Macroeconomic Analysis

Lecture notes (4) on: *Dynamic macroeconomic models of real-financial interactions*

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The rational expectations hypothesis has been introduced in many macroeconomic models, even outside the framework of the new classical macroeconomics, to investigate the interactions between asset prices and real activity. These notes outline two dynamic models that focus on the determinants of stock prices and exchange rate fluctuations, and their effects on the real economy.

1. The stock market in a dynamic IS-LM model (Blanchard 1981)

This section describes a simplified version of the dynamic $IS - LM$ model in Blanchard (1981), that extends this elementary macroeconomic scheme along two dimensions: (i) by considering dynamics, and (ii) by enlarging the set of financial markets beyond the traditional "money" and "bonds". In the model below, the latter extension is limited to the *stock market*. Stock prices have a *forward-looking* nature, since they incorporate agents' expectations on the future course of the economy (particularly on the determinants of stock market returns: dividends, future stock prices, and interest rates).[1] The changes in stock prices (and, in extended versions of the model, in the prices of other financial assets such as long-term bonds) highlight the role of expectations as driving forces of real variables

[1] In the model, absent any stochastic element, the rational expectations hypothesis coincides with perfect foresight.

dynamics and as determinants of the effects of economic (fiscal and monetary) policies.

The main features of Blanchard's model are:

- the economy is closed, and, as in the static version of the $IS - LM$ model, the goods price is exogenously fixed and constant;
- private investments (a component of aggregate demand) depend positively on the ratio of the market value of the productive capital owned by firms to its replacement cost: this ratio is known as Tobin's q;
- in the model, q is interpreted as the market valuation of firms' capital stock. Such valuation is incorporated into the level of stock prices: an increase in stock prices signals that the market attributes a higher value to the capital stock (relative to its replacement cost), thereby inducing firms to increase investment and enlarge the existing capital stock.

Formally, the aggregate demand side of the model is represented by equations describing goods demand and the equilibrium on financial asset markets (money, bonds, and stocks), and the supply side, with fixed prices, is given by a function capturing the dynamic adjustment of output to excess demand or excess supply in the goods market. In the following, time t is measured by a continuous variable, and all variables (with the exception of the price level p) are functions of time.

1.1. Aggregate demand

The aggregate *demand for goods*, $y^D(t)$, is given by the following linear function:

$$y^D(t) = \alpha\, q(t) + c\, y(t) + g(t) \qquad \alpha > 0\ ,\ 0 < c < 1 \tag{1.1}$$

The level of aggregate demand has three determinants: output $y(t)$ through the consumption function (the parameter c captures the marginal propensity to consume), the level of stock prices $q(t)$ (here the only determinant of firms' investments), and a measure of the fiscal policy stance $g(t)$ (e.g. government spending less taxation), exogenously set by the policymaker.

Three financial assets are available: money, short-term bonds, and stocks. Therefore, a complete description of the simultaneous equilibrium on all financial markets is given by two equilibrium conditions. On the money market, equilibrium is described by a standard LM curve:

$$\frac{m(t)}{p} = h_0 + h_1\, y(t) - h_2\, r(t) \tag{1.2}$$

where real money supply (nominal money $m(t)$ -used as the monetary policy instrument, perfectly controlled by the central bank- divided by the exogenous price level p) is equal to money demand, that depends positively on output (for transaction motives: purchases of goods and payments to productive inputs) and negatively on the interest rate on short-term bonds $r(t)$.[2] For analytical convenience, we assume that the short-term bonds have "instantaneous" (infinitesimal) maturity. Thus, the rate of return obtained on those bonds coincides with the interest rate r with no additional capital gain (or loss) component due to changes in the market price of bonds.

Short-term bonds and stocks are seen as perfect substitutes in investors' portfolios: therefore, their rates of return must be equal in equilibrium. Investors' arbitrage operations on the two financial markets ensures that any return differentials is immediately eliminated, restoring at any time t the following equilibrium condition (known as a "*no-arbitrage condition*"):

$$\frac{\pi(t)}{q(t)} + \frac{\dot{q}(t)}{q(t)} = r(t) \tag{1.3}$$

where the left-hand side is the (instantaneous) rate of return on stocks, made up of dividends paid to shareholders $\pi(t)$ (and, by assumption, equal to firms' profits) and by the capital gain (or loss) due to stock price changes. Such changes are captured by $\dot{q}(t) \equiv \frac{dq(t)}{dt}$, where the dot over q denotes the time derivative: $\dot{q}(t) > 0$ is a capital gain, whereas $\dot{q}(t) < 0$ a capital loss.[3] The no-arbitrage condition (1.3) imposes equality in each moment in time between the rate of return on stocks and the short-term interest rate $r(t)$.[4]

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[2] The assumption of fixed prices implies a zero expected inflation rate. Therefore, there is no need to distinguish between the nominal and the real interest rates.

[3] The absence of stochastic elements in the model allows to equate the *expected* capital gain $\dot{q}^e(t)$ (that should enter the definition of the rate of return on stocks in (1.3)) with the *realized* capital gain $\dot{q}(t)$. In the absence of uncertainty, the rational expectations hypothesis coincides with perfect foresight: the expected and realized changes in shock prices are equal.

[4] In the original version of the model (Blanchard 1981), also long-term bonds are introduced. In this case, a second no-arbitrage condition would be necessary to ensure that the return on those bonds (made up of two elements -interest payments and price changes- as for stocks) is equal to the short-term rate $r(t)$.

Finally, profits (entirely paid out as dividends to shareholders) are positively related to output:

$$\pi(t) = a_0 + a_1\, y(t) \tag{1.4}$$

1.2. Aggregate supply

On the supply-side of the economy, with fixed prices, the output level adjusts to clear the goods market. The evolution of output over time is given by the following dynamic (differential) equation:

$$\dot{y}(t) = \beta\,(y^D(t) - y(t)) \qquad \beta > 0 \tag{1.5}$$

where $\dot{y} \equiv \frac{dy(t)}{dt}$ is the (instantaneous) change of output in response to the excess demand for goods, $y^D(t) - y(t)$: when aggregate demand $y^D(t)$ is higher than current output $y(t)$, firms meet goods demand by decumulating inventories and increasing production only gradually over time. Therefore, changes in inventories ensure continuous clearing of the goods market, whereas output reacts only gradually to excess demand (or excess supply) on the market. Under this assumption on dynamic adjustment, output $y(t)$ has the nature of a "*predetermined*" variable: it cannot show instantaneous (i.e. "discrete") adjustments in response to changes in market demand conditions.[5]

1.3. Steady-state equilibrium and dynamics

The two differential equations (1.3) e (1.5) describe the time dynamics of stock prices $q(t)$ and output $y(t)$. In a *steady-state equilibrium* both variables do not change over time: $\dot{q}(t) = \dot{y}(t) = 0$. Off the steady-state equilibrium, $q(t)$ and $y(t)$

[5] Formally, the differential equation (1.5) can be solved to express the *level* of output in any given instant T as a function of goods demand. In the case of (1.5), the negative coefficient on $y(t)$ $(-\beta)$ guarantees that a *backward-looking* solution (making output depend only on *past* demand levels) is stable. It is then possible to express output at T as a weighted average of all past demand levels:

$$y(T) = \int_{-\infty}^{T} y^D(t)\, \beta\, \mathrm{e}^{-\beta(T-t)} dt\,,$$

with values of $y^D(t)$ in the distant past having decreasing weights. The weights sum to one: $\int_{-\infty}^{T} \beta\, \mathrm{e}^{-\beta(T-t)} dt = 1$. Therefore, in each moment in time, output is determined by the whole past goods demand history. A current excess demand can only trigger a gradual adjustment over time, and cannot determine an immediate adjustment leading instantaneously to the equality between production and demand: $y(t) = y^D(t)$.

change over time according to their dynamic equations; moreover, in each moment in time, the money market is in equilibrium, satisfying (1.2), and aggregate demand and firms' profits are at the levels given by (1.1) and (1.4).

