Macroeconomic Analysis

Lecture notes (2) on: *Rational expectations and the "New Classical Macroeconomics"*

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The rationalization of the different nature of the short- and long-run relation between inflation and unemployment put forward by Friedman and Phelps, centered around the concept of the natural rate of unemployment, features a key role for agents' *expectations*. However, the process of expectations formation that is (implicitly) assumed is essentially "*adaptive*": people (especially workers) form expectations about the price level (and therefore the real wage) on the basis of past experience, gradually "correcting" over time any past forecast error. The immediate consequence of this behavior is the possibility of long periods in which systematic mistakes occur, with repeated under- (or over-)estimates of the inflation rate.

Since the 1970s a different hypothesis on expectation formation is introduced into macroeconomic models. The *rational expectations* hypothesis seems more satisfactory (because it rules out systematic mistakes) and consistent with the degree of rationality attributed to economic agents in other decision-making processes, such as profit and utility maximization. The adoption of the rational expectations hypothesis in models that show a "natural" rate of output has had important implications on business cycle thoery and macroeconomic policy.

These notes start with a comparison between the rational expectations and the adaptive expectations hypotheses; then, some formal business cycle models incorporating the rational expectations assumption are illustrated. Such models are typical of the so-called *new classical macroeconomics* school usually associated to the names of R. Lucas, T. Sargent, R. Barro and N. Wallace.

1. Adaptive expectations vs. rational expectations: a comparison

The fact that economic agents (consumers, firms, workers, financial investors) base their current choices on what they expect on the future course of some economic variables is a key ingredient of many economic theories (concerning, for example, consumption, investment, labor supply, portfolio allocation). The mechanism whereby expectations are formulated has been specified in various ways.

A first, simple, hypothesis is that of **adaptive expectations**, implicitly adopted in the Friedman-Phelps account of the absence of a long-run inflation-unemployment trade-off. This assumption is formalized by the following equation

$$p^e_{t,\,t+1} = p^e_{t-1,\,t} + \lambda\,(p_t - p^e_{t-1,\,t}) \qquad 0 < \lambda < 1 \tag{1.1}$$

where $p^e_{t,\,t+1}$ is the expectation of p_{t+1} (the log of the price level) formulated at time t. The value of p expected for the next period is modified between t and $t+1$ by a fraction λ of the forecast error occurred at t, $p_t - p^e_{t-1,\,t}$. Such mechanism generates the possibility of "systematic" forecast errors.

The alternative assumption of **rational expectations** has been adopted to avoid the possibility that agents make systematic mistakes without altering the expectation formation mechanism in (1.1). Under rational expectations

$$p^e_{t,\,t+1} = E(p_{t+1} \mid \Omega_t) \equiv E_t p_{t+1} \tag{1.2}$$

where E_t denotes the *mathematical expectation* of p_{t+1} conditional upon the information set available to agents at time t, Ω_t. The key feature of rational expectations is that the forecast error (or "surprise") at time $t+1$ has a zero expected value conditional on t:

$$E_t\,(p_{t+1} - E_t p_{t+1}) = 0$$

In the models that adopt the rational expectations hypothesis, the precise definition of the content of the information set Ω_t plays a crucial role. Usually, Ω_t includes past and current (time t) values of all variables and the structure of the economy, as described by the equations of the model. Rationally formed expectations are therefore consistent with the structure of the model describing the economy, "as if" agents made use of the model itself in forming expectations on future values of the endogenous variables.

To clarify this point, highlighting the consequences on the equilibrium dynamics of macroeconomic variables under either mechanism, we use a simple, discrete-time, version of the classic *hyperinflation* model due to P. Cagan (1956), assuming adaptive expectations, and used by T. Sargent and N. Wallace (1973) with rational expectations.

1.1. A simple model

The model considers an economy in which in each period a consumption good can be exchanged for money. In the presence of only two markets (for the only good and for money) only one market-clearing condition is necessary to describe the simultaneous equilibrium between demand and supply on both markets. Focusing on the money market, the equilibrium condition equates the real money supply to the real money demand as follows:

$$\underbrace{m_t - p_t}_{\text{money supply}} = \underbrace{-\alpha\left(p^e_{t,\,t+1} - p_t\right)}_{\text{money demand}} \qquad \alpha > 0 \tag{1.3}$$

In (1.3) all variables are in logarithms: m is the quantity of nominal money in the economy, exogenously set by a monetary authority (central bank), and p is the price level of the consumption good; $p^e_{t,\,t+1} - p_t$ is therefore the inflation rate expected between t and $t+1$. Since the model is focused on the dynamics of the price level, the amount of the consumption good produced in each period is assumed to be constant (and not affected by monetary variables), and therefore its level is omitted from money demand. No stochastic elements (random "shocks") are added to the money demand and money supply functions.[1]

[1]Equation (1.3) may be interpreted more generally as a traditional LM curve, where an exogenous money supply is equated in equilibrium to a money demand which depends positively on output and negatively on the nominal interest rate. Letting upper-case letters denote the levels of the corresponding variables (whereas lower-case letters denote logarithms, with the only exception of the interest rate) we can write the equilibrium condition at time t as:

$$\frac{M_t}{P_t} = \bar{Y}\,\mathrm{e}^{-\alpha(\bar{r}+\pi^e_{t,t+1})} \tag{*}$$

where output is assumed to be constant at level $\bar{Y}$ and the nominal interest rate $i = \bar{r} + \pi^e_{t,t+1}$ is the sum of the real interest rate $\bar{r}$ (assumed constant) and expected inflation. The specific formulation of the money demand function is chosen for analytical tractability. Taking logarithms of both sides of (*) yields:

$$m_t - p_t = \left(\log\bar{Y} - \alpha\,\bar{r}\right) - \alpha\,\pi^e_{t,t+1}$$

where the term in brackets is a constant and can be omitted from the equation without altering the economic interpretation of the results. Finally, approximating $\pi^e_{t,t+1}$ with $p^e_{t,\,t+1} - p_t$ we get

Rearranging (1.3), the equilibrium price level at time t may be expressed as a function of current money supply and the price level expected for the next period:

$$p_t = \frac{1}{1+\alpha} m_t + \frac{\alpha}{1+\alpha} p^e_{t,\,t+1} \tag{1.4}$$

In order to solve the model, an assumption on the mechanism that agents use to form expectations on the future price level is required.

