Employment Fluctuations with Equilibrium Wage Stickiness

By ROBERT E. HALL*

Following a recession, the aggregate labor market is slack–employment remains below normal and recruiting efforts of employers, as measured by help-wanted advertising and vacancies, are low. A model of matching friction explains the qualitative responses of the labor market to adverse shocks, but requires implausibly large shocks to account for the magnitude of observed fluctuations. The incorporation of wage stickiness vastly increases the sensitivity of the model to driving forces. I develop a new model of the way that wage stickiness affects unemployment. The stickiness arises in an economic equilibrium and satisfies the condition that no worker-employer pair has an unexploited opportunity for mutual improvement. Sticky wages neither interfere with the efficient formation of employment matches nor cause inefficient job loss. Thus the model provides an answer to the fundamental criticism previously directed at sticky-wage models of fluctuations. (*JEL* E24, E32, J64)

Modern economies experience substantial fluctuations in aggregate output and employment. In recessions, employment falls and unemployment rises. In the years immediately after a recession, the labor market is slack—unemployment remains high and the vacancy rate and other measures of employer recruiting effort are abnormally low. Unemployment is determined by the rate at which workers lose jobs and the rate at which the unemployed find jobs. I develop a model of fluctuations with a matching friction and sticky wages. The incorporation of wage stickiness makes employment realistically sensitive to driving forces. My characterization of wage stickiness is rather different from earlier ideas of wage rigidity and more closely integrated with the matching process. The model describes an economic equilib-

rium and overcomes the arbitrary disequilibrium character of earlier sticky-wage models.

A line of research starting with Peter Diamond (1982), Dale Mortensen (1982), and Christopher Pissarides (1985)—nicely summarized in Pissarides (2000) and in Robert Shimer (2005)—provides an account of unemployment as a productive use of time. I adopt many of the elements of their model—the DMP model—in this paper. The DMP model views the labor market in terms of an economic equilibrium where workers and employers interact purposefully. A friction in matching unemployed workers to recruiting employers accounts for the existence of unemployment. Variations in the economic environment lead to fluctuations in unemployment. The DMP model portrays wage determination as a Nash bargain, where employers receive a constant fraction of the match surplus. The payoff to recruiting activity—the employers' share of the surplus—is not very sensitive to driving forces. Hence the DMP model cannot explain the magnitude of movements in recruiting activity. In reality, the labor market slackens substantially in recessions and workers encounter difficulty in finding jobs, but the DMP model with Nash-bargain wage determination suggests stability in job-finding rates under plausible variations in the driving forces.

In a model with matching frictions, the bargaining set for wage determination is relatively wide, because the difficulty in locating matches creates match capital the moment a tentative match is made. The value of the match capital determines the gap between the minimum wage acceptable to the worker and the maximum wage acceptable to the employer. From the perspective of bilateral bargaining theory in general, any wage within the bargaining set could be an outcome of the bargain. The Nash bargain sets the wage at a weighted average of the limiting wages, with a fixed weight over time. The alternative I offer permits variations over time in the position of the wage within the bargaining set. When the wage is relatively high—closer to the employer's maximum—the employer anticipates less of the surplus from new matches and puts correspondingly less effort into recruiting workers. Jobs become hard to find, unemployment rises, and employment falls.

In the sticky-wage model I develop, when temporary changes in the economic environment shift the boundaries of the bargaining set, the wage remains constant, provided it remains inside the bargaining set. The wage adjusts over time in response to nonstationary changes in the environment. This mechanism guarantees that wage rigidity never results in an allocation of labor that is inefficient from the joint perspective of worker and employer. Consequently, the model provides a full answer to the condemnation of sticky-wage models in Robert Barro (1977) for invoking an inefficiency that intelligent actors could easily avoid. Unlike stickiness portrayed as an essentially arbitrary restriction on the ability to set wages or prices—such as the well-known model for prices of Guillermo Calvo (1983)—the stickiness considered here arises within an economic equilibrium. It satisfies the criterion that no employer-worker pair forgoes bilateral opportunities for mutual improvement. Peter Howitt (1986) made this point.

Although wage stickiness has no effect on the formation of a job match once worker and employer meet and no effect on the continuation of the match, stickiness does have a profound influence on the search process. If wages are toward the upper end of the bargaining set, the incentives that employers face to look for additional workers are low. I start the paper with evidence about the remarkably strong procyclical movements of help-wanted advertising and vacancies. This evidence supports the mechanism proposed here.