How the economy works can be illustrated graphically by means of the *stationary equations* (or "stationary loci") for $y(t)$ and $q(t)$, capturing the relationships that hold between output and stock when we impose $\dot{y}(t) = 0$ and $\dot{q}(t) = 0$ respectively on (1.5) and (1.3). The relationships so obtained can then be plotted on a $(q,\, y)$ plane.

Let us start by imposing $\dot{y}(t) = 0$ in (1.5). Using the aggregate demand equation (1.1) to substitute out $y^D(t)$, we get the following relationship between output and stock prices:

$$y(t) = \frac{\alpha}{1-c}\, q(t) + \frac{1}{1-c}\, g(t) \tag{1.6}$$

For a given value of the fiscal measure $g(t)$, equation (1.6) describes all the combinations of output $y(t)$ and stock prices $q(t)$ that ensure equality between goods demand and supply. The positive coefficient on $q(t)$ means that a higher value of $q(t)$, by increasing private investment, stimulates aggregate spending, and an increase in production $y(t)$ is necessary to restore the stationary equilibrium on the goods market (a situation in which firms do not have to decumulate inventories to meet goods demand, and output is constant over time). The relation (1.6), plotted in Figure 1(a), is the equivalent of the IS schedule in a more traditional $IS - LM$ model, linking negatively the interest rate to output. For each level of output, there exists a unique value of $q(t)$, given by (1.6), for which output equals aggregate spending. Higher values of $q(t)$ determine larger investment flows and a corresponding excess demand for goods, $y^D(t) > y(t)$; according to the dynamic equation (1.5), output starts to increase, gradually adjusting to the higher demand: $\dot{y}(t) > 0$. This output adjustment occurs at all points above the $\dot{y}(t) = 0$ curve (or "locus"), and is graphically depicted by the arrows pointing to the right (note that, since we are dealing with the dynamics of output only, keeping $q(t)$ constant, the arrows are horizontal). Symmetrically, at all points below (1.6), there is excess supply of goods ($y^D(t) < y(t)$) and output decreases over time ($\dot{y}(t) < 0$), as shown by the arrows pointing to the left.

Similarly, the stationary locus for $q(t)$ is derived by setting $\dot{q}(t) = 0$ in (1.3):

$$q(t) = \frac{\pi(t)}{r(t)} = \frac{a_0 + a_1\, y(t)}{\frac{h_0}{h_2} + \frac{h_1}{h_2}\, y(t) - \frac{1}{h_2}\,\frac{m(t)}{p}} \tag{1.7}$$

where the last equality is obtained using (1.4) and (1.2) to substitute out $\pi(t)$ and $r(t)$. (1.7) shows the combinations of $y(t)$ and $q(t)$ for which the money market is in equilibrium and the returns on stocks and bonds are equal without the need for capital gains or losses (i.e. with constant share prices, $\dot{q}(t) = 0$). The sign of this relationship is not uniquely determined: as output increases, profits and dividends increase, raising $q(t)$; also the interest rate $r(t)$, at which future profits are discounted, increases, with a depressing effect on stock prices. The slope of the $\dot{q}(t) = 0$ locus then depends on the relative strength of those two effects; in what follows we assume that the "interest rate effect" dominates, and consequently draw a downward-sloping stationary locus for $q(t)$ in Figure 1(b); this assumption corresponds to the "bad news" case in Blanchard (1981).[6] The dynamics of $q(t)$ out of its stationary locus are governed by the no-arbitrage condition (1.3). For each level of output (that uniquely determines dividends and the interest rate from (1.4) and (1.2), only the value of $q(t)$ on the stationary locus is such that $\dot{q}(t) = 0$. Higher values of $q(t)$ (as in points above the curve) reduce the dividend component of the rate of return on shares, and a capital gain, implying $\dot{q}(t) > 0$, is needed to fulfil the no-arbitrage condition between shares and bonds: $q(t)$will then move upwards, as shown by the arrows in the figure (note that, since we are dealing with the dynamics of share prices only, keeping $y(t)$ constant, the arrows are vertical). Symmetrically, at all points below the $\dot{q}(t) = 0$ curve, capital losses are needed to equate returns, and therefore $\dot{q}(t) < 0$.

[6] Formally, the negative slope of the stationary locus for $q(t)$ depends on the following conditions on parameters:

$$\left.\frac{dq}{dy}\right|_{\dot{q}=0} < 0 \Leftrightarrow a_1 < q\,\frac{h_1}{h_2}$$

Moreover, as shown in Figure 1(b), the $\dot{q}(t) = 0$ curve has the following asymptote (as $y \to \infty$):

$$\lim_{y\to\infty} q|_{\dot{q}=0} = \frac{a_1\,h_2}{h_1}$$

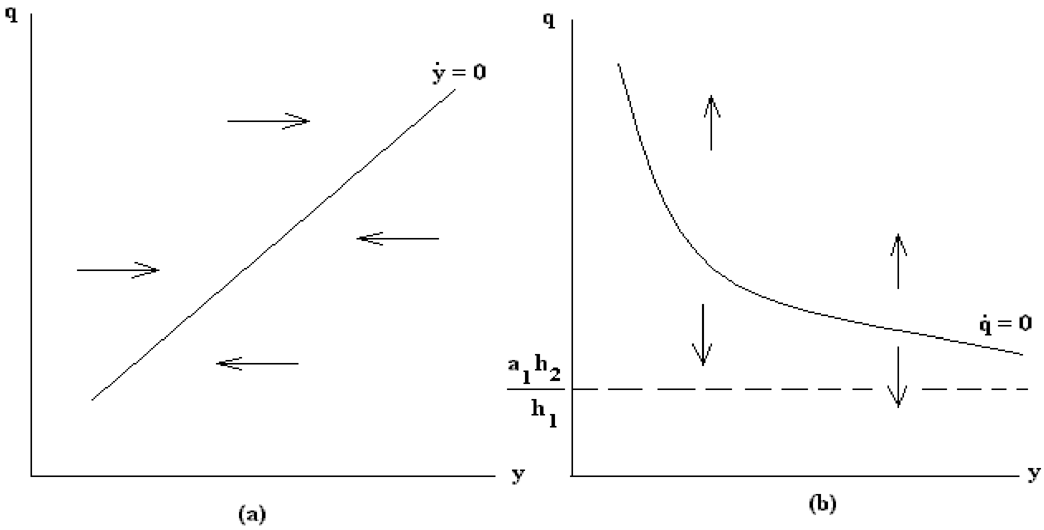

Figure 1

A complete graphical description of the joint dynamic behavior of $q(t)$ and $y(t)$ is obtained by superimposing the two pictures in Figure 1, getting the *phase diagram* shown in Figure 2. The unique *steady state* equilibrium of the system is found at the point where the two stationary loci given by (1.6) and (1.7) cross, and output and stock prices are constant over time. The arrows of motion describe, for each region of the diagram, the joint effect of the dynamics of $y(t)$ and $q(t)$ already (separately) analyzed in Figure 1. In the regions above and below the steady state equilibrium, the dynamics follow divergent paths, with $y(t)$ and $q(t)$ either increasing towards infinitely large values (in the upper region, with arrows pointing right and upwards), or decreasing towards economically nonsensical (negative) values (in the lower region, with arrows pointing left and downwards). Those paths, not converging to the steady-state equilibrium, are hardly interpretable from the economic point of view. Moreover, also trajectories starting from the left and right regions in Figure 2 may not lead the system towards the steady-state equilibrium; this happens when the dynamic paths cross the $\dot{y}(t) = 0$ locus (vertically) or the $\dot{q}(t) = 0$ locus (horizontally), and then, instead of pointing towards the steady state, proceed in the regions where arrows point away from it. Two such paths are shown in Figure 2, starting from points B and C.

