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Financial factors, macroeconomic information and the Expectations Theory of the term structure of interest rates

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Abstract

In this paper we develop on the VAR framework, originally proposed by Campbell and Shiller (J. Political Econom 95 (1987) 1062) to evaluate the Expectations Theory, along three dimensions: the use of a testing method based on a real-time procedure, the measurement of the risk premium, the specification of the implicit monetary policy maker's reaction function. We use financial factors and macroeconomic information to construct a test of the theory based on simulating investors' effort to use the model in 'real-time' to forecast future monetary policy rates. The application of our approach to a monthly sample of US data from the eighties onward leads us to conclude that the deviation from the ET are very rarely significant and that fluctuations of risk premia are not large.

JEL classification: E43; E44; E47

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1. Introduction

The objective of this paper is to provide new evidence on the Expectations Theory (ET) of the term structure of interest rates.

How is this possible?

Our starting point is the widely cited work by Campbell and Shiller (1987)(CS), where they implement a bi-variate vector autoregression (VAR), which is different from the bulk of the available literature which rejects the ET within a single-equation, limited information approach (see, for example, Campbell, 1995; Fama and Bliss, 1987; Cochrane, 2001). CS implement a test which still rejects the ET but their analysis of the data leads them to conclude that there is an important element of truth to the expectations theory of the term structure.

We develop on the CS framework along three dimensions: the use of a testing method based on a real-time procedure in which the econometrician is given the same information available to market participants when they make their decisions on portfolio allocation, the specification of the implicit monetary policy maker's reaction function, the measurement of the risk premium in case of rejection of the null of the ET.

First, CS test the restrictions imposed by the ET on a VAR model in the spread between long- and short-term interest rates and the change of short-term interest rates and by using only in-sample information. Such procedure cannot simulate the investors' effort to use the model in 'real-time' to forecast future monetary policy rates: the information from the whole sample is used to estimate parameters while investors can use only historically available information to generate (up to n-period ahead) predictions of policy rates. Moreover, the within sample test understates the uncertainty of agents who forecast policy rates by out-of-sample projections. In this paper we use the present value framework to generate real-time forecast for future policy rates. At each point in time we estimate, using the historically available information, a model and then we use it to project out-of-sample policy rates up to the nth-period ahead. Given the path of simulated future policy rates, we can construct yield to maturities consistent with the Expectations Theory. Using the historically available information on uncertainty we perform dynamic stochastic simulations and construct confidence bounds around the ET-consistent long-term rates. These bounds reflect explicitly the uncertainty associated with out-of-sample projections. It then becomes natural to test the ET by checking if the observed long-term rates fluctuate within the bounds.

Second, by having an explicit model for the short rate in their testing framework CS circumvent one of the main assumptions of the single-equation approach to the ET, namely the use of ex-post realized returns as a proxy for ex-ante expected returns. In a recent paper, Elton (1999) clearly asserts that there is ample evidence against the belief that information surprises tend to cancel out over time and hence realized returns cannot be considered as an appropriate proxy for expected returns. Interestingly, Campbell (1995) finds strong effects of expectations errors on the single-equation tests, which are confirmed by a number of papers which concentrates on expectations errors by relating them to "peso problems" or to the very low predictability of short-term interest rates. In a famous study Mankiw and Miron (1986), using data on a 3- and 6-month maturity, found evidence in favor of the expectation theory prior to the founding of the Federal Reserve System in 1915. They show that the shift in regime occurred with the founding of the Fed led to a remarkable decrease in the predictability of short-term interest rates. Rudebusch (1995), and Balduzzi et al. (1997) expand on this evidence by looking at more recent data. As a consequence of the use of ex-post realized returns as a proxy for ex-ante expected returns the single-equation approach cannot identify if the empirical failure of the model is due to systematic expectations errors, or to shifts in the risk premia. CS have an implicit model to construct expectations, they find much milder evidence against the ET but they do not exploit their model to construct a measure of risk premium.

By implementing our simulation based procedure we can explicitly measure deviations from the ET and, under the null that our proposed model delivers expected future policy rates not different from those expected by the market, interpret them as a measure of risk premium.

Third, on a different, but clearly related, ground McCallum (1994) is the first to argue that the limited information approach might cause bias in the estimates due to simultaneity. He shows that the anomalous empirical findings based on a single equation evidence can be rationalized with the expectations theory by a recognition of an exogenous term premium plus the assumption that monetary policy involves smoothing of the policy rates together with the responses to the prevailing level of the spread. Interestingly, the bi-variate framework considered by CS matches exactly the scenario used by McCallum to illustrate the simultaneity bias in the single-equation approach. However, McCallum himself notes that a reaction function according to which the Fed reacts to the spread only represents a simplification relative to the actual behaviour of the Fed, which almost certainly responds to recent inflation and output or employment movements, as well as to the spread. In fact, both the financial literature and the macroeconomic literature point to potential mis-specification of the simple reaction function used by CS.

There is ample empirical evidence that a three-factor model is needed to accurately describe the term structure and that the use of term structure related factors is of considerable help in modelling monetary policy rates (see, for example, Ang and Piazzesi (2003)), it is easy to see that in the CS approach only two factors are considered. The success of Taylor rules (Taylor, 1993; Clarida et al., 1998, 1999, 2000) points out an obvious potential misspecification of the original Campbell–Shiller framework: the omission of macroeconomic variables to which the monetary policy maker reacts. We shall assess potential mis-specification effects by using an extended VAR which includes three factors for the term structure and macro-economic variables used in Taylor rules.

The paper is organized as follows. Section 1 illustrates the testing framework by contrasting the Present Value approach with our simulation based alternative. Section 2 illustrates our testing framework and our extension of the information set. Section 3 presents the empirical evidence. Section 4 contains an assessment of the robustness of our results to the use of a different sample and of a different method for updating parameter estimates upon accrual of new information. Section 5 concludes.

2. Testing framework

We introduce our testing framework by comparative evaluation of the traditional present value approach and of proposed simulation based approach.

