities are ruled out, whereas they featured prominently in the macroeconomic models of the neoclassical synthesis)

[1] The assumption of perfect competition is not essential to Friedman's view but simplifies the formal analysis.

- in the labor market, both demand (by producers) and supply (by workers) depend on the level of the *real wage*
- in this otherwise neoclassical framework, an *informational imperfection* is introduced: workers find it hard to gather information on the actual (goods) price level. Consequently, they base their work-leisure decisions on an *expected* ( or "perceived") price level, and therefore, on an *expected real wage*. On the other hand, producers are assumed to easily observe the actual price level. Therefore, firms base their demand for labor decisions on the *actual* level of the *real wage*.

### 1.1. Labor demand

The market demand for labor is obtained by aggregating the outcomes of individual firms' profit-maximization problems in a perfectly competitive environment (on the product and labor markets). Focusing on a "representative firm" and assuming that labor is the only input into production, according to the (well-behaved) production function

$$Y = f(L) \qquad f' > 0\,,\; f'' < 0 \tag{1.1}$$

where $Y$ is the amount of output and $L$ is the quantity of labor input, the optimal quantity of labor demanded on the market is chosen in order to maximize profits:

$$\max_{L} \quad \Pi = P\, f(L) - W\, L \tag{1.2}$$

where $P$ is the market price of the good produced by the firm and $W$ is the nominal wage paid to each unit of labor input (both prices are taken as given by perfectly competitive firms).

The first-order condition for this problem defines the *demand for labor* $L^D$

$$\frac{W}{P} = f'(L^D) \tag{1.3}$$

Firms demand as much labor as it is necessary to equate the marginal productivity of labor to the real wage.

### 1.2. Labor supply

Workers decide the quantity of labor to supply $L^S$ as a function of the *expected* real wage, obtained by deflating the (observed) nominal wage $W$ with the perceived

(expected) price level $P^e$, which is based on the incomplete information at their disposal. Formally, the labor supply function has the form:

$$L^S = g(\frac{W}{P^e}) \qquad g' > 0$$

from which we can write (by inverting $g(.)$):

$$\frac{W}{P^e} = h(L^S) \qquad h(.) \equiv g(.)^{-1} \quad , \quad h' > 0 \tag{1.4}$$

### 1.3. Equilibrium

The equilibrium on the labor market is obtained by equating demand and supply: $L^D = L^S$. From (1.3) and (1.4) we get the equilibrium condition

$$\begin{aligned} P\, f'(L) &= P^e\, h(L) \\ \Rightarrow \frac{P}{P^e}\, f'(L) &= h(L) \end{aligned} \tag{1.5}$$

The equilibrium level of employment $L$ depends on the ratio of the *actual* price level $P$ (observed by firms) to the price level *expected* (or perceived) by workers $P^e$. When $P = P^e$ the equilibrium condition defines an employment level $L^*$ (such that $f'(L^*) = h(L^*)$) that depends only on the properties of the production and labor supply functions, and therefore on the available technology and workers' preferences. When $P \neq P^e$ there are deviations of equilibrium employment from the value $L^*$. Given the properties of $f'(.)$ and $h(.)$, if $P > P^e$ then $L > L^*$ and if $P < P^e$ then $L < L^*$. Equilibrium employment is therefore an increasing function of the ratio $P/P^e$:[2]

$$L = l\,(\frac{P}{P^e}) \qquad l' > 0 \tag{1.6}$$

[2]This result can be easily proved by totally differentiating the equilibrium condition (1.5) with respect to $L$ and $P/P^e$, getting:

$$f'(L)\, d(\frac{P}{P^e}) + \frac{P}{P^e}\, f''(L)\, dL = h'(L)\, dL$$

from which

$$\frac{dL}{d(\frac{P}{P^e})} = \frac{f'(L)}{h'(L) - f''(L)\,\frac{P}{P^e}} > 0$$

The working of the labor market is graphically depicted in Figure 1, which relates employment $L$ and the real wage $W/P$. Labor demand, given by (1.3), is negatively sloped, reflecting decreasing marginal productivity. Labor supply, from (1.4), is represented as $\frac{W}{P} = \frac{P^e}{P} h(L^S)$. This equation describes a set of (positively sloped) functions, whose position depends on the $P^e/P$ ratio. When $P^e = P$ the point at which the two curves cross identifies the "*natural*" (in Friedman's terminology) level of employment $L^*$ and the corresponding level of equilibrium real wage $(W/P)^*$. For a given labor force $\bar{L}$ , the ratio $u^* \equiv (\bar{L} - L^*)/\bar{L}$ defines the *natural rate of unemployment.*