In the figure, however, a pair of dynamic paths is drawn that start from points to the left and right of the steady state and continue towards it, at gradually decreasing speed,[7] without ever meeting the system's stationari loci. All points along such path are compatible with convergence towards the steady state, and together form the *saddlepath* of the dynamic system. For any given level of output, only one level of stock prices puts the system on a trajectory converging to the steady state. For example, given the output level y_0 lower than steady-state output, only the level of share prices corresponding to point A in Figure 2 allows the system to travel along a path leading to the steady state.All other values of $q(t)$, such as those corresponding to points B and C in the figure, would put the economy on an "explosive" path, gradually diverging from the steady state at an increasing speed..[8] In what follows, for reasons of economic interpretability, we will focus attention only on "stable" trajectories, converging to the unique steady-state equilibrium of the system.

[7] From the law of motion of $y(t)$ given by (1.5), the output change over time is larger the larger is the excess demand for goods $y^D(t) - y(t)$, which is gradually reduced along the path approaching the steady state. Similarly, from the dynamic equation for $q(t)$ (1.3), the change in stock prices is larger the bigger is the diference between the interest rate and the dividend component of share returns $r(t) - \frac{\pi(t)}{q(t)}$, that again gradually decreases along the converging path.

[8] Formally, the dynamic properties of $y(t)$ and $q(t)$ off their stationary loci, and the relative slopes of the $\dot{y}(t) = 0$ and $\dot{q}(t) = 0$ curves ensure the uniqueness of the saddlepath converging to the steady state.

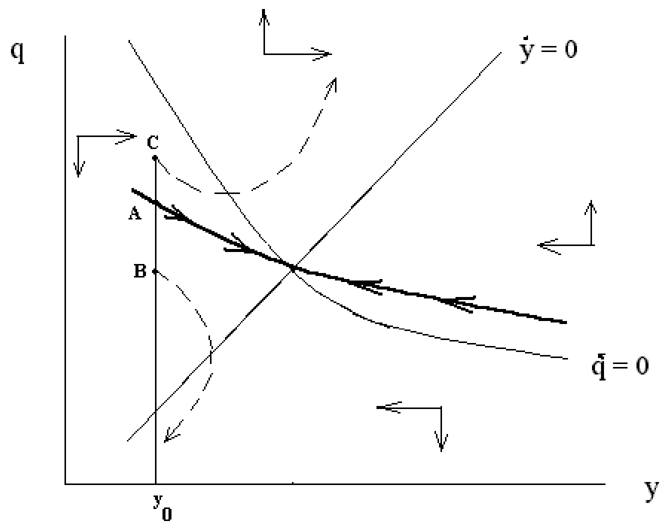

Figure 2

To rationalize the negative slope of the saddlepath, let us consider again point A in the figure, where output y_0 is lower than its steady-state level. The associated level of stock prices on the saddlepath is higher than the value of $q(t)$ on the stationary locus $\dot{y}(t) = 0$ corresponding to $y(t) = y_0$. Therefore, there is excess demand for goods owing to a high level of investment, and output gradually increases towards its steady-state value. As $y(t)$ increases, the demand for money increases as well, and, with a given money supply, the interest rate $r(t)$ rises. The level of stock prices $q(t)$ is affected by the future behavior of both output (through higher profits and therefore dividends) and the interest rate. The first effect pushes $q(t)$ upwards, whereas the "interest rate effect" on share prices (since future dividends are discounted at higher interest rates) is negative.Under our maintained assumption that the latter effect dominates, $q(t)$ then declines over time towards its steady-state value.

Formally, the behavior of share prices in any instant of time t_0 can be obtained by solving forward the differential equation (1.3), yielding the value of $q(t_0)$ as the present discounted value of future dividends:[9]

$$q(t_0) = \int_{t_0}^{\infty} \pi(t)\, \mathrm{e}^{-\int_{t_0}^{t} r(s)\, ds} dt \tag{1.8}$$

Over time, $q(t)$ changes for two reasons: on the one hand, stock prices are positively affected by the increase in dividends (resulting from higher output),; on the other, higher interest rates increase the discount factor applied to future dividends (given, in continuous time, by the negative exponential function $\mathrm{e}^{-\int_{t_0}^{t} r(s)\, ds}$), with a negative effect on $q(t)$.

Using the dynamic model of the economy developed above, we can study the effects of macroeconomic policies on output, share prices, and the interest rate. The dynamic adjustment of output and share prices, governed by the two dynamic equations (1.5) and (1.3), have fundamentally different features. As mentioned previously, output, being a predetermined variable, reacts only gradually to the excess demand for goods. Instead, with agents forming expectations rationally, stock prices are a *forward-looking* variable, immediately reacting to changes in expected future interest rates and output levels over the whole infinite horizon; therefore, $q(t)$ can display instantaneous, or discrete, adjustments ("jumps") in the face of events or news that change the expected future path of interest rates and output, for example as a consequence of the implementation (or even the announcement) of monetary and fiscal policy measures.

1.4. The effects of macroeconomic policies

Since agents, endowed with rational expectations, use all available information to form expectations on the future course of the relevant macroeconomic variables, whether the policy measures under study are unexpected or previously anticipated, and permanent or transitory, are features of crucial importance in the evaluation of their effects on the system's dynamic adjustment to a new steady state equilibrium. In this section we derive the dynamic response of output, share prices, and the interest rate to monetary and fiscal policy actions.

[9] In solving the equation, the "transversality condition"

$$\lim_{t\to\infty} \pi(t)\mathrm{e}^{-\int_{t_0}^{t} r(s)ds} = 0$$

is imposed, thereby ensuring convergence to the steady state.

1.4.1. Monetary policy

Let us consider a monetary policy *restriction*, implemented at time t_0 by means of a *permanent* reduction of the quantity of money m; such reduction is *unexpected* by agents. Up to t_0 the economy is in a steady-state equilibrium at point A in Figure 3(a). To pin down the new steady state equilibrium, with a lower quantity of money m, we note that a reduction of the money stock leaves the position of the $\dot{y}(t) = 0$ curve unchanged, while shifting the $\dot{q}(t) = 0$ locus downwards. [10] The new steady-state equilibrium is then determined at point C in the figure: in the long run, a monetary restriction causes output and share prices to decline; moreover, as in the static $IS - LM$ model, a monetary restriction increases the interest rate.

Figure 3(a) shows also the unique dynamic adjustment path converging to the new steady state along the saddlepath. At t_0, when money supply is reduced, with output still at the initial equilibrium level $y(t_0)$, the interest rate r increases to clear the money market, according to (1.2). To restore the no-arbitrage condition between bonds and stocks, share prices decrease at t_0(point B), reaching the saddlepath converging to the new equilibrium. From t_0 onwards, the dynamic adjustment follows the saddlepath (from B to C), with output declining and increasing share prices. A level of $q(t_0)$ lower than the initial equilibrium reduces investment and the aggregate goods demand, triggering a gradual decline in output. Also the interest rate, after the increase in t_0 in the face of the money supply reduction, starts decreasing, due to the lower demand for money. The dynamic paths of all variables are displayed in Figure 3(b).