2.1. The Present Value approach

We describe the Present Value approach by adopting the linearized expectations model of Shiller (1979) in the bi-variate framework proposed by CS.

We start by imposing a no-arbitrage condition, according to which the expected one-period holding returns from long-term bonds must be equal to the risk-free short-term interest rate plus a term premium. For long-term bonds bearing a coupon $C, H_{t,T}$ is a non-linear function of the yield to maturity $R_{t,T}$. Shiller (1979) proposes a linearization which takes the approximation in the neighborhood $R_{t,T} = R_{t+1,T} = R = C$, in which case we have

$$\mathrm{E}[H_{t,T} \mid I_t] = \mathrm{E}\left[\frac{R_{t,T} - \gamma_T R_{t+1,T}}{1-\gamma_T} \mid I_t\right] = r_t + \phi_{t,T}, \tag{1}$$

where $H_{t,T}$ is the one-period holding return of a bond with maturity date T, I_t is the information set available to agents at time t, r_t is the short-term interest rate, γ_T is a constant of linearization which depends on the maturity of the bond and $\phi_{t,T}$ is a term premium defined over a one-period horizon for holding the bond with residual maturity $T-t$. Consider the above expression for a very long-term bond, by recursive substitution, under the terminal condition that at maturity the price equals the principal, we obtain

$$R_{t,T} = R^*_{t,T} + \mathrm{E}[\Phi_{t,T} \mid I_t] = \frac{1-\gamma}{1-\gamma^{T-t}} \sum_{j=0}^{T-t-1} \gamma^j \mathrm{E}[r_{t+j} \mid I_t] + \mathrm{E}[\Phi_{t,T} \mid I_t], \tag{2}$$

where $\lim_{T\to\infty} \gamma_T = \gamma = 1/(1+R)$ and $\Phi_{t,T}$ is the term premium over the whole life of the bond:

$$\Phi_{t,T} = \frac{1-\gamma}{1-\gamma^{T-t}} \sum_{j=0}^{T-t-1} \gamma^j \phi_{t+j,T}.$$

CS tests the ET[1] by using Eq. (2) in considering the case of the risk free rate and a very long-term bond. In such case, the null of the ET is imposed in strong form by imposing that $\mathrm{E}[\Phi_{t,T} \mid I_t]$ is zero and in weak form by imposing that $\mathrm{E}[\Phi_{t,T} \mid I_t]$ is captured by a constant. CS consider de-meaned variables, and hence test a weak

[1] In fact CS use de-meaned-variables, that is equivalent to test a weak form of the Expectations Theory, in the sense that de-meaning eliminates a constant risk premium.

form of the ET by considering the following restriction:

$$R_{t,T} = R^*_{t,T} \approx (1-\gamma) \sum_{j=0}^{T-t-1} \gamma^j \mathrm{E}[r_{t+j} \mid I_t], \tag{3}$$

which could be re-written in terms of spread between long- and short-term rates, $S_{t,T} = R_{t,T} - r_t$:

$$S_{t,T} = S^*_{t,T} = \sum_{j=1}^{T-t-1} \gamma^j \mathrm{E}[\Delta r_{t+j} \mid I_t]. \tag{4}$$

Eq. (4) shows that a necessary condition for the ET to hold puts constraints on the long-run dynamics of the spread. In fact, the spread should be stationary being a weighted sum of stationary variables. Obviously, stationarity of the spread implies that, if yields are non-stationary, they should be cointegrated with a cointegrating vector $(1, -1)$. However, the necessary and sufficient conditions for the validity of the ET impose restrictions both on the long-run and the short-run dynamics.

Assuming[2] that $R_{t,T}$ and r_t are cointegrated with a cointegrating vector $(1, -1)$, CS construct a bivariate stationary VAR in the first difference of the short-term rate and the spread

$$\begin{aligned} \Delta r_t &= a(L)\Delta r_{t-1} + b(L)S_{t-1} + u_{1t}, \\ S_t &= c(L)\Delta r_{t-1} + d(L)S_{t-1} + u_{2t}. \end{aligned} \tag{5}$$

Stack the VAR as

$$\begin{bmatrix} \Delta r_t \\ \cdot \\ \cdot \\ \Delta r_{t-p+1} \\ S_t \\ \cdot \\ \cdot \\ S_{t-p+1} \end{bmatrix} = \begin{bmatrix} a_1 & \cdot & \cdot & a_p & b_1 & \cdot & \cdot & b_p \\ 1 & \cdot & \cdot & 0 & 0 & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & 0 & 0 & \cdot & \cdot & 0 \\ 0 & \cdot & 1 & 0 & 0 & \cdot & \cdot & 0 \\ c_1 & \cdot & \cdot & c_p & d_1 & \cdot & \cdot & d_p \\ 0 & \cdot & \cdot & 0 & 1 & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & 0 & 0 & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & 0 & 0 & \cdot & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta r_{t-1} \\ \cdot \\ \cdot \\ \Delta r_{t-p} \\ S_{t-1} \\ \cdot \\ \cdot \\ S_{t-p} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ \cdot \\ \cdot \\ 0 \\ u_{2t} \\ \cdot \\ \cdot \\ 0 \end{bmatrix}. \tag{6}$$

This can be written more succinctly as

$$z_t = Az_{t-1} + v_t. \tag{7}$$

The ET null puts a set of restrictions which can be written as

$$g'z_t = \sum_{j=1}^{T-1} \gamma^j h' A^{j'} z_t, \tag{8}$$

[2]In fact, the evidence for the restricted cointegrating vector which constitutes a necessary condition for the ET to hold is not found to be particularly strong in the original CS work.

where g' and h' are selector vectors for S and Δr correspondingly (i.e., row vectors with 2p elements, all of which are zero except for the p + 1st element of g' and the first element of h' which are unity). Since the above expression has to hold for general z_t, and, for large T, the sum converges under the null of the validity of the ET, it must be the case that

$$g' = h'\gamma A(I - \gamma A)^{-1}, \tag{9}$$

which implies

$$g'(I - \gamma A) = h'\gamma A \tag{10}$$

and we have the following constraints on the individual coefficients of VAR(5):

$$\{c_i = -a_i, \forall i\}, \quad \{d_1 = -b_1 + 1/\gamma\}, \quad \{d_i = -b_i, \forall i \neq 1\}. \tag{11}$$

The above restrictions are testable with a Wald test. By doing so using US data between the fifties and the eighties Campbell and Shiller (1987) rejected the null of the ET. However, when CS construct a theoretical spread $S^*_{t,T}$, by imposing the (rejected) ET restrictions on the VAR they find that, despite the statistical rejection of the ET, $S^*_{t,T}$ and $S_{t,T}$ are strongly correlated.