I then turn to the model. I adopt the matching friction of the DMP model. But as Shimer (2005) and Marcelo Veracierto (2003) have stressed, the DMP model and others with the same basic view of the labor market do not offer a plausible explanation of observed fluctuations in unemployment. The magnitude of changes in driving forces needed to account for the rise in unemployment and decline in recruiting effort during slumps is much too large to fit the facts about the U.S. economy. For this reason—and following Shimer's suggestion—I introduce wage stickiness into the DMP setup. The resulting model makes recruiting effort, job-finding rates, and unemployment remarkably sensitive to changes in determinants. A small decline in productivity results in a slump in the labor market. With wage stickiness, these changes depress employer returns to recruiting substantially. The immediate effect is a decline in recruiting efforts, a lower job-finding rate, and a slacker labor market with higher unemployment.

I focus on the points that sticky wages can arise in a full economic equilibrium and that stickiness results in high volatility of employment fluctuations. I do not venture into the territory of explaining why the economy appears to choose sticky wages from the wide variety of alternative equilibrium wage patterns. In addition, I do not try to demonstrate that aggregate or individual wages actually are sticky. The reason is simple: as the paper shows, the difference between the sticky wage and the corresponding flexible, Nash-bargain wage is small. The model proposes that the sticky wage varies over time, but not by as much as does the Nash wage. I do not believe that this type of wage movement could be detected in aggregate data.

I. Variations in Recruiting Effort

The DMP model portrays recruiting effort in terms of job vacancies. Prior to the beginning of the Job Openings and Labor Turnover Survey (JOLTS) in December 2000, no direct measures
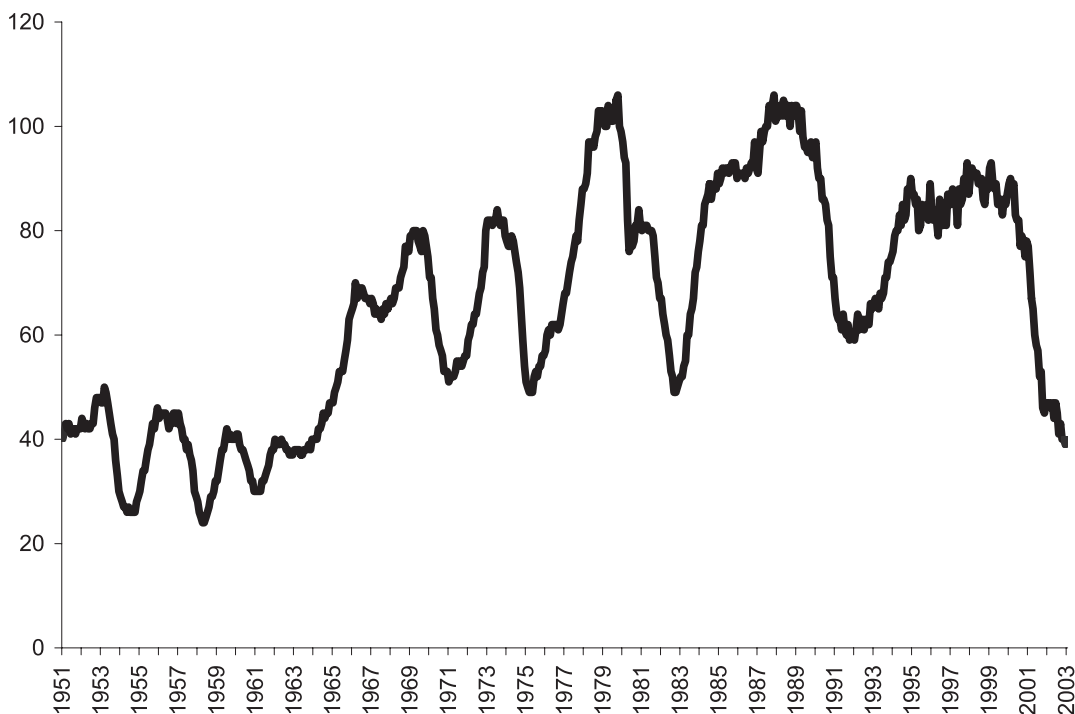
FIGURE 1. INDEX OF HELP-WANTED ADVERTISING

Source: The Conference Board, http://www.globalindicators.org

of vacancies had been available for the U.S. labor market. Previously, authors have suggested—reasonably persuasively—that data on help-wanted advertising provided good evidence about variations in vacancies over time. Figure 1 shows the Conference Board's index of help-wanted advertising since 1951. Recruiting effort as measured by advertising is remarkably volatile. It is not uncommon for advertising to fall by 50 percent from peak to trough, as it did from 2000 to 2003.