The dynamic adjustment of share prices can be rationalized in terms of the behavior of output (and therefore of profits and dividends) and the interest rate. With rational, forward-looking agents, when the *unexpected* monetary restriction takes place at t_0, share prices immediately incorporate the future course of $y(t)$ (and therefore $\pi(t)$) and $r(t)$. At t_0 agents understand that, from then on, the interest rate will always be higher than in the initial equilibrium, and output and dividends will always be lower. Both effects contribute to lower the level of stock prices $q(t_0)$ needed to equate returns. As shown by (1.8), lower future profits and higher future interest rates both determine a lower level of q at the very

[10] Formally, from (1.7), we have

$$\left.\frac{\partial q}{\partial m}\right|_{\dot{q}=0} > 0$$

whereas m does not enter the stationary locus $\dot{y}(t) = 0$ (1.6).

moment of the monetary policy restriction (point B). On the subsequent dynamic path (along the saddlepath from B to C), even though output and dividends are decreasing, the effect of declining interest rates dominates, making share prices to increase towards their new steady-state level. Along this path, $\dot{q}(t) > 0$: capital gains ensure the equality of the rates of return on bonds and stocks.

In this model, the effects of a restrictive monetary policy are characterized not only in the long run (increase of the interest rate, decrease of share prices and output), but also in the adjustment dynamics of all the variables, that in some cases display a change in their direction of motion. In fact, both share prices and the interest rate show an impact response to the monetary restriction which leads their values at t_0 respectively below and above the final steady-state equilibrium levels. Afterwards, together with the gradual output decline, we observe decreasing interest rates and increasing share prices, a behavior that could be hardly explained in the absence of agents forming rational expectations

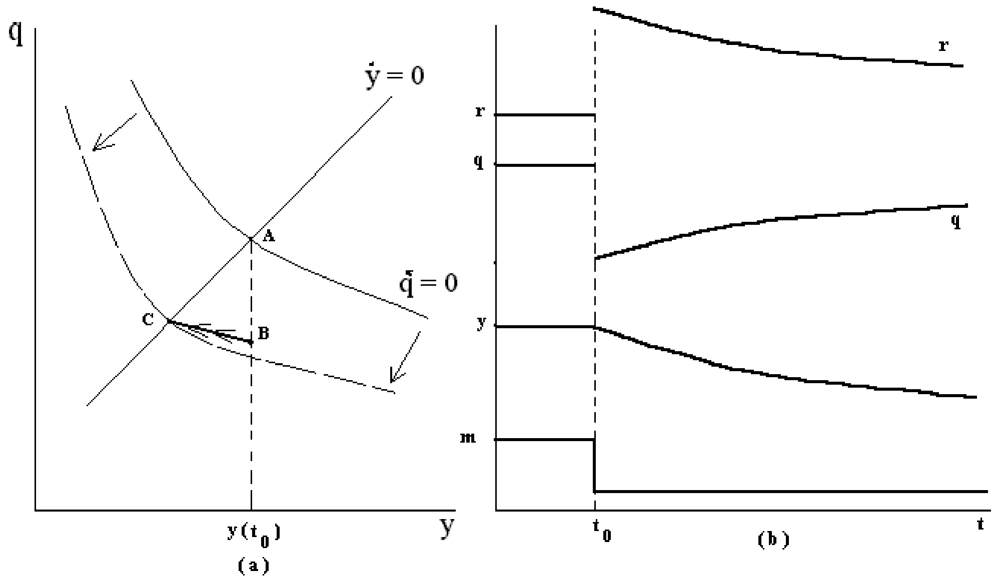

Figure 3

1.4.2. Fiscal policy

Suppose that at time t_0 a future fiscal *restriction* is *announced*, to be implemented at time $t_1 > t_0$: public spending, that is initially constant at g_0, will be decreased to $g_1 < g_0$ at t_1 and will then remain permanently at this lower level. The information on the *permanent* nature of the fiscal restriction is included in the policy announcement.

The effects of this anticipated fiscal restriction on the steady state levels of output and the interest rate are immediately clear from a conventional $IS - LM$ (static) model: in the new steady state both y and r will be lower. Both changes affect the new steady-state level of q: lower output and dividends depress stock prices, whereas a lower interest rate raises q. Again, the latter effect is assumed to dominate, leading to an increase of the steady-state value of q. Only the position of the stationary locus for output, $\dot{y}(t) = 0$, is affected by changes in g: at t_1 (the implementation date of the fiscal policy measure), this curve shifts upwards along an unchanged $\dot{q}(t) = 0$ schedule, leading to a higher q and a lower y in steady state, as shown by point D in Figure 4(a).

In order to characterize the dynamics of the system, we note that, from time t_1 onwards, no further change in the exogenous variables occurs: to converge to the steady state, the economy must then be on the saddlepath portrayed in the diagram. Accordingly, from t_1 onwards, along the saddlepath from C to D in the figure, output decreases (since the lower public spending causes aggregate demand to fall below current production) and q increases (due to the dominant effect of the decreasing interest rate).

What happens from the fiscal policy announcement at t_0 to its delayed implementation at t_1? At t_0, when the future policy becomes known, agents in the stock market anticipate lower future interest rates (they also foresee lower dividends but this effect is relatively weak); consequently, they immediately shift their porfolios towards shares, bidding up their price. Then, at the announcement date, with output and the interest rate still at their initial steady state levels, q increases, as at point B in the figure. The ensuing dynamics from t_0 up to the implementation date t_1 follow the equations of motion in (1.5) and (1.3) referred to the initial steady state (i.e. the unchanged $\dot{q}(t) = 0$ locus, and the *initial* $\dot{y}(t) = 0$ curve). A higher value of q stimulates investment, causing an excess demand for goods; starting from t_0, then, output gradually *increases*, and so does the interest rate (due to the increase in money demand). The dynamic adjustment of output and q is such that when the fiscal policy is implemented at t_1 (and the stationary locus $\dot{y}(t) = 0$ shifts upwards) the economy is exactly on the saddlepath (point C) leading to the new steady state: aggregate demand falls and output starts decreasing along with the interest rate, whereas q and investment continue to rise. Figure 4(b) displays the dynamic adjustment of all relevant variables.

Therefore, an apparently "perverse" effect of fiscal policy (an expansion of investment and output following the announcement of a future fiscal restriction) can be explained by the forward-looking nature of stock prices, anticipating future lower interest rates.