2.2. A new testing framework with an extended information set

We extend the CS approach along two dimensions: the specification of the VAR and the construction of a test based on information available in real time.

Both the financial and the macroeconomic empirical literature suggest that the parsimonious model consisting of the spread and the change in the short-term rate could be in fact too parsimonious to fit the data. The financial literature has shown that the construction of a satisfactory model of the term structure requires at least three factors, usually labelled as level, slope and curvature. Rethinking the CS empirical work in this framework makes clear that they have included in their bivariate VAR some proxy for the level and the slope of the term structure, but they have omitted the curvature. Interest rate rules, which feature (very) persistent policy rates responding to central bank's perceptions of (expected) inflation and output gaps (Taylor, 1993; Clarida et al., 1998, 1999, 2000) not only track the data well but are also capable of explaining the high inflation in the seventies in terms of an accommodating behaviour towards inflation in the pre-Volcker era.

Interestingly, Fuhrer (1996) uses a simple Taylor-rule type reaction function, the expectations model and reduced-form equations for output and inflation to solve for the reaction function coefficients that delivers long-term rates consistent with the Expectations Theory. He finds that modest and smoothly evolving time-variation in the reaction functions parameters is sufficient to reconcile the expectations model with the long-bond data. Favero (2002) extends Fuhrer framework to derive standard errors for long-term rates consistent with the ET. Our approach of extending the VAR framework is also related to recent work by Roush (2003). Roush considers a VAR model with macro and financial variables to show that the expectations theory of the term structure holds conditional on an exogenous change in monetary policy. The paper adds to the picture the important issue of identification but it does not provide evidence on the impact of the extension of the original CS information set on the outcome of the test for the unconditional validity of cross-equation restrictions; moreover, the attention is limited to the within-sample evidence.

The bivariate CS approach has an implicit reaction function according to which the only determinant of policy rates are long-term rates, therefore we have a potential mis-specification due to the omission of macroeconomic factors.

However, we think that our main contribution is not the augmentation of the original dimension of the VAR but the proposal of a new approach to test the ET based on information available in real time. To show our point, consider a cointegrated VAR framework, in which the original set of variables used by CS is extended by including a vector of variables **X**. Such vector includes financial factors and macroeconomic variables. At each point in time we estimate, using the historically available information, the following model:

$$\begin{aligned} \Delta r_t &= a_0 + a_1(L)\Delta r_{t-1} + a_2(L)S_{t-1} + a_3(L)\mathbf{X}_{t-1} + u_{1t}, \\ S_t &= b_0 + b_1(L)\Delta r_{t-1} + b_2(L)S_{t-1} + b_3(L)\mathbf{X}_{t-1} + u_{2t}, \\ \mathbf{X}_t &= c_0 + c_1(L)\Delta r_{t-1} + c_2(L)S_{t-1} + c_3(L)\mathbf{X}_{t-1} + \mathbf{u}_{3t}, \end{aligned}$$

$$\begin{bmatrix} u_{1t} \\ u_{2t} \\ \mathbf{u}_{3t} \end{bmatrix} \sim N[0, \Sigma].$$

We then simulate the estimated model forward, to obtain projection for all the relevant policy rates and to construct ET-consistent spreads as follows:

$$\hat{S}^*_{t,T} = \sum_{j=1}^{T-t-1} \gamma^j \mathrm{E}[\Delta r_{t+j} \mid \Omega_t], \tag{12}$$

where, $\mathrm{E}[\Delta r_{t+j} \mid \Omega_t]$ are the VAR-based projections for the future changes in policy rates, hence Ω_t is the information set used by the econometrician to predict on the basis of the estimated VAR model. Given this simulation-based version of the ET consistent spread we can also construct a confidence interval around it. Confidence intervals around simulated series are usually constructed by adopting stochastic simulation techniques. In a standard stochastic simulation the model is simulated forward repeatedly for N draws of its stochastic components. In general, there are two sources of uncertainty: residuals and coefficient uncertainty. Residuals are drawn from a multivariate normal distribution $N(0, \hat{\Sigma})$ where $\hat{\Sigma}$ is the estimated variance–covariance matrix of residuals of our VAR. Similarly, VAR coefficients are drawn from a multivariate normal distribution with the vector mean given by the point estimates of coefficient and the variance–covariance matrix given by the parameters' variance–covariance matrix. However, the confidence interval constructed by allowing for residuals and coefficient uncertainty will be a confidence interval for the evolution of $\sum_{j=1}^{T-t-1} \gamma^j [\Delta r_{t+j} \mid \Omega_t]$ which is very different, and certainly larger, than a confidence interval for $\hat{S}^*_{t,T} = \sum_{j=1}^{T-t-1} \gamma^j \mathrm{E}[\Delta r_{t+j} \mid \Omega_t]$[3]. However, it is immediate to construct bounds for $\hat{S}^*_{t,T}$ by performing the stochastic simulation allowing only for coefficients uncertainty. While future realized policy rates are affected both by parameters uncertainty and shocks, future expected policy rates are not affected by shocks, hence the only source of uncertainty for the ET consistent spread is parameters' uncertainty. ET consistent yields are calculated applying Eq. (12) to each of the N simulated paths of future expected short-term rates: among these, the 0.5th, 0.05th, and 0.95th quantiles represent respectively, the median ET-consistent yield and its 90% confidence bounds. The estimation window is then enlarged by one observation and simulation horizon is shifted one period ahead and the same steps are repeated.