The qualifier *natural* is used to stress the *real* (as opposed to purely nominal) character of the determinants of $L^*$ and $u^*$. In fact, they reflect the available production technology, agents' work-leisure preferences, and other structural and institutional features of the product and labor markets (such as deviations from perfect competition, stochastic variability in demand and supply, costs and frictions in collecting information about job opportunities and available workers, the existence of unemployment subsidies, the structure of taxation, costs of mobility, etc.; all those elements are not explicitly modelled here but, in principle, they should affect the position of the labor demand and/or labor supply schedule, determining $L^*$).

Whenever $P^e > P$ the real wage perceived by workers is lower than $(W/P)^*$ and the amount of labor supplied decreases: the labor supply schedule in Figure 1 shifts upwards, causing employment to decrease below its natural level. The opposite outcome occurs when $P^e < P$: labor supply shifts downwards and employment increases beyond its natural level.

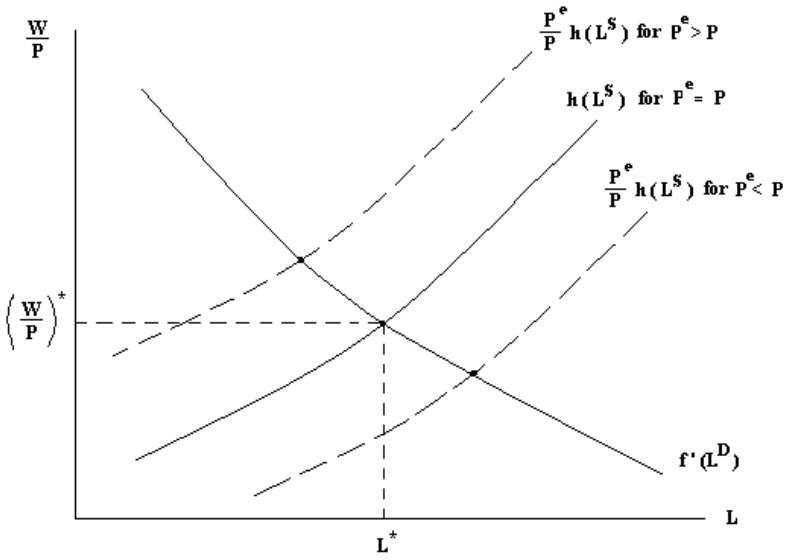


Figure 1

## 2. Policy implications

The simple model of the labor market outlined above yields important implications as to the relationship between inflation and unemployment (the "Phillips curve") and the role of economic (in particular monetary) policy.

### 2.1. Aggregate supply

From the production function (1.1) and equilibrium employment (1.6) we get the aggregate supply of output $Y$ as a function of the price level ratio:

$$Y = f\left(l\,(\frac{P}{P^e})\right) = Y^S(\frac{P}{P^e}) \qquad Y^{S\prime} > 0 \tag{2.1}$$

This equation is an aggregate supply function whereby the level of output depends positively on the *unanticipated* component (or "*surprise*") in the general price level, given by the difference between the actual and the expected (by workers) price levels. A way of writing (2.1) frequently used in the macroeconomic literature is:

$$y = y^* + \alpha\,(p - p^e) \qquad \alpha > 0 \tag{2.2}$$

where lower-case letters denote the logarithm of the corresponding variables (output and the price level) and a linear functional form for $f(l(.))$ is assumed.[3] The *natural* level of output (in logs), occurring when $p = p^e$ and employment is $L^*$, is given by $y^*$.

As shown in Figure 2, relating the price level $p$ and output $y$, when the actual and expected price levels are equal ($p = p^e$), the aggregate supply function ($AS$) is vertical at the natural level of output $y^*$. Since the equality between realized and expected values is a plausible feature of an equilibrium state that the economy tends to reach over time, this $AS$ function is often called "*long-run* aggregate supply". Instead, in the short run, for a *given* expected price level $p^e$, if $p$ is different from $p^e$, employment and output differ from their natural equilibrium levels: the corresponding *short-run* aggregate supply is positively sloped.