1.2. Adaptive expectations

The *adaptive* expectations hypothesis (1.1) can be rewritten as:

$$p^e_{t,\,t+1} = \lambda\, p_t + (1-\lambda)\, p^e_{t-1,\,t} \tag{1.5}$$

By repeated substitution of past values of the expected price level p^e obtained by lagging (1.5) we get:

$$\begin{aligned} p^e_{t,\,t+1} = {} & \lambda\, p_t + \lambda(1-\lambda) p_{t-1} + (1-\lambda)^2 p_{t-2} + \ldots + \\ & + \lambda\,(1-\lambda)^{T-1}\, p_{t-T+1} + (1-\lambda)^T\, p^e_{t-T,\,t-T+1} \end{aligned} \tag{1.6}$$

Since, for $T \to \infty$, $(1-\lambda)^T \to 0$, (assuming also a finite value for $p^e_{t-T,\,t-T+1}$) the last term in (1.6) can be neglected, getting:

$$p^e_{t,\,t+1} = \lambda \sum_{i=0}^{\infty} (1-\lambda)^i\, p_{t-i} \tag{1.7}$$

Finally, substituting (1.7) into (1.4) and solving for per p_t yields the solution for the equilibrium price level at time t:

$$p_t = \frac{1}{1+\alpha(1-\lambda)} m_t + \frac{\alpha\lambda}{1+\alpha(1-\lambda)} \sum_{i=1}^{\infty} (1-\lambda)^i\, p_{t-i} \tag{1.8}$$

The current price level depends only on its past history and on the current value of the money stock m_t. As a consequence, a transitory change in m_t has the same influence on p_t as a permanent change of the same amount (as will be shown in detail below).

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the equilibrium condition (1.3) in the text.

1.3. Rational expectations

If the *rational* expectations hypothesis is introduced in the model as in (1.2), the resulting price level is:

$$p_t = \frac{1}{1+\alpha} m_t + \frac{\alpha}{1+\alpha} E_t p_{t+1} \tag{1.9}$$

It is now necessary to solve the model to get the (model-consistent) expectation for p_{t+1}. Leading (1.9) forward by one period and taking the expected value at time t we get:

$$E_t p_{t+1} = \frac{1}{1+\alpha} E_t m_{t+1} + \frac{\alpha}{1+\alpha} E_t p_{t+2} \tag{1.10}$$

Note that : (i) the information set Ω_t on which the expected value E_t is based includes the current-period values of m and p and the structure of the economy, as summarized by (1.3); (ii) the *law of iterated expectations* (whereby $E_t(E_{t+1} p_{t+2}) = E_t p_{t+2}$) has been applied to the last term of $E_t p_{t+1}$. Substituting (1.10) into (1.9) yields:

$$p_t = \frac{1}{1+\alpha} m_t + \frac{\alpha}{1+\alpha} \left(\frac{1}{1+\alpha} E_t m_{t+1} + \frac{\alpha}{1+\alpha} E_t p_{t+2} \right)$$

and repeating the same steps for $E_t p_{t+i}$ ($i = 2, 3, ..., T-1$) we get:

$$p_t = \frac{1}{1+\alpha} \sum_{i=0}^{T-1} \left(\frac{\alpha}{1+\alpha} \right)^i E_t\, m_{t+i} + \left(\frac{\alpha}{1+\alpha} \right)^T E_t\, p_{t+T} \tag{1.11}$$

Since, if $T \rightarrow \infty$, $\left(\frac{\alpha}{1+\alpha}\right)^T \rightarrow 0$, the last term in (1.11) can be neglected (assuming that $E_t\, p_{t+T}$ is finite). Then, the equilibrium price level under rational expectations is:

$$p_t = \frac{1}{1+\alpha} \sum_{i=0}^{\infty} \left(\frac{\alpha}{1+\alpha} \right)^i E_t\, m_{t+i} \tag{1.12}$$

No variable dated $t-1$ or earlier affects p_t in equilibrium: the current price level is entirely determined by the current value of money supply that agents observe ($E_t m_t = m_t$) and by future values of m_{t+i} ($i = 1, ...$) expected as of time t. Any piece of information available to agents at t and useful to forecast the future path of m_{t+i} has an immediate effect on the price level. Therefore, the solution of the model with rational expectations is *forward-looking*, whereas the solution with adaptive expectations in (1.8), depending only on past and currently observed variables, is *backward-looking*.

1.4. Examples

Using alternatively (1.8) or (1.12) we can now study the behavior of the price level in response to changes in the quantity of money m. In particular, we will examine the following cases: (i) a *permanent* increase of m at time t_1 from $\bar{m}$ to $\bar{m}+k$; (ii) a *temporary* increase of m by the same amount from t_1 up to t_2; (iii) a temporary increase of m (as in (ii)) but previously *announced* at time t_0 (and therefore *expected* by agents).

Permanent increase of m (Figure 1). Sarting from an initial situation in which the quantity of money has always been equal to $\bar{m}$, a *permanent* increase from $\bar{m}$ to $\bar{m}+k$ occurs at time t_1. With *adaptive* expectations, up to period t_1-1 the price level is $\bar{m}$; at t_1, following the current-period increase in m, from (1.8) the price level increases to:

$$p_{t_1} = \frac{1}{1+\alpha(1-\lambda)}k + \bar{m} \tag{1.13}$$

After t_1 a gradual increase in the price level occurs, according to (1.8) with $m_t = \bar{m}+k$ for $t > t_1$, and p asymptotically reaches the new stationary equilibrium level $\bar{m}+k$. On the contrary, with *rational* expectations, according to (1.12), p immediately adjusts to its new equilibrium level $\bar{m}+k$.

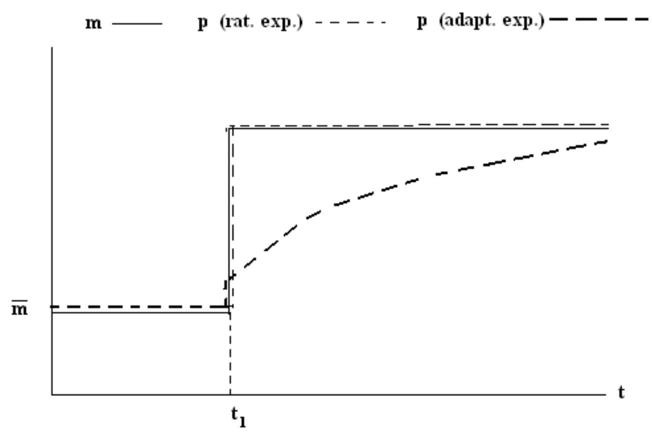

Figure 1: Permanent increase of m

Temporary increase of m **(Figure 2).** In this case money is $\bar{m}$ up to $t_1 - 1$, then increases to $\bar{m}+k$ only from $t_1$to $t_2 - 1$, to finally go back (forever) to the previous level $\bar{m}$ at t_2. The temporary nature of the increase of money becomes known to agents in t_1. Figure 2(a) describes the reaction of p with *adaptive* expectations. At t_1 the price increase is again given by (1.13): indeed, since agents do not take into account the future path of m in forming expectations on p, there is no difference between the response of the price level to a transitory and a permanent money increase. At t_2, when money decreases to the initial level $\bar{m}$, the price level reacts in the opposite way and decreases according to (1.8); after t_2 expectations are gradually adjusted downwards and the price level gradually decreases towards the initial equilibrium level $\bar{m}$.