Importantly, in implementing our procedure the econometrician uses the same information available to market participants in real time. Future policy rates at time t are constructed using information available in real time for parameters estimation and forward projection of the model. Point forecasts and their confidence bounds define a region inside which the actual long-term rates should lie if the ET holds.

Interestingly, by simulating the VAR coefficients from a multivariate normal distribution we treat parameters as random and the data fixed. This is obviously different from the classical bootstrap or Monte Carlo methods that simulate data for fixed parameters. This might give rise to a Bayesian interpretation of our paper, according to which our relevant distribution is the posterior for the relevant parameters when the prior is non-informative and the sample error covariance matrix is assumed to coincide with the true distribution. Importantly, in this interpretation the flat prior imposes stationarity of the VAR.[4] This is the reason why, as in Campbell–Shiller, we always adopt a stationary representation of our system, in which the variables are transformed either by taking first differences or by taking stationary linear combinations of non-stationary variables (cointegrating vectors).[5] As a result of our specification choice very few (less than one percent) of the simulated VAR long-run matrices contain one eigenvalue that lie on or outside the unit circle, we discard these simulations before constructing the relevant distribution.[6] Using a full Bayesian framework with appropriate specification of prior distributions is on our agenda for research.

By combining (4) and (12), we have:

$$S_{t,T} = \sum_{j=1}^{T-t-1} \gamma^j \mathrm{E}_t[\Delta r_{t+j} \mid I_t] + \mathrm{E}[\Phi_{t,T} \mid I_t], \tag{13}$$

[3]We thank a referee for pointing this out. In fact, bounds constructed by allowing both for residuals and coefficients uncertainty could be thought of as a simulation equivalent of the volatility bounds proposed by Shiller (1979).

[4]We thank (again) an anonymous referee for pointing out this interpretation to us.

[5]For example, our VAR does never contain the level of the term structure, which clearly a very persistent variable, but rather the spread between the level and the intercept which we interpret as cointegrating vector with parameters $(-1, 1)$. The intercept is then considered in first differences.

[6]In our simulations we also impose the constraint that nominal yields are non-negative.

$$\widehat{S^*_{t,T}} = \sum_{j=1}^{T-t-1} \gamma^j \mathrm{E}_t[\Delta r_{t+j} \mid \Omega_t], \tag{14}$$

$$S_{t,T} - \widehat{S^*_{t,T}} = \left(\sum_{j=1}^{T-t-1} \gamma^j \mathrm{E}_t[\Delta r_{t+j} \mid I_t] - \sum_{j=1}^{T-t-1} \gamma^j \mathrm{E}_t[\Delta r_{t+j} \mid \Omega_t] \right) + \mathrm{E}[\Phi_{t,T} \mid I_t],$$

$$S_{t,T} - \widehat{S^*_{t,T}} = \xi_t + \mathrm{E}[\Phi_{t,T} \mid I_t]. \tag{15}$$

Eq. (15) makes clear that deviation of $S_{t,T}$ from $\widehat{S^*_{t,T}}$ can be explained by the effect of the risk premia or by differences between model based forecasts, which are derived by using the information set used by the econometrician Ω_t, and agents' expectations, which are formed given the information set I_t, unknown to the econometrician. Under the assumption that the first term is negligible (statistically) significant deviations of $S_{t,T}$ from $\widehat{S^*_{t,T}}$ do offer a measurable counterpart of the risk premium.

3. The empirical evidence

We shall present our empirical evidence in three sub-sections. The first sections discusses our data-set, and our choice of sample for estimation and simulation, the second section presents the replica of the CS procedure on our data-set and an application of our simulation based procedure on the CS specification, while the third section illustrates the extension of the original specification to include financial factors and macroeconomic variables.

3.1. The data-set

Our basic data set consists of a set of zero-coupon equivalent US yields, provided by Brousseau and Sahel (2001). In particular, we consider data on zero-coupon equivalent yields for US data measured at the following maturities:[7]

1-month, 2-month, 3-month, 6-month, 9-month, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year.

From this data-set we construct financial factors by estimating at each point of our time series t, by non-linear least squares, on the cross-section of eleven yields, the following Nelson–Siegel model (Nelson and Siegel, 1987):

$$y_{t,t+k} = L_t + SL_t \frac{1-\exp(-k/\tau_1)}{k/\tau_1} + C_t \left(\frac{1-\exp(-k/\tau_1)}{k/\tau_1} - \exp(-k/\tau_1) \right), \tag{16}$$

[7]The data were kindly made available by the ECB, and they are posted on Favero's website at the following address: http:/www.igier.uni-bocconi.it/personal/favero in the section working papers.

which is implicit in the instantaneous forward curve

$$f_{tk} = L_t + SL_t \exp\left(-\frac{k}{\tau_1}\right) + C_t \frac{k}{\tau_1} \exp\left(-\frac{k}{\tau_1}\right). \tag{17}$$

The parameter τ_1 is kept constant over time,[8] as this restriction decreases the volatility of the β parameters, making them more predictable in time. As discussed in Diebold and Li (2002) the above interpolant is very flexible and capable of accommodating several stylized facts on the term structure and its dynamics. In particular, L_t, SL_t, C_t, which are estimated as parameters in a cross-section of yields, can be interpreted as latent factors. L_t has a loading that does not decay to zero in the limit, while the loading on all the other parameters do so, therefore this parameter can be interpreted as the long-term factor, the level of the term-structure. The loading on SL_t is a function that starts at 1 and decays monotonically towards zero; it may be viewed a short-term factor, the slope of the term structure. In fact, $r_t^{\mathrm{rf}} = L_t + SL_t$ is the limit when k goes to zero of the spot and the forward interpolant. We naturally interpret r_t^{rf} as the risk-free rate. Obviously SL_t, the slope of the yield curve, is nothing else than the minus the spread in Campbell–Shiller. C_t is a medium term factor, in the sense that their loading start at zero, increase and then decay to zero (at different speed). Such factor captures the curvature of the yield curve. In fact, Diebold and Li show that it tracks very well the difference between the sum of the shortest and the longest yield and twice the yield at a mid range (2-year maturity). The repeated estimation of loadings using a cross-section of yields at different maturities allows to construct a time-series for our factors. We report in Fig. 1 the three factors, while Fig. 2 shows the goodness-of-fit of the Nelson and Siegel interpolation for all yields considered in our sample. The extreme good performance of the Nelson–Siegel interpolant for our observed data shows that the fact that we have fitted the Nelson–Siegel model to zero coupon equivalent yields rather than to individual yields should not be a cause of concern for the problem at hand.