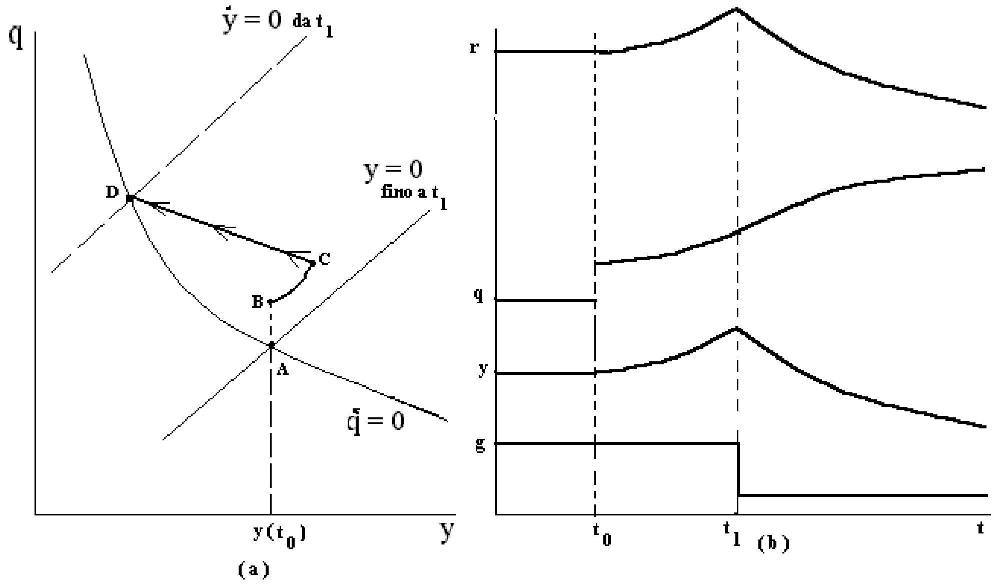

Figure 4

2. Expectations and exchange rate dynamics (Dornbusch 1976)

The wide fluctations of nominal and real exchange rates observed under flexible exchange rate regimes (perticularly in the 1970s) motivated the analysis of simple dynamic macroeconomic models where the exchange rate is determined by equilibrium conditions on financial markets, and agents are endowed with rational expectations. This section outlines a version of the most influential model of this kind, due to R. Dornbusch (1976), that shows the possibility of an over-reaction of the exchange rate to unexpected monetary policy actions (a result known as *exchange rate overshooting*).

The main features of the model are the following:

(i) the model describes a relatively "small" open economy, under a flexible exchange rate regime, and with perfect capital mobility;

(ii) on the financial markets (for domestic bonds, denominated in local currency, and foreign bonds, denominated in foreign currency) the *uncovered interest rate parity* holds, and agents form rational expectations on the future course of the exchange rate;

(iii) the (perfectly flexible) exchange rate ensures that equilibrium occurs at any instant on financial markets; on the contrary, the goods price level, though not fixed (as in the simple $IS - LM$ scheme and in Blanchard's model), adjusts only gradually in response to deviations of output from its "natural" level. Therefore, the nominal rigidity in the model takes the form of a slower adjustment of the goods price level with respect to the exchange rate.

We now specify the structure of the economy and show the dynamic response of output, the interest rate, and the exchange rate to fiscal and monetary policy measures.

2.1. The structure of the economy

The model is set in continuous time: therefore, all variables are functions of time t (for notational simplicity, this dependence will not be made explicit in the following equations). All variables, with the exception of interest rates, are in logarithms, and all parameters are positive.

In the *goods market*, output y is determined by aggregate demand according to:

$$y = -\alpha\, r + \beta\, (e + p^* - p) + \gamma\, g \tag{2.1}$$

where demand has three determinants: the nominal interest rate r, the real exchange rate $e + p^* - p$, and government spending g. Since the price level p is not fixed, the inclusion of the *nominal* instead of the *real* interest rate (i.e. $r - \dot{p}$) as a (negative) demand determinant is allowed only for the sake of analytical simplicity, without any qualitative change in the model's conclusions. The real exchange rate is defined as the ratio of the foreign goods price level converted into domestic currency (that is the nominal exchange rate E times the -exogenously set and constant- foreign price level P^*) to the domestic price level P: then $\frac{EP^*}{P}$, in logs $e + p^* - p$ as in (2.1). Given this definition, an increase in the real exchange rate corresponds to a depreciation of the domestic currency, which stimulates exports and reduces imports: the overall effect on aggregate demand is therefore positive ($\beta > 0$).

On the domestic *money market*, the standard equilibrium condition between money demand (determined by output y and the nominal domestic interest rate r) and money supply holds:

$$m - p = y - h\,r \tag{2.2}$$

where a unit elasticity of money demand to income has been assumed (for simplicity), and $-h$ measures the (semi)elasticity of money demand to the interest rate.

On the supply side of the economy, the *price level* p chnages gradually over time in response to possible deviations of current output y from its natural level $\bar{y}$, according to the dynamic equation:

$$\dot{p} = \theta\,(y - \bar{y}) \tag{2.3}$$

When current output is above (below) $\bar{y}$, the price level increases (decreases) at a gradual pace (since the value of θ is finite, $0 < \theta < \infty$).[11]

Finally, equilibrium on *asset markets* for domestic and foreign bonds is imposed by means of a no-arbitrage condition, known as *uncovered interest rate parity*:

$$r = r^* + \dot{e}^e \tag{2.4}$$

where r and r^* are the interest rates on domestic and foreign bonds respectively, and $\dot{e}^e$ is the expected change of the exchange rate. Assuming that the denomination currency is the only difference between the two bonds, and that agents are risk-neutral (and therefore consider only expected returns and not return variabilities in their portfolio allocation decisions), the rates of return on domestic and foreign bonds, both measured in domestic currency, must be equal in equilibrium. The rate of return on domestic bonds is simply the interest rate r, whereas the return on foreign bonds is given by the (exogenous) foreign interest rate r^*

[11] This assumption on price level adjustment is adopted in the last section of the original paper by Dornbusch (1976). The main version of the model adopts the stronger assumption of output fixed at its natural level.

plus the expected depreciation of the domestic currency $\dot{e}^e$. An interest rate differential in favor of domestic bonds ($r > r^*$) can be consistent with no arbitrage on bond markets only if a depreciation of the domestic currency is expected ($\dot{e}^e > 0$). The opposite occurs, with expected appreciation ($\dot{e}^e < 0$), if the interest rate differential favors foreign bonds ($r < r^*$). Immediate arbitrage operations between currencies (with perfect international capital mobility) ensure that the no-arbitrage condition (2.4) holds at each instant in time. Agents are endowed with rational expectations, and in the absence of stochastic elements in the model (as in Dornbusch 1976), this assumption leads to perfect foresight. Therefore, in (2.4) we have

$$\dot{e}^e = \dot{e} \tag{2.5}$$

Using the set of equations (2.1)-(2.5) we can now characterize the steady-state equilibrium of the economy and the adjustment dynamics of all variables in response to changes in the (exogenous) foreign variables p^* and r^*,and in the instruments of monetary and fiscal policy, m and g.

2.2. Steady-state equilibrium

The properties of the steady state of the system, representing the economy's long-run equilibrium, can be easily derived by imposing that all dynamic adjustments of the variables have been completed; therefore, there are no further changes in prices ($\dot{p} = 0$) and in the exchange rate ($\dot{e} = 0$). Imposing those conditions onto (2.3) and (2.4), we can derive the steady-state levels of all variables.

From the two dynamic equations, with $\dot{p} = 0$ and $\dot{e} = 0$, we get

$$\begin{aligned} \dot{p} &= 0 \Rightarrow y = \bar{y} && (2.6) \\ \dot{e} &= 0 \Rightarrow r = r^* && (2.7) \end{aligned}$$

stating that in the steady state, output is at the natural rate and the domestic and foreign interest rates are equal. The long-run values of the nominal variables p and e can be found using the money market equilibrium condition and the aggregate demand equation (both with $r = r^*$ and $y = \bar{y}$):

$$\begin{aligned} p &= m - \bar{y} + h\, r^* && (2.8) \\ e &= p - p^* + \frac{1}{\beta}\,\bar{y} + \frac{\alpha}{\beta}\,r^* - \frac{\gamma}{\beta}\,g && (2.9) \end{aligned}$$

The levels of p and e are then proportional to the quantity of money m. If a monetary policy action makes money supply change by dm, in the long run only a proportional change of the price level and of the exchange rate occur ($dp = de = dm$), with no effect on output, the interest rate, and the real exchange rate: monetary *neutrality* prevails in the long run.