Instead, with *rational* expectations, the reaction of p at t_1 depicted in Figure 2(b) is different from the case of a permanent increase. The increase of p at t_1 can be computed using (1.12) evaluated at t_1:

$$p_{t_1} = \left[1 - \left(\frac{\alpha}{1+\alpha}\right)^{t_2 - t_1}\right] k + \bar{m} \tag{1.14}$$

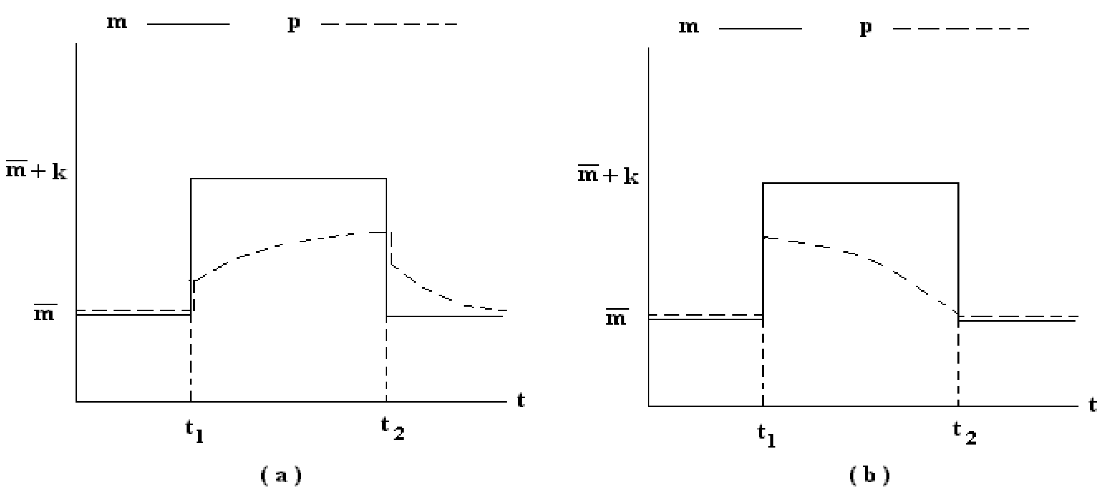

Figure 2: Temporary increase of m. (a) adaptive expect.; (b) rational expect.

Afterwards, between t_1 and t_2, the price level gradually decreases up to the initial equilibrium value $\bar{m}$, that is reached exactly at time t_2. Note that under rational expectations the price level "jumps" (displaying discrete changes) only when unexpected events occur. This is the case of the increase of m at t_1; on the contrary, the decrease of m at t_2 has already been incorporated (at t_1) in the agents' expectations and does not cause any "jump" in the price level.

Expected temporary increase of m (Figure 3)**.** The temporary increase in money supply is the same as in the previous case, but now it is announced to agents in advance, at time t_0. Therefore, when it occurs, the temporary money expansion is perfectly expected on the basis of the previous (credible) announcement of the monetary authorities. With *adaptive* expectations (Fig. 3(a)), the behavior of the price level is the same as in the previous case: the announcement at t_0 of future events does not change agents' expectations and therefore has no effect on the equilibrium price path.

Instead, with *rational* expectations (Fig. 3(b)), agents formulate their expectations in t_0 taking into account the announcement of a future temporary increase of m. Using (1.12) with $m = \bar{m} + k$ between t_1 and t_2, we obtain the equilibrium price level at the announcement date t_0:

$$p_{t_0} = \left(\frac{\alpha}{1+\alpha}\right)^{t_1 - t_0} \left[1 - \left(\frac{\alpha}{1+\alpha}\right)^{t_2 - t_1}\right] k + \bar{m} \tag{1.15}$$

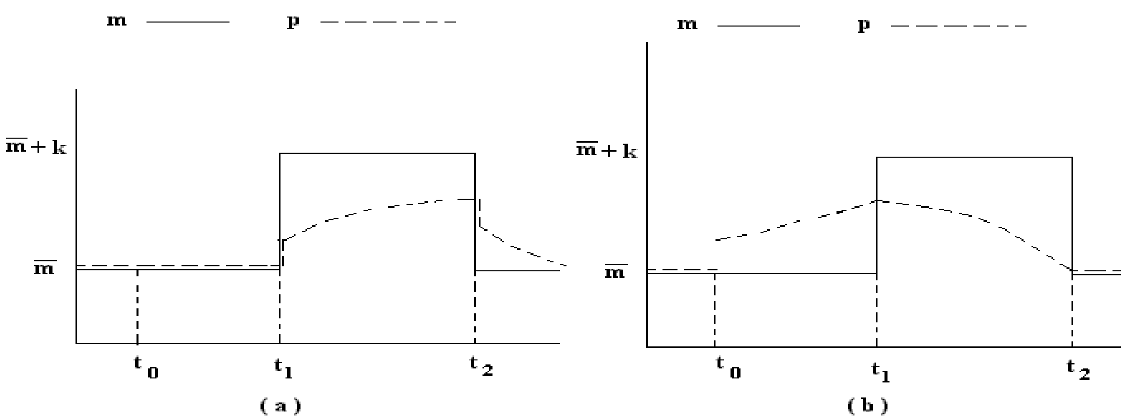

Figure 3: Expected temporary increase of m. (a) adaptive exp.; (b) rational exp. Note: (*i*) the only "jump" of p occurs at the announcement date t_0 (indeed,

only the announcement comes as a "surprise" to agents: everything that happens after the announcement is perfectly anticipated at t_0); at t_1, when the increase of m does take place, p is exactly at the level computed in the previous case (without announcement) and given by (1.14); (*ii*) the equilibriun response of the price level implies a period of inflation between t_0 and t_1 even in the absence of a monetary expansion: inflation is entirely attributable to the fact that agents expect money to increase in the future; then, in the very period in which money is at the higher level (between t_1 and t_2) “deflation” occurs (with the price level gradually decreasing).

1.5. Extension to stochastic shocks under rational expectations

So far, no stochastic element has been introduced in the model. In this case, if the process generating money supply is not subject to random shocks, adopting the rational expectations hypothesis reduces to the (stronger) assumption of *perfect foresight*.

The model can be easily extended to allow for stochastic elements both in the money supply process and on the demand for money side. In the latter case a shock u_t can be added to expected inflation as a determinant of money demand in each period t, yielding the following equilibrium condition on the money market:

$$m_t - p_t = -\alpha\,(p^e_{t,\,t+1} - p_t) + u_t \tag{1.16}$$

where u_t is a money demand shock with the property $E_{t-1}u_t = 0$ (i.e. unforecastable one period earlier).