Note that the fact that we use zero-coupon equivalent yields has a relevant implication for the CS linearization, which should be applied taking the limit of the relevant formulae when γ approaches 1. In particular, we have

$$\begin{aligned} R_{t,T} = R^*_{t,T} + \mathrm{E}[\Phi_{t,T} \mid I_t] &= \lim_{\gamma \to 1} \frac{1-\gamma}{1-\gamma^{T-t}} \sum_{j=0}^{T-t-1} \gamma^j \mathrm{E}[r_{t+j} \mid I_t] + \mathrm{E}[\Phi_{t,T} \mid I_t] \\ &= \frac{1}{T-t} \sum_{j=0}^{T-t-1} \mathrm{E}[r_{t+j} \mid I_t] + \mathrm{E}[\Phi_{t,T} \mid I_t] \end{aligned} \tag{18}$$

and, given that $R^*_{t,T} = \frac{1}{T-t} \sum_{j=0}^{T-t-1} \mathrm{E}[r_{t+j} \mid I_t]$, we then have

$$S^*_{t,T} = R^*_{t,T} - r_t = \sum_{j=1}^{T-t} \left(1 - \frac{j}{T-t}\right) \mathrm{E}\Delta[r_{t+j} \mid I_t].$$

[8] We restrict τ_1 at the value of 0.87, which is the median, over the time series, of the estimated value of τ_1 in a four parameter version of the Nelson–Siegel interpolant.

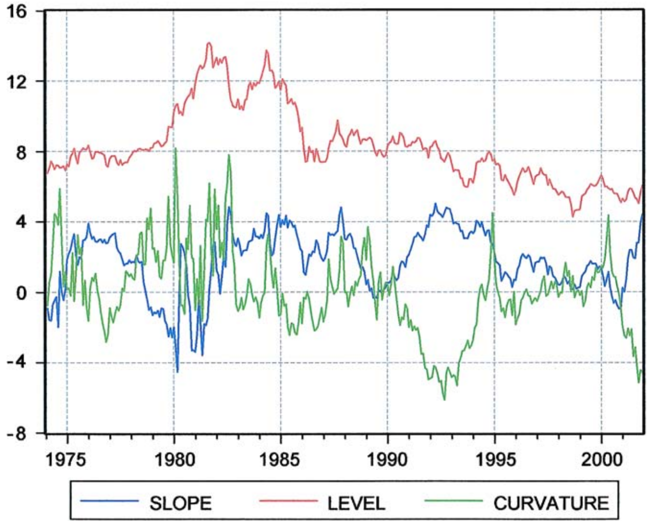

Fig. 1. The time series of the three Nelson–Siegel factors for the US yield curve.

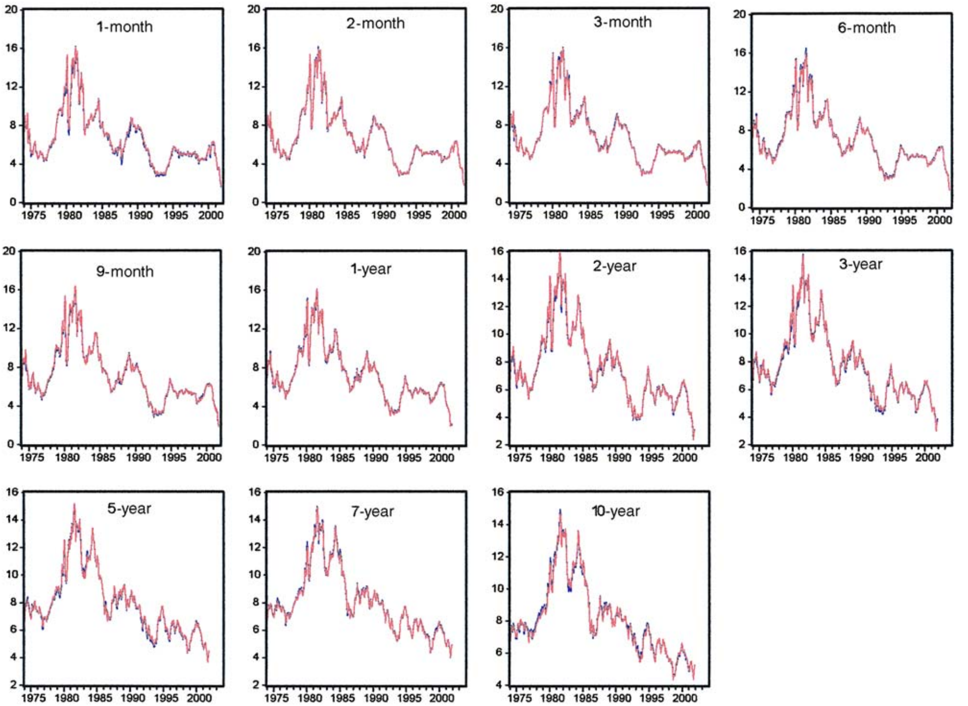

Fig. 2. The time series of yields at different maturities and the Nelson–Siegel interpolants.

Our empirical analysis will be based on a simulation sample starting at the beginning of the eighties. One of the main points of our paper is to construct expected future policy rates by considering explicitly the central bank reaction functions, so we have chosen the initial date of the sample for simulation to concentrate on an era of homogenous monetary policy, i.e., the Volcker–Greenspan era. In fact, there is ample empirical evidence that, from the beginning of the eighties onward, the Fed engaged in interest rate targeting and that the behaviour of policy rates can be successfully described by a Taylor rule. The traditional argument of a Taylor rule are expected inflation and some measure of the output gap. Our framework for simulating policy rates is geared to mimic the decisions of agents in real time. Orphanides (2001) has shown that data revisions could generate misleading inference. For this reason, as suggested by Evans (2003), we consider as macroeconomic factors variables which are not subject to revision: the CPI inflation and unemployment rate.