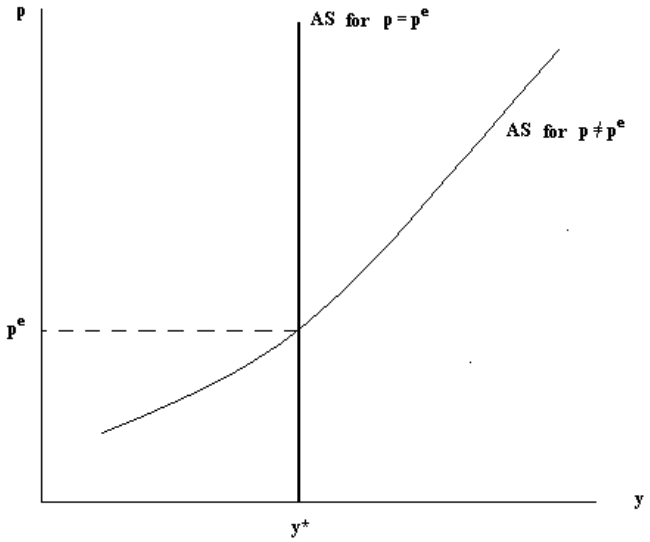


Figure 2

### 2.2. The natural rate of unemployment and the Phillips curve

Given the labor force $\bar{L}$ in the economy, a natural level of employment $L^*$ implies a *natural rate of unemployment* $u^*$. As already noted, this rate depends

[3]For simplicity, the fact that the logarithm of the expected price level, $\log P^e$, is different from the expected value of the logarithm of the price level, $p^e$, is ignored here.

exclusively on *real* phenomena (production technology, workers' preferences, institutional features of the labor and product markets, efficiency of the matching process between vacancies and unemployed workers). The natural rate $u^*$ is not necessarily constant, but can vary over time, reflecting changes in the work-force and in the efficiency of the job matching process in the labor market.. Moreover, $u^*$ can be affected by policy measures concerning some characteristics of the labor market, such as minimum wages, unemployment benefits, the structure of taxation and social security contribution.

The concept of a natural rate of unemployment has important implications for the interpretation of the relationship between inflation and unemployment captured by the "Phillips curve" and its use for policy purposes. To illustrate the main implications, Figure 3 depicts the relationship between the unemployment rate $u$ and the inflation rate $\pi$ according to Friedman's insight. Let us start from a long-run equilibrium position where unemployment is at the natural rate $u^*$ and the actual inflation rate $\pi_0$ is perfectly anticipated (point $A$). From this situation, a boost in aggregate demand (for example, following an expansionary monetary policy) causes an increase of the inflation rate (and of the growth rate of nominal wages) to $\pi_1$, together with an increase in employment which pushes the unemployment rate below the natural rate $u^*$. The increase in employment occurs because workers do not immediately perceive the changed demand conditions and misinterpret the increase in nominal wages as a *real* wage increase, therefore voluntarily supplying more labor. The economy moves from $A$ to $B$, showing a negative relationship between inflation and unemployment of the "Phillips curve" variety.

However, the trade-off between inflation and unemployment is only temporary: over time, workers' expectations adjust to the new (higher) inflation rate and the perceived real wage gradually decreases towards the level consistent with the natural rate of unemployment $u^*$. Graphically, the adjustment of expectations shifts the Phillips curve upwards, leading the economy to the new long-run equilibrium position $C$, again at the natural rate of unemployment but with a higher inflation rate. Therefore, the trade-off described by the Phillips curve holds only in the short run (when expectations have not yet adjusted to the change in aggregate demand), whereas there is no long-run trade-off: the long-run Phillips curve is vertical at the natural rate of unemployment $u^*$.

Analytically, the formulation of the Phillips curve that captures the role of expectations, allowing for a trade-off only in the short run, is of the following

kind:

$$\pi = \pi^e - b\,(u - u^*) \tag{2.3}$$

where $\pi^e$ is the expected inflation rate and $b > 0$. According to (2.3), known as the "*expectations-augmented Phillips curve*", an unemployment rate at its natural level ($u = u^*$) is consistent with any inflation rate, if perfectly predicted ($\pi = \pi^e$). In the short run, for given expected inflation, the relationship in (2.3) allows for a negative trade-off, as in the original formulation of the Phillips curve.