The model is then solved following the same steps as in the non-stochastic case. Given the price level at $t+1$

$$p_{t+1} = \frac{1}{1+\alpha}\,m_{t+1} + \frac{\alpha}{1+\alpha}E_{t+1}\,p_{t+2} - \frac{1}{1+\alpha}\,u_{t+1} \tag{1.17}$$

the expectations of p_{t+1} formed at time t is

$$E_t\,p_{t+1} = \frac{1}{1+\alpha}E_t m_{t+1} + \frac{\alpha}{1+\alpha}E_t p_{t+2} \tag{1.18}$$

which coincides with (1.10) since $E_t u_{t+1} = 0$. Repeating this step for p_{t+2}, p_{t+3}, ... and letting T go to infinity, the final expression for the price level is:

$$p_t = \frac{1}{1+\alpha}\sum_{i=0}^{\infty}\left(\frac{\alpha}{1+\alpha}\right)^i E_t\,m_{t+i} - \frac{1}{1+\alpha}\,u_t \tag{1.19}$$

The price level is now affected not only by the expected values of future money supply, but also by the realization of the money demand shock u_t. In each period t, the realized price level p_t will differ from the expected price level $E_{t-1}p_t$ because of the presence of the disturbance u_t, which is unpredictable at $t-1$, and because the process that generates money supply can have a stochastic (unpredictable) component. Therefore, the forecast error at time t is:

$$p_t - E_{t-1}\, p_t = \frac{1}{1+\alpha}\sum_{i=0}^{\infty}\left(\frac{\alpha}{1+\alpha}\right)^i (E_t\, m_{t+i} - E_{t-1}\, m_{t+i}) - \frac{1}{1+\alpha}\, u_t \qquad (1.20)$$

The price forecast error at time t depends on the current realization of the shock u_t, and on the *revisions* in expectations on the future path of money supply occurred between $t-1$ and t: $E_t\, m_{t+i} - E_{t-1}\, m_{t+i}$. Such revisions will occur any time new information (relevant to predict future money) accrues to agents at t, enlarging the information set on which their (rational) expectations are based, and leading agents to change their expected values for future money supplies. An immediate implication of rational expectations is that such forecast error on p_t is unpredictable on the basis of the information set available to agent at $t-1$; taking expectations of (1.20) at $t-1$ we get:

$$E_{t-1}\left[p_t - E_{t-1}\, p_t \mid \Omega_{t-1}\right] = 0 \qquad (1.21)$$

References

[1] Cagan P. (1956) "The monetary dynamics of hyperinflation", in M. Friedman (ed.) *Studies in the quantity theory of money*, Chicago, University of Chicago Press

[2] Lucas R.J. (1973) "Some international evidence on the output-inflation trade-off", *American Economic Review*, 63

[3] Sargent T.J. and N. Wallace (1973) "The stability of models with money and growth with perfect forecast", *Econometrica*, 41, 6

[4] Sargent T.J. and N. Wallace (1976) "Rational expectations and the role of monetary policy", *Journal of Monetary Economics*, 2

2. Imperfect information and the business cycle: Lucas' "islands" model (1973)

The rational expectations hypothesis, ruling out agents' systematic forecast errors, has been considered more satisfactory from a theoretical viewpoint and has been adopted by several macroeconomic models aimed at explaining the basic mechanisms originating cyclical fluctuations, and particularly the positive correlation between nominal variables (such as monetary aggregates) and real quantities (output, employment). One of the first business cycle models making use of rational expectations and attributing the correlation between nominal and real variables to informational imperfections (of a different variety from those assumed by Friedman) is due to R. Lucas (1972, 1973).

The ideal economy described by Lucas is composed of a large number of geographically dispersed markets, where producers decide the amount of output in each period on the basis of the comparison between the price of the good in the local market in which they operate and the general price level in the whole economy.[2] Only an increase of the *local* price *relative* to to the economy-wide price level induces producers to increase output.

In this economy there are two sources of stochastic disturbances, both from the demand side: (a) an *aggregate demand* shock, hitting all markets in the same way; (b) a *local demand* shock, hitting any individual market. Only aggregate disturbances affect the economy-wide price level, whereas local prices are affected by both aggregate and local demand shocks. If producers in each market could always observe *both* their local price and the general price level, they would react only to differences between the two (a "relative" price), that would be a clear signal that a purely local demand shock has occurred. Instead, the Lucas model is built on the crucial hypothesis that producers have *imperfect information* on price changes: they observe the price on their local market but do *not* observe the general, economy-wide, price level. Therefore, output decisions depend on the difference between the locally observed price and the general price level that producers (rationally) *expect* on the basis of their imperfect information. This insight can be formalized in a simple model (following Lucas 1973).

[2] The view of the economy as made up of many separated markets justifies the denomination of Lucas "islands" model often used is the literature.

2.1. The model

The economy is composed of N separated markets, indexed by $z = 1, 2, ...N$, for a homogeneous good, operating under conditions of perfect competition. In each market, a perfectly flexible *local* price $p_t(z)$ ensures equilibrium between demand and supply in each period t (i.e. there is continuous market-clearing).[3] The amount of output on each market z, $y_t(z)$, is given by the following *local supply function*

$$y_t(z) = y^* + \gamma\left(p_t(z) - E(p_t \mid I_t(z))\right) \qquad \gamma > 0 \tag{2.1}$$

where y^* is the "natural" level of output, determined by long-run real driving forces such as capital accumulation and population growth (here, for simplicity, y^* is assumed constant over time and equal across all markets), and γ is a "structural" parameter, reflecting the properties of technology and workers' preferences on the local market (again, those structural factors are assumed to be equal across all markets). Producers increase output beyond y^* if the price observed locally is higher than the general price level p_t expected by local producers on the basis of their available information. Such information set, denoted by $I_t(z)$, includes the *observed* level of the local price $p_t(z)$ and the characteristics of the probability distributions of p_t e $p_t(z)$. More specifically, let us assume that

$$\begin{aligned} p_t &= \bar{p}_t + v_t \qquad \text{with} \quad v_t \sim N(0, \sigma^2) && (2.2) \\ p_t(z) &= p_t + z_t \qquad \text{with} \quad z_t \sim N(0, \tau^2) && \\ &= \bar{p}_t + v_t + z_t && (2.3) \end{aligned}$$

The general price level p_t is normally distributed around a mean $\bar{p}_t$ (perfectly known by local producers); v_t is an *aggregate* demand shock, with zero mean and variance σ^2 (again, known by producers). The local price $p_t(z)$ is the sum of p_t and the *local* demand disturbance z_t (normally distributed with zero mean and variance τ^2), with the property that the sum across markets of all local shocks is zero, $\sum_1^N z_t = 0$, and therefore does not affect the general price level; moreover, there is no correlation between v_t and z_t. The distribution of $p_t(z)$ is then normal around $\bar{p}_t$ and has a variance of $\sigma^2 + \tau^2$.

To choose the optimal output level, local producers make use of all available information on the distribution of p_t in (2.2) and the observation of their local price $p_t(z)$ to form an expectation of the general price level. This expectation is

[3] In the model, lower-case letters denote logarithms of the corresponding variable: e.g. $p_t(z) \equiv \log P_t(z)$.