We present our empirical evidence in three parts: a replica on our data-set of the original Campbell–Shiller results, an application of our simulation based procedure on the CS specification, the extension of the original specification to include financial factors and macroeconomic variables.

3.2. Testing the ET with a bivariate VAR

The discussion of the measurement of financial factors makes clear that the closest model to CS original specification in our framework is the following:

$$\begin{bmatrix} \Delta r_t^{\mathrm{rf}} \\ S_t \end{bmatrix} = A(L) \begin{bmatrix} \Delta r_{t-1}^{\mathrm{rf}} \\ S_{t-1} \end{bmatrix} + u_t, \tag{19}$$

where $r_t^{\mathrm{rf}} = L_t + SL_t$, and $S_t = -SL_t$. Our specification differs from CS in that they take the 1-month rate as the short-term rate and the yield to maturity on 10-year bonds as the long-term rate. Interestingly the level factor, L_t, is the asymptote of the term structure, hence cross-equation restrictions on the VAR hold exactly for the spread constructed by using this factor while they are just approximate for the spread constructed using a 10-year yield.[9] We also estimate our model recursively, allowing for a smooth evolving path in the estimated coefficients. This procedure might capture historical shifts in market perceptions of the policy target for inflation, which have been shown (Kozicki and Tinsley, 2001) to be important to achieve a satisfactory specification of agents' expectations. We report the results of the application of the CS testing methodology, based on a four-lag VAR, in Fig. 3. Fig. 3 reports the results of the test for the ET cross-equation restrictions, which is conducted recursively after using the sample 1974:4–1991:12 for initialization. The ET restrictions are consistently rejected, however, as in the original work of CS the actual spread has a correlation of with the spread obtained by imposing the invalid

[9] As a matter of fact we have tested that for simulation based on our VAR specification 10 years is sufficiently far in the future to give a good approximation of infinite.

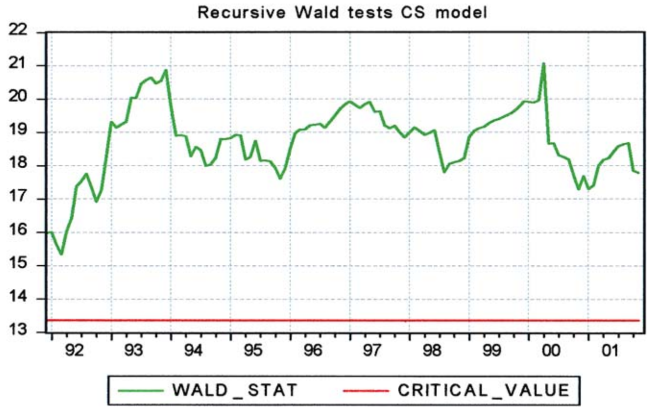

Fig. 3. Recursive tests (and five percent critical value) for the validity of the cross-equation restrictions implied by the Expectations Theory in a four-lags VAR with two financial factors (change in policy rates and slope of the yield curve, as in CS).

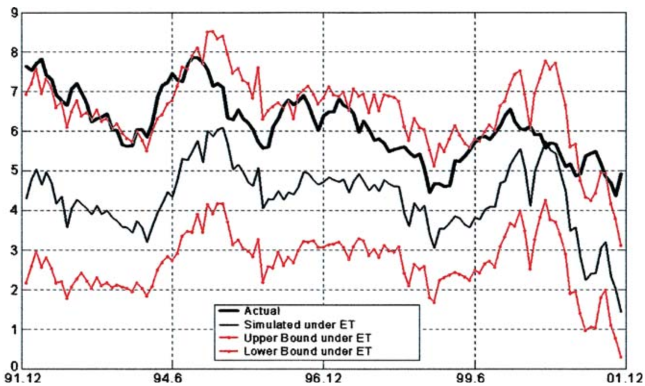

Fig. 4. Simulated ET-consistent 10-year yields to maturity based on the CS model, with lower and upper bond of its 90% Confidence Interval.

restrictions of .96. This is the evidence that leads CS to conclude that "... deviations from the present value model for bonds are transitory...", however no measurement of the risk premium is explicitly provided by the two authors.

We report in Fig. 4 the results of our simulation based test of the ET. We use our model to simulate ET consistent 10-year yields to maturity and their associated confidence intervals. Fig. 4 ET consistent yields to maturity along with their associated confidence interval and the actual yields. Under the null of the ET the observed yields should fall within the bounds. In fact, the actual yields lie consistently above the simulated ones, but they are outside the 90 percent confidence intervals, constructed under the null of the ET, only in a short subsample covering the period 1991–1994. Interestingly, a positive risk difference between actual and simulated yields is what we should observe in the presence of risk premium, when the impact of the difference between the information sets used by the agents and the econometrician is negligible. Overall, we attribute the difference between the results of our simulation based methodology and the traditional CS to the fact that the tests for the cross-equations restrictions understates the uncertainty faced by the agents in real time and therefore uses a too tight statistical criterion. Our evidence of non-rejection of the EH is consistent with the evidence proposed by CS of the very high correlation between the actual spread and the spread obtained by simulating imposing the restrictions (rejected by the Wald test). Our results confirm the much less strong evidence against the EH generated by models in which expectations are explicitly derived rather than taking the ex-post realized returns as a proxy for ex-ante expected returns. Interestingly, Beakaert et al. (2001) find the same results from a different perspective: the use of small sample distribution of the relevant tests in VAR models leads to much less strong evidence against the ET.

We believe that it is important to assess this first set of results against those obtained by enlarging the information set of the VAR following the available empirical evidence form studies on the term structure and on the empirical analysis of monetary policy. In particular, the difference between actual and simulated rates is sizeable when significant and we think that it would be interesting to see how this distance is affected by the enlargement of the information set which we shall implement in the next section.