Also a fiscal policy measure, changing government expenditure by dg, does not affect output and the interest rate in the long run. Moreover, from (2.8) we note that also the price level p is unaffected in the long run by changes in g. What is affected is the exchange rate e: from (2.9) we have that $de = -\frac{\gamma}{\beta}dg$; and, since prices are unaffected, also the real exchange rate varies by the same proportion as the nominal exchange rate. Then, a change in fiscal policy, though unable to affect the overall level of output in the long run, can change the output composition: higher government spending makes net exports to decrease by the same amount (due to the real exchange rate appreciation).

2.3. Dynamics

Let us now study the dynamics of the system outside the steady state. As a first step, we separately analyze the two dynamic equations describing the adjustment of prices (2.3) and of the exchange rate (2.4), to get the two stationary loci relating p and e that can be plotted in a phase diagram.

Starting from the *stationary locus for the price level* p, setting $\dot{p} = 0$ into (2.3) and using the money market equilibrium condition (2.2) to eliminate the interest rate r from the output equation (with $y = \bar{y}$) we get

$$\bar{y} = \beta(e + p^* - p) - \frac{\alpha}{h}\bar{y} + \frac{\alpha}{h}(m - p) + \gamma g$$

from which the following relationship between p and e can be derived

$$e = \frac{\beta h + \alpha}{\beta h}\, p - \frac{\alpha}{\beta h}\, m - \frac{\gamma}{\beta}\, g + \frac{h + \alpha}{\beta h}\, \bar{y} - p^* \tag{2.10}$$

The stationary locus for p is shown in Figure 5(a) as a positively sloped line. Along the curve, output is at its natural level and the interest rate is at the level needed to keep the equilibrium on the money market. Starting from a point on the curve (where $y = \bar{y}$), a higher p (for a given e) has two negative effects on output: on the one hand, through a real exchange rate appreciation, it decreases net exports; on the other hand, through a reduction in real money supply, it causes an increase of the interest rate, which makes investment, and therefore aggregate demand, to decline. Output is now below the natural level and an exchange rate depreciation (i.e. an increase of e) is needed to restore the equality $y = \bar{y}$. This explains the positive slope of the $\dot{p} = 0$ curve.[12]

Off the stationary locus, production is different from natural output. Indeed, at all points above the curve, $y > \bar{y}$ (because an increase of e for a given p stimulates net exports, whereby increasing output) and the price level increases over time according to (2.3): $\dot{p} > 0$, as shown by the horizontal arrows in Figure 5(a). The opposite dynamics occurs at all points below the curve, where $y < \bar{y}$ and therefore $\dot{p} < 0$. Given the assumption of gradual adjustment, the price level behaves in the model as a *predetermined* variable: when current output deviates from its natural level, the price level does not adjust instantaneously (no "jump" in p is allowed in response to unanticipated events) but follows a dynamic path pointing towards its stationary locus.

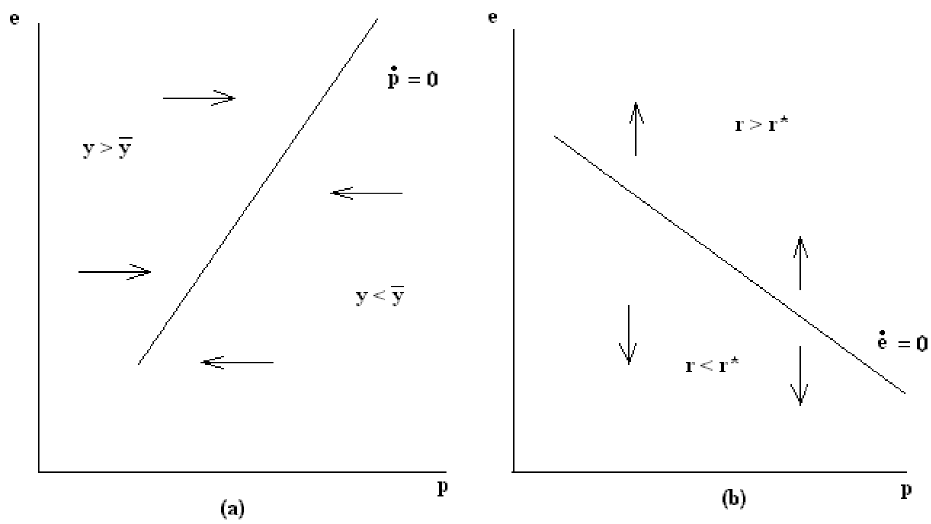

Figure 5

Similarly, we obtain the *stationary locus for the exchange rate* e by setting $\dot{e} = 0$ in (2.4) and using again the money market condition (2.2) to eliminate the

[12]From (2.10) we note that the slope is greater than 1 if $\alpha > 0$. In fact, when $\alpha = 0$, the interest rate does not affect aggregate demand and the second negative effect on output described in the text does not occur. In this case, the necessary variation in the exchange rate is simply proportional to the change in p, yielding a unitary slope of the stationary locus.

interest rate r:

$$\frac{1}{h}y - \frac{1}{h}(m-p) = r^*$$

Now, using the aggregate demand equation (with $r = r^*$) to substitute out y, and rearranging terms, we get a second relation between the price level and the exchange rate as

$$e = \frac{\beta - 1}{\beta}p + \frac{1}{\beta}m - \frac{\gamma}{\beta}g + \frac{h+\alpha}{\beta}r^* - p^* \qquad (2.11)$$

Along this stationary locus for e the domestic and the foreign interest rates are equal and the money market is in equilibrium. Differently from the case of $\dot{p} = 0$, here the slope of the stationary locus $\dot{e} = 0$ depends on the sign of the coefficient on p:

$$\left.\frac{de}{dp}\right|_{\dot{e}=0} = \frac{\beta - 1}{\beta} \gtrless 0 \quad \text{if} \quad \beta \gtrless 1$$

This ambiguity stems from the twofold effect of an increase of p (for a given level of e) on the demand and the supply of money. On the demand side, through the real exchange rate appreciation, leading to a reduction of y, money demand decreases, pushing the interest rate r downwards. On the supply side, the increase in p reduces real money supply, with an upward pressure on the interest rate. The slope of the stationary locus for the exchange rate then depends on which effect dominates. To proceed, we assume that the dominant effect is on the money supply side: an increase in the price level causes a rise in the domestic interest rate. A reduction in the exchange rate is then necessary to bring the interest rate back to a level equal to the (exogenous) foreign interest rate r^*. This offsetting effect works through the appreciation of the real exchange rate and the ensuing reduction of output and of the demand for money. Thus, in what follows, we assume that $\beta < 1$ and draw the $\dot{e} = 0$ locus as a downward sloping line in Figure 5(b).[13] Off the stationary locus, the uncovered interest rate parity (no arbitrage) condition still holds, but in the presence of a non-zero interest rate differential, and therefore with a compensating expected change in the exchange rate. At all points above the curve, since an increase in e (for a given p) increases output and the demand for money, then $r > r^*$ and an expected depreciation of the domestic currency is needed to restore equilibrium: $\dot{e} > 0$, as shown by the arrows pointing

[13] In the original paper by Dornbusch (1976), the assumption of output fixed at the natural level determines a negative slope of the stationary locus for e without any additional condition on the model's parameters.