rational since it is obtained as the (mathematical) expected value of p_t conditional on $p_t(z)$ and on the properties of the relevant probability distributions: we denote this expectation as $E(p_t \mid p_t(z))$. Given the properties in (2.2) and (2.3) the two variables are jointly normally distributed as follows:

$$\begin{pmatrix} p_t \\ p_t(z) \end{pmatrix} \sim N \left[\begin{pmatrix} \bar{p}_t \\ \bar{p}_t \end{pmatrix}, \begin{pmatrix} \sigma^2 & \sigma^2 \\ \sigma^2 & \sigma^2 + \tau^2 \end{pmatrix} \right] \tag{2.4}$$

where the matrix collects the variances and the covariance between p_t e $p_t(z)$. Given (2.4) we can derive the conditional expected value as :[4]

$$\begin{aligned} E(p_t \mid p_t(z)) &= \bar{p}_t + \frac{\sigma^2}{\sigma^2 + \tau^2} (p_t(z) - \bar{p}_t) \\ &= \frac{\tau^2}{\sigma^2 + \tau^2} \bar{p}_t + \frac{\sigma^2}{\sigma^2 + \tau^2} p_t(z) \\ &\equiv \theta \, \bar{p}_t + (1 - \theta) \, p_t(z) \end{aligned} \tag{2.5}$$

where $\theta \equiv \frac{\tau^2}{\sigma^2+\tau^2}$, with $0 < \theta < 1$. To infer the general price level p_t producers use optimally the information contained in the local price. The expected value of p_t is then obtained as a weighted average of the local price $p_t(z)$, and the unconditional mean of the general price level, $\bar{p}_t$. The weight on local price is given by $1 - \theta \equiv \frac{\sigma^2}{\sigma^2+\tau^2}$ and depends negatively on the variance of the local shock τ^2 and positively on the variance of the aggregate disturbance σ^2. Intuitively, if the aggregate shock has a high variance relative to the local disturbance, the local price $p_t(z)$ contains information which is very useful in forming expectations on p_t and will carry a relatively large weight in (2.5). The opposite case occurs when τ^2 is relatively high: movements in the local price mainly reflect local disturbances

[4] To obtain (2.5) we made use of the following property of the joint normal distributionof two generic random variables x and y:

$$\begin{pmatrix} x \\ y \end{pmatrix} \sim N \left[\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}, \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix} \right]$$

The distribution of x conditional on y is normal with (conditional) mean and variance given by:

$$x \mid y \sim N \left[\bar{x} + \frac{\sigma_{xy}}{\sigma_y^2} (y - \bar{y}) \, , \, \sigma_x^2 - \frac{(\sigma_{xy})^2}{\sigma_y^2} \right]$$

and $p_t(z)$ will be given a lower weight in determining p_t. Here producers face a classic problem of *signal extraction*: they have to "extract" an estimate of the "signal" they are interested in (in our case p_t) from the observation of the local price $p_t(z)$ which contains also an element of "noise" (the local disturbance z_t).

Substituting (2.5) into the local supply function (2.1) we get:

$$y_t(z) = y^* + \gamma\,\theta\,(p_t(z) - \bar{p}_t) \tag{2.6}$$

which relates the fluctuations of local output around the natural value to deviations of the local price from the average general price level $\bar{p}_t$. The "slope" of this supply curve is given by $\gamma\,\theta$ and therefore depends not only on the structural parameter γ but also, through θ, on the relative variances of the local and aggregate demand shocks. The relative magnitude of the variability of v relative to the variability of z determines the effect on output of movements in local prices. This feature of the model would not arise in the case of perfect information, in which local producers can directly observe, beside $p_t(z)$, also the general price level p_t.

To get a measure of the *aggregate* supply in the whole economy we can average over the N markets the local output given by (2.6), getting (using the property that $\sum_1^N z_t = 0$, and (2.2) to set $v_t = p_t - \bar{p}_t$):

$$\begin{aligned} \frac{\sum_1^N y_t(z)}{N} &= y^* + \gamma\,\theta\,\frac{\sum_1^N (p_t(z) - \bar{p}_t)}{N} \\ &= y^* + \gamma\,\theta\,\frac{\sum_1^N v_t + \sum_1^N z_t}{N} \\ &= y^* + \gamma\,\theta\,v_t \\ \Rightarrow \qquad y_t &= y^* + \gamma\,\theta\,(p_t - \bar{p}_t) \end{aligned} \tag{2.7}$$

Equation (2.7), known as the "*Lucas supply curve*", shows that fluctuations of output around the natural level depend positively on unanticipated movements in the aggregate price level p_t. As for the local supply functions, also the "slope" of the aggregate output supply (2.7) depends not only on the structural parameter γ but also on θ and therefore on the relative variance of demand shocks. The larger the variance of the aggregate disturbance (relative to the local shock), the more "vertical" the supply function in a traditional $(p,\, y)$ plane, since movements in local prices are mainly attributed to changes in the general price level p_t, having only a limited effect on output.

In modelling the supply side of the economy we assumed that producers base their expectations on the *true* probability distribution of the price level p_t. To check the "rationality" of the expectations formed according to (2.5) we have to get the general price level in equilibrium and derive its distribution. To this aim, we add to the model an (extremely simple) *aggregate demand* function

$$y_t = m_t - p_t \tag{2.8}$$

capturing the essential property of aggregate demand, that is a negative relationship between output and the price level. m_t can be interpreted as the nominal quantity of money supplied by monetary authorities or,more generally, as a measure of *nominal* aggregate demand.[5] Equating aggregate supply and demand, given by (2.7) and (2.8) we get the equilibrium price level:

$$p_t = \frac{1}{1+\gamma\,\theta}\,m_t - \frac{1}{1+\gamma\,\theta}\,y^* + \frac{\gamma\,\theta}{1+\gamma\,\theta}\,\bar{p}_t \tag{2.9}$$

The expected value of the general price level, $\bar{p}_t \equiv E(p_t)$, can be obtained by taking the expected value $E(.)$ of both sides of (2.9) and solving for $\bar{p}_t$:[6]

$$\bar{p}_t = E(m_t) - y^* \tag{2.10}$$

Thus, the expected price level depends on the anticipated component of money (or nominal demand), and by the (known) level of natural output. Substituting (2.10) into (2.9) and rearranging, we get the final form of the equilibrium price level:

$$p_t = \underbrace{E(m_t) - y^*}_{\bar{p}_t} + \underbrace{\frac{1}{1+\gamma\,\theta}\,(m_t - E(m_t))}_{v_t} \tag{2.11}$$

Equation (2.11) gives an economic content to the deviations of p_t from its mean value $\bar{p}_t$: they are determined by the unexpected component of money (or nominal demand), $m_t - E(m_t)$. Finally, using (2.10) and (2.9) into aggregate supply (2.7) we obtain the equilibrium output:

$$y_t = y^* + \frac{\gamma\,\theta}{1+\gamma\,\theta}\,(m_t - E(m_t)) \tag{2.12}$$

[5]Rewriting (2.8) as $y_t + p_t = m_t$ highlights the interpretation of m_t as an indicator of the nominal level of aggregate demand, since $y_t + p_t$ ($\equiv \log(Y_t P_t)$) is output in nominal terms. This interpretation is adopted by Lucas (1973).

[6]Note that $\bar{p}_t$ is the *unconditional* expected price level, obtained on the basis of available *aggregate* information only, and not conditioned on the observation of a local price level.

Only the unexpected component of aggregate demand affects output, whereas the perfectly anticipated component $E(m_t)$ affects only the price level p_t. The economic mechanism whereby purely nominal disturbances (the unexpected component of m_t) have real effects is based on the producers' imperfect information about the nature of the shocks hitting the local price level. With imperfect information, an unexpected increase of nominal demand is *partly* attributed to a local disturbance, making the local price change relative to the general price level. The optimal response of all producers is then to increase supply, which generates an aggregate output increase. This response is stronger the higher is the (relative) variability of local shocks (given by the ratio τ^2/σ^2) and therefore the larger is θ.