3.3. Testing the ET in a model with financial factors and macroeconomic variables

Our VAR with financial factor and macroeconomic variables takes the following specification:

$$\begin{bmatrix} \Delta r_t^{\mathrm{rf}} \\ -S_t \\ C_t \\ \pi_t \\ UN_t \end{bmatrix} = A(L) \begin{bmatrix} \Delta r_{t-1}^{\mathrm{rf}} \\ -S_{t-1} \\ C_{t-1} \\ \pi_{t-1} \\ UN_{t-1} \end{bmatrix} + u_t. \tag{20}$$

We consider the three factors obtained via the application of the Nelson–Siegel interpolant together with CPI inflation, π_t, and the unemployment rate, UN_t, which are our proxies for the variables normally entered as arguments of Taylor rules. Importantly, our macroeconomic variables are not subject to revision, consistently with our intention of using the model to replicate the decision process of agents in real time. As in the VAR with financial factors our representation is stationary and it allows for the cointegrating relationship which constitute a necessary condition for the ET to hold, being also consistent with the presence of a stationary risk

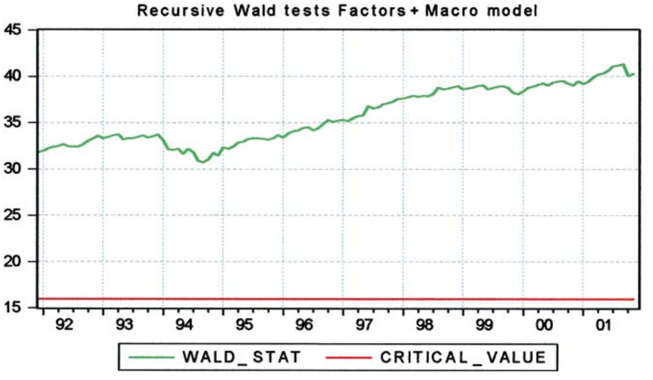

Fig. 5. Recursive tests (and five percent critical value) for the validity of the cross-equation restrictions implied by the Expectations Theory in a VAR with three financial factors and two macroeconomic variables.

premium.[10] Estimation is conducted on the same sample with the two variables VAR and, on the basis of the traditional lag selection criteria, we adopt a VAR of length two.[11] The results of the recursive within sample test and of the simulation based out-of-sample procedure are reported respectively in Figs. 5 and 6. The results of the Wald tests are very similar to those obtained in the basic model. However, the enlargement of the information set generates some notable modification in the simulation based procedure. In fact, the difference simulated yields get much closer to actual yields and there no evidence of violation of the ET. The peak in the differences between observed yields and simulated yields under the null of ET is in 1994, a period which has been widely cited in the literature as featuring an episode of "inflation scare" (see, for example, Rudebusch, 1998). We interpret these results as evidence for the importance of the VAR enlargement to achieve a better identification of the expectations for the future path of the financial and macroeconomic variables relevant to determine monetary policy.

4. Robustness

The results on the size and the significance of risk premium delivered by our simulation based approach call for some robustness analysis. In particular, we want to make sure that our sample initialization is not inappropriate in that our initial

[10]The trace statistics for the null of at most four cointegrating vectors yielded an observed values of 6.35, for the estimation on the full sample and of 5.2 for the estimation on the shortest sample used in the recursive approach, while the five percent critical value is 3.76 (we allowed for a constant restricted to belong to the cointegrating vector).

[11]The lag length criteria do not uniformly favour two lags for all possible sample splits. So we have analyzed the robustness of our results to the adoption of a four-lags VAR. The evidence, available upon request, shows that moving from a lag length of two to a lag length of four leaves our results unaltered.

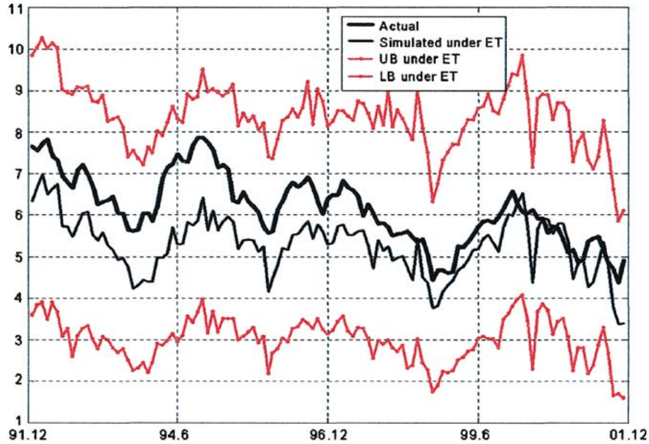

Fig. 6. Simulated ET-consistent 10-year yields to maturity based on the model with financial factors and macroeconomic variables, with lower and upper bond of its 90% confidence interval.

VAR estimates are not contaminated by large residuals. In fact, after the Volcker disinflation, the volatility of macroeconomic variables have decreased remarkably in the eighties. We conduct our robustness check by concentrating on our five variable VAR specification, by considering as a benchmark the recursive estimation approach with initial sample 1974:6–1991:12 discussed in the previous section and by considering as an alternative estimation strategy a rolling estimation with initialization 1974:6–1991:12 and a fixed window of 210 observations. The alternative estimation method is chosen to evaluate the impact of our choice of initialization for the recursive estimation. In fact, the last sample for our rolling estimation approach is 1984:6–2001:12 and covers a very different period from the initial one in terms of (unconditional) volatility of all variables included in the VAR. Moreover, our rolling estimation could also provide evidence against the potential objection that some estimates (see, for example, Bernanke and Mihov, 1998) suggest that the starting period of the Volcker Greenspan era should be located at the beginning of the 1984, and simulation and tests based on post 1984 data could be different from those based on pre 1984 data.

We find the results of the application of the Wald tests and of the simulation based procedure, reported in Figs. 7 and 8, interesting.