(vertically) upwards in the figure. The opposite reasoning applies to all points below the curve, with $r < r^*$ and an equilibrating appreciation of the exchange rate, $\dot{e} < 0$.

Now, superimposing the two pictures in Figure 5, we can draw in Figure 6 the phase diagram of the system, showing the dynamic path of the whole economy at all points in the (e, p) plane. The steady-state equilibrium is at the point where the two stationary loci (2.10) and (2.11) cross. Here the price level and the exchange rate are constant over time, output is at its natural level, and the domestic and foreign interest rates are equal. The arrows of motion show, in each region of the picture delimited by the two curves, the joint effect of the forces driving the dynamics of p and e already separately illustrated in Figure 5. As in the model by Blanchard (1981), in the regions above and below the steady state, the dynamic path points away from equilibrium, and such explosive trajectories are hardly interpretable from the economic point of view. Moreover, even starting from points located in the two regions at the left and right of the steady state (such as B and C in Figure 6) the dynamic path may cross either the $\dot{p} = 0$ locus (vertically) or the $\dot{e} = 0$ curve (horizontally), ending up in a "unstable" region, again on a non-convergent trajectory. In the figure, however, a pair of dynamic paths is drawn that start from points to the left and right of the steady state and continue towards it, at gradually decreasing speed,[14] without ever meeting the system's stationari loci. All points along such path are compatible with convergence towards the steady state, and together form the *saddlepath* of the dynamic system. For any given price level p, only one level of the exchange rate e puts the system on a trajectory converging to the steady state. For example, given the price level p_0 lower than the steady-state level, only the level of the exchange rate corresponding to point A in Figure 6 allows the system to travel along a path leading to the steady state.All other values of e, such as those corresponding to points B and C in the figure, would put the economy on an "explosive" path, gradually diverging from the steady state at an increasing speed.[15] Also in analyzing this model, for reasons of economic interpretability, from now on we will focus attention only on "stable" trajectories, converging to the unique steady-state equilibrium of the system.

[14] From the laws of motion of p and e (given by (2.3) and (2.4)), their changes are larger, the larger are, respectively, the deviation of output from its natural level $(y - \bar{y})$, and the domestic-foreign interest rate differential $r - r^*$.

[15] Also in this model, the dynamic properties of p and e off their stationary loci, and the relative slopes of the $\dot{p} = 0$ and $\dot{e} = 0$ curves ensure the uniqueness of the saddlepath converging to the steady state.

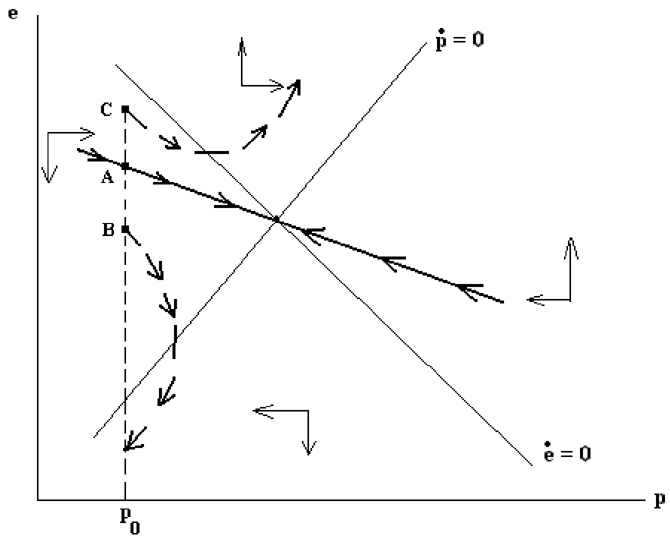

Figure 6

Starting from point A in Figure 6 (where $y > \bar{y}$ and $r < r^*$), the dynamic behavior of the economy along the saddlepath may be rationalized as follows. A level of production above natural output pushes the price level up, and an interest rate differential in favor of foreign assets causes an appreciation of the nominal exchange rate. Both effects determine an appreciation of the real exchange rate, which tends to bring output back to $\bar{y}$. Moreover, the increase in p reduces real money supply, with an upward pressure on the domestic interest rate, which reduces the interest rate differential. Travelling along the saddlepath, the deviation of y from $\bar{y}$ and of r from r^* gradually decrease and smaller and smaller changes of p and e are necessary to lead the economy towards the steady-state equilibrium.

Using the dynamic model of the economy developed above, we can study the effects of macroeconomic policies on output, prices, the interest rate and the exchange rate. In Dornbusch's model, the dynamic adjustment of prices and of the exchange rate, governed by the two dynamic equations (2.3) and (2.4), have fundamentally different features. As mentioned previously, the price level is a predetermined variable, reacting only gradually to deviations of current from natural output. Instead, with agents forming expectations rationally, the exchange rate is a *forward-looking* variable, immediately reacting to changes in expected future differentials between domestic and foreign interest rates over the whole infinite horizon. Therefore, e can display instantaneous adjustments ("jumps") in the face of events or news that change the expected future path of interest rates, for example as a consequence of the implementation (or even the announcement) of monetary and fiscal policy measures.

2.4. Monetary policy

Let us consider an *expansionary* monetary policy, implemented by means of an *unexpected permanent* increase of the money supply m at time t_0. Up to t_0 the economy is in a steady-state equilibrium at point A in Figure 7(a). The steady-state properties of the system studied above ensure that in the long run money neutrality prevails: an increase in m entails an increase in p and e of the same magnitude. The new steady state equilibrium is at point C in the figure. The shifts of the stationary loci caused by the money supply increase can be formally derived from (2.10) and (2.11). We have:

$$\left.\frac{de}{dm}\right|_{\dot{p}=0} = -\frac{\alpha}{\beta h} < 0$$

which implies a downward shift of the $\dot{p} = 0$ locus,[16] and

$$\left.\frac{de}{dm}\right|_{\dot{e}=0} = \frac{1}{\beta} > 0$$

which implies an upward shift of the $\dot{e} = 0$ curve. Then, in the long run, the monetary expansion causes a depreciation of the nominal exchange rate and an increase of the price level, so that the real exchange rate $(e + p^* - p)$ is brought back to its initial level.

The adjustment towards the final steady state follows the saddlepath shown in Figure 7(a). Along the path, the nominal exchange rate appreciates (e decreases) and p gradually increases: both effects make the real exchange rate to appreciate as well. At t_0, when the unanticipated monetary policy is implemented, the price level does not react (as a predetermined variable, it cannot respond immediately to deviations between current production and natural output), whereas the nominal

[16] Note that in the case of $\alpha = 0$, the stationary locus for p has a unitary slope and does not shift in the face of changes in m. Both the initial and the final steady-state equilibria are on the same (unchanged) $\dot{p} = 0$ curve.

exchange rate e instantaneously adjusts, jumping to point B onto the saddlepath converging to the new steady state. At t_0, then, a *depreciation* of the nominal (and real) exchange rate occurs, with e reaching a higher level then in the long-run equilibrium (at C). This (apparently) excessive impact response of the exchange rate with respect to its long-run level is the most notable result of the Dornbusch's model, known as *exchange rate overshooting*.