2.2. Implications

Lucas model has relevant implications for policy and econometric practice.

The output-inflation trade-off. The model provides microeconomic foundations to Friedman's insight that informational imperfections may generate a positive correlation between movements of nominal (money or, more generally, nominal aggregate demand) and real variables (employment, output).

Differently from Friedman's model, the Lucas model is not based on an adaptive expectations mechanism, but assumes that agents form expectations rationally using all available information. Again, a *Phillips curve* arises, linking unforeseen changes in nominal demand to deviations of output from its natural level. The adoption of the rational expectations hypothesis leads to the substitution of the distinction between a short-run and a long.run Phillips curve (as in Friedman's model) with that between a (positively sloped) Phillips curve which applies to *unexpected* changes in demand, and a vertical Phillips curve which applies to *anticipated* demand changes: the latter curve does not show any trade-off between output and inflation.

The "Lucas critique". The derivation of the supply curve and the equilibrium levels of output and prices given above showed that the relationship between changes in nominal and real variables depends on both "structural" and "non structural" parameters. The structural parameters capture features of the economy, such as technology and agents' preferences, that do not vary when demand-management policies, affecting the process generating aggregate demand, change; those parameters are then *policy-invariant*, as γ in the model. Instead, non structural parameters change when the properties of aggregate demand and economic policies vary. This is the case of parameter θ in the model, which (together with

γ) determines the effect of unanticipated fluctuations in aggregate demand on output (i.e. the "slope" of supply function). θ depends on the relative variance of local and aggregate shocks, on which monetary policy can have a direct influence. In fact, a policy characterized by large unpredictable fluctuations in money supply determines a relatively large variance of aggregate demand disturbances σ^2, thereby affecting the ratio θ. In this case, the price-output relation is not a policy-invariant feature of the economy but depends also on (in particular monetary) policymakers' decisions. This basic insight has been put forward by Lucas (1976) and has become known as the *Lucas critique* to traditional economic (and econometric) models, where behavioral relationships were implicitly considered as invariant to changes in economic policies.

Lucas (1973) provides a simple empirical application which is inspired by his own critique. From (2.12), the coefficient linking equilibrium output to unexpected aggregate demand can be expressed (writing θ in terms of the variances σ^2 and τ^2) as:

$$\frac{\gamma\,\theta}{1+\gamma\,\theta}=\frac{\gamma\,\tau^2}{\sigma^2+(1+\gamma)\,\tau^2} \tag{2.13}$$

For given τ^2 and γ this coefficient decreases in σ^2. The basic idea of the test is to compare economies characterized by a different variability of aggregate demand (σ^2), and see whether there is a systematic negative correlation between the estimates of σ^2 and the estimates of the "slope" of the supply function (2.12). Overall, the empirical evidence presented by Lucas supports this implication of the model.

The key point of the Lucas critique has a general validity, and, in models with rational expectations, derives immediately from the fact that agents, in forming expectations on the relevant variables, use also what they know about how economic policies are conducted. It follows that parameters that describe such policies play a role in determining agents' behavior. From the perspective of econometric practice, policy changes affect the parameters of equations describing agents' behavior: therefore, values for those parameters estimated on past data are inappropriate to simulate the effects of new policy measures.

3. New Classical Macroeconomics: the policy ineffectiveness proposition

The fundamental insights of Lucas (1973) model are at the heart of the so-called "*New Classical Macroeconomics*" (NCM). This school of thought (associated mainly to the names of Lucas, Sargent, Wallace and Barro) made an effort to offer theoretical underpinnings to macroeconomic fluctuations, particularly to the comovements between output and inflation, entirely grounded on the assumption of market-clearing on all markets, in open contrast with traditional macroeconomic models of keynesian inspiration, based on price and wage rigidities that slow down or completely hinder the equilibrium adjustment of demand and supply. Combining the market-clearing assumption with the rational expectations hypothesis the NCM derives sharp implications on the ineffectiveness of economic policies based on aggregate demand management (known as the *policy ineffectiveness proposition*).

In general, the essential elements of NCM models are:

- the existence of a *natural rate of output* (and therefore unemployment) determined exclusively by real forces;
- continuous and instantaneous *market-clearing* on all markets due to perfectly flexible prices;
- *rational expectations.*

3.1. A typical NCM model

What follows is a simplified version of the model by Sargent and Wallace (1976), which formally derives the ineffectiveness proposition. Moreover, it provides a clear application of a solution technique for models with rational expectations that will be used again later on in the course.

The demand side of the model is based on the elementary macroeconomic $IS-LM$ structure and on the three assumptions mentioned above. The IS and LM curves have a traditional form (in the following equations all variables, with the exception of the nominal interest rate, are in logarithms):

$$
\begin{aligned}
y_t &= -b\,(i_t - E_{t-1}(p_{t+1}-p_t)) + v_{1t} &\quad (3.1)\\
m_t - p_t &= y_t - d\,i_t + v_{2t} &\quad (3.2)
\end{aligned}
$$

where y, m and p are aggregate output, the quantity of money, and the price level, respectively. For simplicity, the natural level of output is assumed constant over time and normalized to zero (in logs): y can then be interpreted as a deviation from the natural level. Finally, i is the nominal interest rate and v_1 and v_2 are two shocks with the property: $E_{t-1}v_{1t} = E_{t-1}v_{2t} = 0$.

Equation (3.1) describes a simple negative relation between demand for goods and the real interest rate (expressed as the nominal interest rate at time t less the inflation rate between t and $t+1$ rationally expected on the basis of the available information at the end of period $t-1$). Equation (3.2) is the equilibrium condition on the money market, imposing equality between real money supply and real money demand (the latter depending positively on output y -with unit elasticity- and negatively on the nominal interest rate i). v_1 and v_2 have the interpretation of shocks to the demand for goods and to the demand for money respectively. Combining the IS (3.1) and the LM (3.2) functions to eliminate the nominal rate i_t, we get thw following *aggregate demand* function (AD):

$$y_t = \alpha\ (m_t - p_t) + \beta\, E_{t-1}(p_{t+1} - p_t) + v_t \tag{3.3}$$

where $\alpha \equiv \frac{b}{b+d}$, $\beta \equiv \frac{bd}{b+d}$, and $v_t \equiv \left(\frac{1}{b+d}\right)(d\, v_{1t} - b\, v_{2t})$ is a composite disturbance to aggregate demand.

The supply side of the economy is described by a Lucas-type *aggregate supply* function, with deviations of output from its natural level due only to the unexpected component of the price level and to a supply disturbance u (reflecting shocks to technology and preferences), with the property $E_{t-1}u_t = 0$:

$$y_t = \gamma\ (p_t - E_{t-1}p_t) + u_t \tag{3.4}$$

Monetary policy is conducted by setting the nominal money supply m_t according to a *feedback rule* of the following form:

$$m_t = m_{t-1} - \delta y_{t-1} + \varepsilon_t \tag{3.5}$$

Money reacts systematically to past deviations of output from its natural level in a "countercyclical" fashion (given the negative coefficient $-\delta$ on y_{t-1}); ε_t denotes the non-systematic, unpredictable ($E_{t-1}\varepsilon_t = 0$) component of money supply.