The uniform rejection of the theory obtained by the recursive approach based on the initialization on the large sample is not confirmed by the rolling approach, which does not lead to rejection of the theory for an estimation sample of 210 observations ending after the end of 1999. Very differently, the results of the simulation based approach in the five variables VAR are very robust to the two different estimation strategies. We report in Fig. 8 the difference between actual 10-year yields and 10-year yields simulated under the null of the ET, obtained by projections based on rolling and recursive estimation for the five variables VAR and the two variables VAR. The results derived using the five factor models are very robust to the choice of

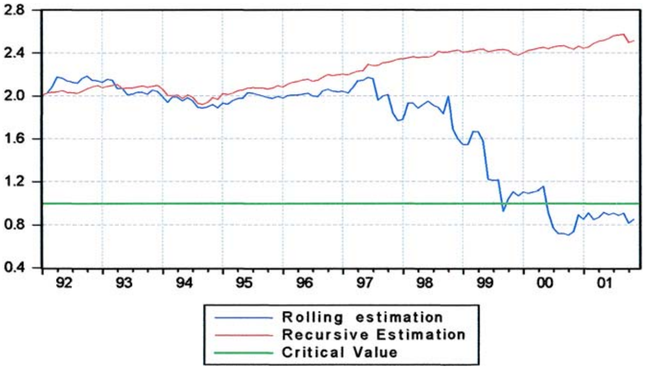

Fig. 7. Wald tests for the EH restrictions on the VAR with financial factors and macroeconomic variables. The reported tests, scaled by their 95 percent critical value, are recursively computed for all end sample points from 1992:1 to 2001:12. Initial sample points are chosen by two different methods: recursive estimation is based on anchoring the first observation to 1974:6, rolling estimation is results based on a rolling estimation with initialization 1974:6–1991:12 and a fixed window of 210 observations.

Fig. 8. Time-series of the difference between actual and simulated yields under ET. Yields are simulated based respectively, on recursive and rolling estimation of a five-variables VAR, and of a two-variables VAR.

the rolling and recursive estimation techniques, delivering differences that reach their peaks during the inflation scare of 1994. The results from the two variables VAR are instead sensitive to the estimation technique. In this case the rolling method delivers series which fluctuate at a level consistently lower than the recursive technique and closer to the series obtained from the five variables VAR. This evidence can be naturally interpreted as indicating mis-specification caused by omitted variables in the more parsimonious model. Interestingly, the results from the five variables VAR are consistent with the evidence, originally reported in CS, that the correlation between the actual spread and the spread obtained under the null of EH is very high even when the null is rejected. Our interpretation of these facts is that the uncertainty faced by the agents in simulating the model to obtain path for the relevant variables to forecast monetary policy is rather stable in a sufficiently parameterized model, even if the coefficients in the estimated VAR do vary over time.

5. What have we learned? A discussion of our results and their relation to the literature

In this paper we have simulated the real-time decision of agents who forecast policy rates by projecting forward a model including financial factors and macro variables to generate long-term rates consistent with the expectations theory along with a confidence interval reflecting the uncertainty associated to out-of-sample forecasting. Our evidence shows that, for different specifications of the information set, the observed long-term yields are, with very few exceptions, contained in the confidence interval generated by our model. Our procedure delivers an observable counterpart of the deviation of the long-term rates from those consistent with the ET. Upon significance of such deviations we can interpret this variable as a proxy for risk premium under the null hypothesis that model based forecasts are not different from agents' expectations. Our empirical results show that a better specification of the VAR used to forecast future monetary policy delivers more credible estimates of the risk premium.

The standard response in finance to the empirical rejection of the Expectations Theory has been modelling the term structure based on the assumption that there are no riskless arbitrage opportunities among bonds of various maturities. The standard model is based on three components: a transition equation for the state vector relevant for pricing bonds, made traditionally of latent factors, an equation which defines the process for the risk-free one-period rate and a relation which associates the risk premium with shocks to the state vector, defined as a linear function of the state of the economy. In such structure, the price of a j-period nominal bond is a linear function of the factors. Unobservable factors and coefficients in the bond pricing functions are jointly estimated by maximum likelihood methods (see, for example, Chen and Scott, 1993). This type of models usually provides a very good within sample fit of different yields but do not perform well in forecasting. Duffee (2002) shows that the forecasts produced by no-arbitrage models with latent factors do not outperform the random walk model.

Recently the no-arbitrage approach has been extended to include some observable macroeconomic factors in the state vector and to explicitly allow for a Taylor-rule type of specification for the risk-free one-period rate. Ang and Piazzesi (2003) and Ang et al. (2003) show that the forecasting performance of a VAR improves when no-arbitrage restrictions are imposed and that augmenting non-observable factors models with observable macroeconomic factors clearly improves the forecasting performance. Hordahl et al. (2003) and Rudebusch (2003) use a small scale macro model to interpret and parameterize the state vector; forecasting performance is improved and models have also some success in accounting for the empirical failure of the Expectations Theory.

No-arbitrage models with observable factors feature a complicated parameterization and cannot accommodate time variation in the parameters of the state vector relevant for pricing bonds. Within this approach, the failure of ET is entirely abscribed to the presence of a time-varying risk premium, which is modelled as a linear function of the state of the economy. There is a lot in common between the latest developments of the no-arbitrage approach and the approach to the term structure proposed in our paper. We share the view on the importance of augmenting the information set with macroeconomic and financial factors to model the yield curve but we concentrate directly on a VAR model for all the relevant factors and we derive risk premium as a residual. The main cost of our approach is that our derived proxy for the risk premium is valid only under the assumption that the difference between the agents information set and the econometrician' information set does not lead to different future projected short-term rates. The main advantage is a much more parsimonious (and linear) parameterization, which easily accommodates time-variation in the parameters describing the state vector relevant for pricing bonds. Our findings suggests that the importance of fluctuations of risk premia in explaining the deviation from the ET might be reduced when some forecasting model for short-term rates is adopted and a proper evaluation of uncertainty associated to policy rates forecast is considered. We believe that improving the forecasting model for policy rates within a no-arbitrage approach is an important step to assess the relative weight of forecasting errors and risk premia in explaining deviations from the Expectations Theory. This is on our agenda for future research.

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