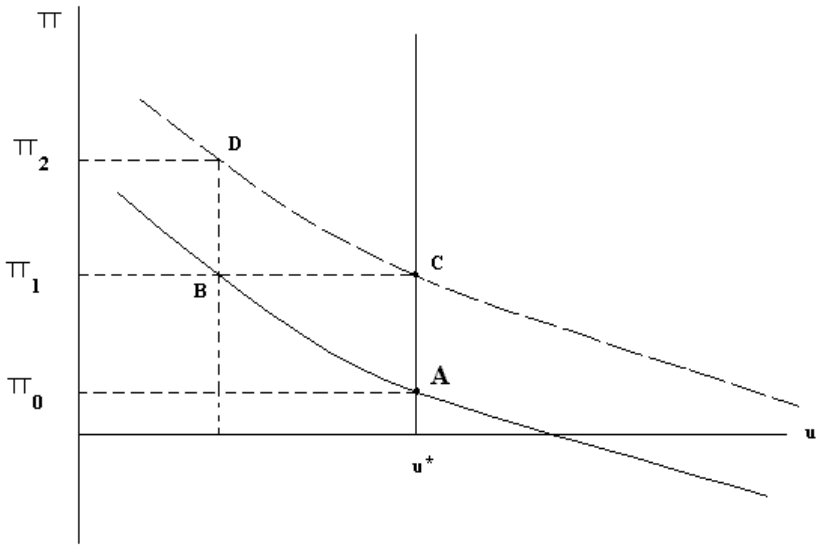


Figure 3

### 2.3. The "accelerationist" hypothesis

The example above shows that, during (demand-driven) expansions, a positive correlation between inflation and output arises. However, this feature of the business cycle does not enable policymakers to keep output over time at a level higher than its natural rate (and unemployment at a level lower than its natural rate). In the long run, when expectations have fully adjusted, the economy tends to go back to its natural (i.e. determined by real driving forces) equilibrium.

In principle, the only way to obtain a permanent trade-off between inflation and unemployment would be to increase the inflation rate over time (making the dynamics of the price level to accelerate), generating in each period of time

a difference between the actual inflation rate (influenced by monetary policy) and the expected rate, so as to maintain $u$ below its natural rate $u^*$. Looking again at Figure 3, if policymakers wanted to keep the unemployment rate at the level in $B$, they should raise the inflation rate to $\pi_2$ in $D$, on a higher Phillips curve (corresponding to the expected inflation rate $\pi_1$). Since the adjustment of expectations to the higher rate $\pi_2$ would lead unemployment back to the natural rate, a further (unexpected) increase of the inflation rate would be required to keep unemployment at the desired level corresponding to $B$ and $D$.

### 2.4. Further considerations

The introduction of a key role for agents' expectations in determining shifts in the relation between inflation and unemployment has far-reaching implications for policymaking. The fact that current inflation depends not only on current unemployment but also on expected inflation yields a general insight (particularly stressed by Phelps 1967) on the *intertemporal* nature of the trade-off between inflation and unemployment. Changes in expected inflation shift the position of the Phillips curve over time, affecting the inflation-unemployment trade-off. From the policymaker's perspective, any action aimed at exploiting the short-run trade-off will change the inflation-unemployment relation in the future, whereby affecting future policy decisions.

One clear example of this intertemporal dimension of policy decision-making is given by the choice of a *disinflation* path for the economy. If the economy is in an initial long-run equilibrium with a relatively high inflation rate $\pi_1$ (as in point $C$ of Figure 3) and policymakers want to permanently reduce the inflation rate to a lower level $\pi_0$ they face a choice between two intertemporal strategies. One strategy is to aggressively reduce aggregate demand (by means of a contractionary monetary policy), obtaining a sizeable reduction in the inflation rate at the cost of a large rise of the unemployment rate beyong the natural level (the economy is moving along a short-run Phillips curve for an expected inflation of $\pi_1$). Subsequently, expectations will adjust to the much lower inflation rate (i.e. the Phillips curve has a large downward shift), allowing for unemployment to quickly revert to the natural rate. The long-run, lower-inflation, equilibrium is reached by means of a deep but relatively short recession. An alternative strategy, using a less aggressive contractionary monetary policy, obtains the same result in the long run with a lower increase in unemployment but for a longer period of time. The choice between the two disinflation strategies will therefore depend on the weight that the

goal of keeping inflation low has in the "objective function" of the policymaker, relative to the social costs generated by a period of high unemployment.

## References


[1] Friedman M. (1968) "The role of monetary policy", *American Economic Review*, 58

[2] Friedman M. (1977) "Nobel lecture: Inflation and unemployment", *Journal of Political Economy*, 85

[3] Phelps E.S. (1967) "Phillips curves, expectations of inflation and optimal unemployment over time", *Economica*, 34