To solve the model made up of (3.3), (3.4) and (3.5) we employ one of the standard solution techniques for models with rational expectations, namely the *method of undetermined coefficients.*

By equating aggregate demand and supply and using the monetary rule (3.5), we obtain a first expression for the equilibrium price level:

$$p_t = \left(\frac{1}{\alpha+\gamma}\right)\left[(\gamma-\beta)\,E_{t-1}p_t + \beta\,E_{t-1}p_{t+1} + \alpha\,m_{t-1} - \delta\alpha\,y_{t-1} + \alpha\,\varepsilon_t + v_t - u_t\right] \tag{3.6}$$

Here the price level is a function of past variables (m_{t-1} and y_{t-1}), current disturbances (ε_t, v_t and u_t) and expectations of the price level itself at time t and $t+1$ (formed on the basis of available information at $t-1$). The adopted solution technique assumes a (linear) solution for the price level p_t with coefficients to be determined in order to satisfy equation (3.6).

Given the set of variables which appear on the right-hand side of (3.6), we can *guess* a solution for the price level of the following form:

$$p_t = \pi_1 m_{t-1} + \pi_2 y_{t-1} + \pi_3 \varepsilon_t + \pi_4 v_t + \pi_5 u_t \tag{3.7}$$

where π_1, ... π_5 are coefficients to be determined. From (3.7) we can derive the expected values $E_{t-1}p_t$ e $E_{t-1}p_{t+1}$ (using the fact that ε_t, v_t and u_t are shock unpredictable one period earlier):

$$\begin{aligned} E_{t-1}p_t &= \pi_1 m_{t-1} + \pi_2 y_{t-1} && (3.8) \\ E_{t-1}p_{t+1} &= \pi_1 E_{t-1}m_t + \pi_2 E_{t-1}y_t \\ &= \pi_1 m_{t-1} - \pi_1 \delta\, y_{t-1} && (3.9) \end{aligned}$$

where we used (3.4) and (3.5) to express the expected values at $t-1$ of m_t and y_t. It is now possible to equate our guess solution (3.7) to (3.6), making use of (3.8) e (3.9) to substitute for the terms involving the expected price level. We get:

$$\begin{aligned} \pi_1 m_{t-1} + \pi_2 y_{t-1} + \pi_3 \varepsilon_t + \pi_4 v_t + \pi_5 u_t = {} & \frac{\gamma-\beta}{\alpha+\gamma}\left(\pi_1 m_{t-1} + \pi_2 y_{t-1}\right) \\ & + \frac{\beta}{\alpha+\gamma}\left(\pi_1 m_{t-1} - \pi_1 \delta y_{t-1}\right) \\ & + \frac{1}{\alpha+\gamma}\left(\alpha m_{t-1} - \alpha\delta y_{t-1} + \alpha\varepsilon_t + v_t - u_t\right) \end{aligned} \tag{3.10}$$

In the last step of the solution procedure we have to find the values of the (as yet undetermined) coefficients π_i that satisfy (3.10). To this aim, we solve the system of equations obtained by equating coefficients on the same variables on the left-hand and right-hand sides of (3.10). We then get:

$$
\begin{aligned}
m_{t-1} \ : \quad \pi_1 \quad &= \quad \frac{\gamma-\beta}{\alpha+\gamma}\,\pi_1 + \frac{\beta}{\alpha+\gamma}\,\pi_1 + \frac{\alpha}{\alpha+\gamma} \\
&\Rightarrow \quad \pi_1 = 1 \\
y_{t-1} \ : \quad \pi_2 \quad &= \quad \frac{\gamma-\beta}{\alpha+\gamma}\,\pi_2 - \frac{\beta\delta}{\alpha+\gamma}\,\pi_1 - \frac{\alpha\delta}{\alpha+\gamma} \\
&\Rightarrow \quad \pi_2 = -\delta \\
\varepsilon_t \ : \quad \pi_3 \quad &= \quad \frac{\alpha}{\alpha+\gamma} \\
v_t \ : \quad \pi_4 \quad &= \quad \frac{1}{\alpha+\gamma} \\
u_t \ : \quad \pi_5 \quad &= \quad -\frac{1}{\alpha+\gamma}
\end{aligned}
$$

Solving this system we obtain the solution for the price level p_t:

$$p_t = m_{t-1} - \delta y_{t-1} + \frac{1}{\alpha+\gamma}\,(\alpha\,\varepsilon_t + v_t - u_t) \tag{3.11}$$

From (3.11) we derive the price "surprise" which, according to (3.4), causes output to deviate from its natural level:

$$p_t - E_{t-1}p_t = \frac{1}{\alpha+\gamma}\,(\alpha\,\varepsilon_t + v_t - u_t) \tag{3.12}$$

Finally, substituting (3.12) into the *AS* function (3.4) we get the final form for output at time t:

$$y_t = \frac{\gamma}{\alpha+\gamma}\,(\alpha\,\varepsilon_t + v_t - u_t) + u_t \tag{3.13}$$

Equilibrium output deviates from the natural rate only because of unanticipated changes in money suply (ε_t) and of shocks to aggregate demand and supply (v_t and u_t). The specific monetary policy rule adopted by the policymaker, captured by the parameter δ (measuring the degree of policy countercyclicality), has no role in determining output. It is then impossible to use systematic monetary policy for output stabilization purposes.

Note that, if the assumption that shocks follow white noise (totally unpredictable) stochastic processes is relaxed, then only the unanticipated part of ε_t and v_t would affect output; moreover, as for the supply shock u_t, beside the effect of the unanticipated component acting through the price surprise, as shown in (3.12), also the anticipated part of the shockwould affect output, since the whole of the disturbance u_t shifts the aggregate supply curve (3.4).

The most widely known proposition associated to the NCM derives directly from (3.13): systematic monetary policy cannot be used to stabilize output. Any monetary *feedback* rule is predictable by agents who base their decisions about labor and output supply on rational expectations that take into account the features of the rule, and therefore cannot cause deviations of output from the natural level. Only unexpected changes in the monetary instruments (here, the supply of money) affect output, at lest in the short run (here, just in the current period). The kind of "Phillips curve" that comes out from NCM models is then vertical with respect to the anticipated part of monetary policy and shows a positive output-inflation correlation only with respect to the unanticipated policy component.

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Problems

1. (*Expectations and price dynamics*) Suppose that the market for a good is described by the following demand and supply functions and by the market-clearing condition:

$$\begin{aligned} q_t^D &= a_0 - a_1 p_t \\ q_t^S &= b_0 + b_1 p_{t-1,t}^e + u_t \\ q_t^D &= q_t^S \equiv q_t \end{aligned}$$

where q_t^D and q_t^S are, respectively, the quantity demanded and supplied of the good at time t, and u_t is a stochastic supply shock which follows a normally distributed white noise process, $u_t \sim N(0, \sigma_u^2)$. All parameters in the model are positive. The demand for the good at t depends deterministically on price p_t (no demand shock is present), whereas supply at t depends positively on the price *expected* one period earlier, $p_{t-1,t}^e$, and is affected by the disturbance u_t.