The depreciation of the nominal exchange rate at t_0 in the face of an increase in money supply can be rationalized by considering what happens on the money market. With p still fixed at its initial level, the increase in m determines an increase in real money supply, with a downward pressure on the domestic interest rate r. In their portfolio allocation decisions, investors will then favor foreign bonds, causing a capital outflow and a depreciation of the domestic currency (i.e. an increase in e). The magnitude of this depreciation is such as to restore the no-arbitrage condition on financial markets: with an interest rate differential in favor of foreign bonds ($r < r^*$), only an expected *appreciation* ($\dot{e} < 0$) can ensure the equality of returns. The initial change (depreciation) of the exchange rate must then be large enough to generate an expected appreciation equal to the interest rate differential: for this reason, at t_0 the exchange rate e increases beyond ("overshoots") its long-run equilibrium level. After t_0, e gradually decreases (appreciates) along the converging saddlepath.

The exchange rate overshooting result is due to the different adjustment speed of p and e to the monetary expansion: the nominal exchange rate reacts immediately to the interest rate differential, whereas p responds only gradually to deviations of y from $\bar{y}$. If also the price level could adjust immediately to the new money supply, then m, p and e would all increase at t_0 by the same amount, and no further dynamic adjustment would be necessary: the economy would reach instantaneously the new steady state. Instead, with an initially fixed price level, the excess money supply must be offset by a decrease of the interest rate r, which induces investors to shift their portfolios towards foreign bonds, making the exchange rate to depreciate. This capital outflow, and the ensuing depreciation, will stop only when the expected future appreciation exactly offsets the interest rate differential, making investors again indifferent between holding foreign or domestic bonds in their asset portfolios.

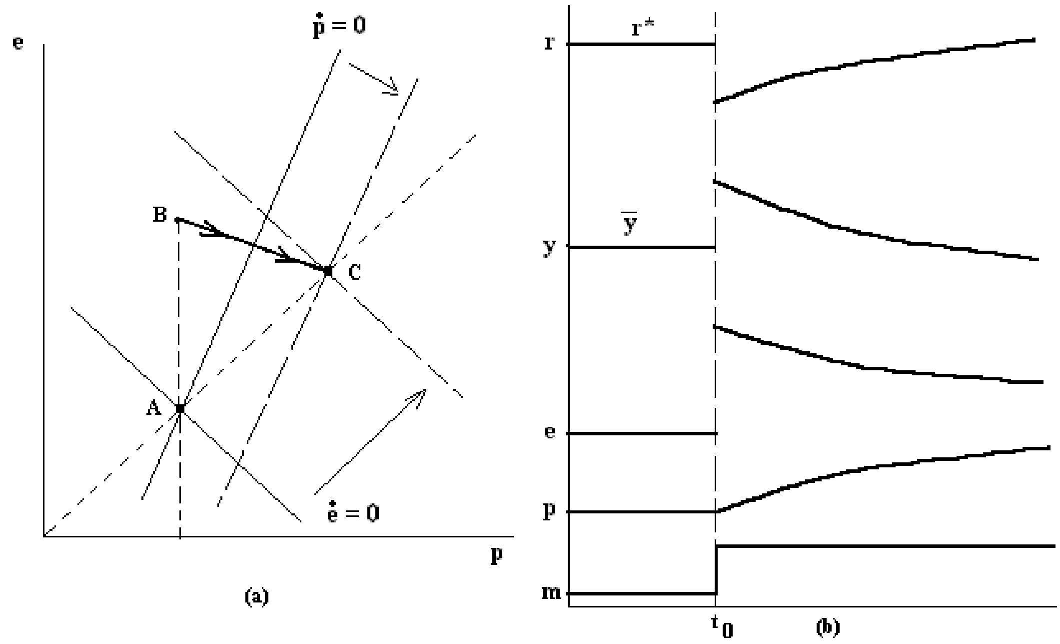

Figure 7

The dynamic behavior of all variables in the model is shown in Figure 7(b). Output increases at t_0 due to the real exchange rate depreciation; the positive deviation from its natural level causes a gradual increase in the price level. On the money market, from t_0 onwards, as p increases, the reduction in real money supply (only partially offset by a reduction in money demand due to a decreasing output) pushes the interest rate r up towards its initial steady-state equilibrium level, gradually eliminating the interest rate differential. At the end of the adjustment process, output will be back at $\bar{y}$, and the real exchange rate and the interest rate will be back at their initial steady-state values.

In conclusion, the model provides a rationale for wide fluctuations in the nominal and real exchange rates in the face of monetary policy actions, with contrasting behavior in the short and long run: in the case of a monetary expansion, a strong depreciation on impact, followed by a prolonged appreciation.

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Problems

1. (*Dynamic IS-LM model with the stock market*) Suppose that, differently from the assumption of Section 1, stocks and short-term bonds are *not* perfect substitutes in investors' portfolios. Now stocks are perceived as riskier than bonds: therefore, investors will be willing to hold stocks only if they are compensated for the higher risk with a rate of return higher than r. Let this compensation, or "risk premium", be $\theta_0 > 0$ in the economy's initial steady-state equilibrium.

    (a) Modify the no-arbitrage condition (1.3) appropriately in the presence of the risk premium on stocks; derive the new stationary locus for q and explain how changes in θ may affect the $\dot{q} = 0$ curve;

    (b) suppose now that at time t_0 an *unexpected* and *permanent* reduction of the risk premium occurs, from θ_0 to θ_1 (with $\theta_0 > \theta_1 > 0$). Derive and explain the effect of this change in θ on the steady-state equilibrium and on the dynamic adjustment of output, share prices, and the interest rate.

2. (*Dynamic IS-LM model with the stock market*) We assumed in Section 1 that the "interest rate effect" dominates on the "dividend effect" in determining stock price changes. Let us now suppose that the "dividend effect" dominates.

    (a) Give a precise characterization of the $\dot{q}(t) = 0$ schedule and of the dynamic properties of the system under the new assumption;

    (b) analyze the effects of an anticipated permanent fiscal restriction (announced at t_0 and implemented at t_1) and contrast the results with those reported in Section 1.

3. (*Expectations and exchange rate dynamics*) Starting from the Dornbusch model of Section 2, let us now assume that $\beta > 1$.

    (a) Under this assumption, plot the stationary locus for the exchange rate and explain its slope;

    (b) draw the phase diagram in this case, and characterize the dynamics of e and p in the various regions of the picture; draw the saddlepath and describe its economics properties;

(c) analyze the effects of an unexpected, permanent expansionary monetary policy and compare the dynamics of the endogenous variables with the case presented in Section 2, explaining the differences.

4. (*Expectations and exchange rate dynamics*) Consider the following modified version of the Dornbusch model:

$$
\begin{aligned}
y^D &= -\alpha\, r + \beta\,(e + p^* - \bar{p}) && \text{(aggregate demand)}\\
m - \bar{p} &= y - h\, r && \text{(money market equilibrium)}\\
r &= r^* + \dot{e} && \text{(uncovered interest rate parity)}\\
\dot{y} &= \varphi\,(y^D - y) && \text{(output adjustment)}
\end{aligned}
$$

where y^D denotes the aggregate demand for goods. The price level is now fixed ($\bar{p}$), as in the $IS - LM$ model, and output gradually adjust over time to excess demand for (or excess supply of) goods, as in Blanchard (1981). The assumption of *perfect foresight* has already been introduced into the uncovered interest rate parity equation.

(a) Derive the properties of the steady-state equilibrium of the model, and of the dynamics (the shape of the stationary loci and of the saddlepath converging to long-run equilibrium);

(b) what are the dynamic effects of an expansionary monetary policy (unexpected and permanent) on the interest rate, output, and the exchange rate?