   (a) Find the equilibrium price for the good p_t and explain its dependence on expected price $p_{t-1,t}^e$.

   (b) Assuming *rational expectations* (such that $p_{t-1,t}^e = E_{t-1} p_t$), find the equilibrium price level p_t. What are the determinants of the forecast error in each period t ?

   (c) Find the equilibrium price level p_t under the alternative assumption of *adaptive* expectations

$$p_{t-1,\,t}^e = \lambda\, p_{t-1} + (1-\lambda)\, p_{t-2,\,t-1}^e \qquad 0 < \lambda < 1$$

   and compare your result with the answer to question (b).

2. (*Cagan's hyperinflation model*) Using the hyperinflation model presented in Section 1 with the assumption of *rational* expectations and with a white noise stochastic disturbance u_t added to money demand:

   (a) find the price level p_t knowing that money supply follows a first-order autoregressive stochastic process:

$$m_t = \rho\, m_{t-1} + \varepsilon_t$$

   where ε_t is a white noise shock and $0 < \rho < 1$.

(b) How and why does the parameter ρ enter the relationship between money and prices? (comment upon the cases $\rho \to 1$ and $\rho \to 0$).

(c) Find the price forecast error at time t and check that it is unpredictable given information available at $t-1$.

3. (*Lucas critique*). Consider a simple macroeconomic model composed by the following aggregate supply (AS) and aggregate demand (AD) functions:

$$\begin{aligned} y_t &= \gamma (p_t - E_{t-1}p_t) + u_t && \text{(AS)} \\ y_t &= m_t - p_t \,, && \text{(AD)} \end{aligned}$$

where all variables are in logs, y is the output level (with $\bar{y} = 0$ for simplicity), p is the price level, m is the nominal quantity of money, and $E_{t-1}p_t$ is the rational expectation of the price level in t formed by agents in the private sector on the basis of all information available at time $t-1$. u is a supply shock such that $E_{t-1}u_t = 0$. Suppose that the monetary authorities set money supply in each period according to the following rule, perfectly known to agents:

$$m_t = \bar{m} + p_{t-1} + \varepsilon_t \,, \qquad \text{(MR)}$$

where $\bar{m}$ is a constant and ε is a stochastic, unpredictable component ($E_{t-1}\varepsilon_t = 0$).

(a) Prove that only the stochastic component of the monetary rule does affect output;

(b) find a relationship between output y_t and the actual inflation rate $\pi_t \equiv p_t - p_{t-1}$. What is the nature of the parameters in this relation?

4. (*Intertemporal substitution and the Lucas critique*) Let the aggregate supply function be of the following form:

$$y_t = \gamma (p_t - E_t p_{t+1})$$

This function captures an "intertemporal substitution" mechanism, whereby producers increase (decrease) output when the current price level p_t is higher (lower) than the price level expected for the next period (based on the information available at t). The aggregate demand function takes the simple form

$$y_t = m_t - p_t$$

and money supply is determined by the following stochastic process

$$m_t = \rho\, m_{t-1} + \varepsilon_t$$

where ε_t is a white noise shock to the (first-order autoregressive) monetary rule and $0 \le \rho \le 1$.

(a) Solve the model for the equilibrium price and output levels [Hint: use the method of undetermined coefficients]. How and why does the response of output to a monetary shock ε_t depend on the parameter ρ?

(b) Find the relationship between y_t and p_t and show that the "slope" of this traditional aggregate supply function depends on the parameter of the policy rule ρ (an example where the Lucas critique applies).

5. (*New Classical Macroeconomics*) Consider a simplified version of the NCM model studied in Section 3, with the following aggregate demand and aggregate supply equations:

$$\begin{aligned} y_t &= \alpha\,(m_t - p_t) + v_t \\ y_t &= \gamma\,(p_t - E_{t-1}p_t) + u_t \end{aligned}$$

where $v_t \sim N(0, \sigma_v^2)$ and $u_t \sim N(0, \sigma_u^2)$ are uncorrelated white noise demand and supply shocks respectively. Monetary policy sets money supply according to the following rule:

$$m_t = m_{t-1} + \delta_1\, u_t - \delta_2\, v_t$$

(with $\delta_1, \delta_2 > 0$). In this setting, *monetary authorities have more information* than private producers, and can set m_t *after* observing the time t realization of the two shocks (whereas producers form expectations on p_t only on the basis of information dated $t-1$).

(a) Solve the model for the equilibrium price level and output and discuss their dependence on the parameters of the monetary rule δ_1 and δ_2;

(b) suppose that monetary authorities want to *stabilize* output around the perfect-information level (with no price surprises), that is $y_t = u_t$. To this aim, they choose the values of the policy parameters δ_1 and δ_2 to minimize the variance of output fluctuations $y_t - u_t$:

$$\min_{\delta_1, \delta_2}\; var\,(y_t - u_t) = var\,(p_t - E_{t-1}p_t)$$

Find the optimal values for the policy parameters and discuss the effectiveness of monetary policy in stabilizing output with respect to demand and supply disturbances.

6. (*New Classical Macroeconomics*) Consider a modified version of the NCM model of Section 3, where private agents in *financial markets* have *more information* than producers and monetary authorities. To capture this information asymmetry the aggregate demand function (obtained by combining the IS and LM schedules) is modified as:

$$y_t = \alpha\,(m_t - p_t) + \beta\,E_t(p_{t+1} - p_t) + v_t$$

where the information set used by agents to form expectations on the inflation rate is dated t (therefore including the current value of the price level p_t) and $v_t \sim N(0, \sigma_v^2)$ is a white noise aggregate demand shock. Aggregate supply is of the traditional (Lucas) "surprise" variety:

$$y_t = \gamma\,(p_t - E_{t-1}p_t) + u_t$$

with $u_t \sim N(0, \sigma_u^2)$ being an aggregate supply disturbance, uncorrelated with the demand shock. Finally, monetary policy follows a *feedback rule* which makes money supply react to $t-1$ realizations of the two disturbances:

$$m_t = \delta_1 u_{t-1} - \delta_2 v_{t-1}$$

(a) Solve the model for the price level and output in equilibrium;

(b) suppose (as in Problem 5) that monetary authorities want to *stabilize* output around the perfect-information level (with no price surprises), that is $y_t = u_t$. To this aim, they choose the values of the policy parameters δ_1 and δ_2 to minimize the variance of output fluctuations $y_t - u_t$:

$$\min_{\delta_1,\delta_2}\ var\,(y_t - u_t) = var\,(p_t - E_{t-1}p_t)$$

Find the optimal values for the policy parameters and discuss the effectiveness of monetary policy in stabilizing output with respect to demand and supply disturbances.

(c) Compare your results with those of Problem 5. Is it possible that monetary policy is effective in stabilizing output even though monetary authorities have less information than (some) other agents in the economy? (provide an economic intuition for this result).