Table 1 shows data from JOLTS on vacancies by industry for the period of slackening of the labor market since late 2000. The figures confirm the high volatility of vacancies suggested by the data on help-wanted advertising. The data show that vacancies have declined in all industries. Although the forces that caused the downturn in the economy disproportionately affected a few industries far more than others—notably computers, software, and telecommunications equipment—the softening of the labor market was economy-

TABLE 1—CHANGE IN VACANCY RATES BY INDUSTRY IN JOLTS, DECEMBER 2000 TO DECEMBER 2002

Industry	Ratio of vacancy rates in 12/02 and 12/00
Mining	0.36
Construction	0.38
Durables	0.45
Nondurables	0.48
Transportation and utilities	0.80
Wholesale trade	0.52
Retail trade	0.60
Finance, insurance, and real estate	0.79
Services	0.68
Federal government	0.54
State and local government	0.70

wide. The new data strongly confirm the position of Katharine Abraham and Lawrence Katz (1986) that recessions are times when the labor markets of almost all industries slacken—not times when workers move from industries with slack markets

to others with tight markets. I conclude that a realistic model of the labor market needs to invoke a market-wide force that has powerful effects on the recruiting efforts of employers.

II. Model of the Labor Market

A. *The Matching Process and Recruiting Effort*

I adopt the standard view of the matching friction in the labor market. The flow of candidate matches results from the application of a constant-returns matching technology to vacancies, v, and unemployment, u (both are expressed as ratios to the labor force). Let x be the ratio of vacancies to unemployment and let $\phi(x)$ be the per-period probability that a searching worker will find a job. Let $\rho(x) = \phi(x)/x$ be the per-period probability that an employer will fill a vacancy. ϕ is an increasing function and ρ is a decreasing function. Employers open vacancies and initiate the recruiting process whenever it is profitable to do so.

The vacancy/unemployment ratio, x, serves as the indicator of labor-market conditions in the model. In a tight market with a high ratio of vacancies to unemployment, the unemployed find it easy to locate new jobs, so the job-finding rate $\phi(x)$ is high. Employers find it difficult to locate new workers, so the job-filling rate $\rho(x)$ is low. The matching model gives a precise meaning to the notion of tight and slack markets.

A standard specification for the matching technology is

$$(1) \qquad \phi(x) = \omega x^{\alpha}.$$

The parameter ω controls the efficiency of matching and the parameter α splits the variation between changes in job-finding rates and changes in job-filling rates. The underlying matching function gives an elasticity of α to vacancies and $1 - \alpha$ to unemployment.

B. *Separations*

For simplicity, I assume a fixed hazard, δ, that a job will end. In the U.S. labor market, separations that result in unemployment appear to rise somewhat when unemployment rises, but separations involving direct reemployment in new jobs decline. JOLTS measures the sum of the two flows; the sum declined moderately from December 2000 through the most recently reported data (see Hall, 2005). The situation is further complicated by the flows into unemployment of people who were previously out of the labor force and the flows of unemployed people back out of the labor force (see Olivier Blanchard and Diamond, 1990). My model in its present form does not claim to do justice to these aspects of labor-market dynamics. It is straightforward to extend the model to make separations endogenous. The key properties considered here would not be altered by that extension. Higher separations in slack markets would require higher vacancies to maintain stochastic equilibrium in the market, and this influence could flatten the Beveridge curve unrealistically (see Shimer, 2005; Hall, 2005).

In addition to ruling out endogenous movements of the separation rate, my assumption also rules out exogenous movements. That is, I do not take spontaneous fluctuations in the separation rate as a driving force in the model. A spontaneous burst of separations raises both unemployment and vacancies, and shifts the Beveridge curve outward. The stability of the Beveridge curve argues against the importance of such a driving force (see Abraham and Katz, 1986).

C. *Equilibrium with Matching Friction*

The following is derived fairly directly from Pissarides (2000) and Shimer (2005). I use discrete time and a discrete random driving force to facilitate computations. Initially, I consider a stationary economy perturbed by a technology shock drawn from a distribution that does not change over time. I let λ be the value a worker enjoys when searching (leisure value and unemployment compensation). The random state of the economy is s and the productivity of labor is z_s. The economy transits from state s to state s' with probability $\pi_{s,s'}$. The price of output is normalized at one. It costs k in recruiting costs to hold a vacancy open for one period. Workers and firms are risk-neutral and discount the future at rate β.

The model is conveniently specified in terms of Bellman value-transition equations. Let U_s be the value a worker associates with being unemployed and searching for a new job when the state of the economy is s, and let V_s be the value the worker associates with being in a job after receiving that period's wage payment, w_s. Let J_s be the value the employer associates with a filled job after making the wage payment. I assume, as is standard in this literature, that employers expand recruiting effort to the point of zero profit, so the value associated with an unfilled vacancy is zero.

The value transition equations are:

$$(2) \quad U_s = \lambda + \beta \sum_{s'} \pi_{s,s'} [\phi(x_s)(w_{s'} + V_{s'})$$

$$+ (1 - \phi(x_s))U_{s'}].$$

(3)

$$V_s = \beta \sum_{s'} \pi_{s,s'}[(1 - \delta)(w_{s'} + V_{s'}) + \delta U_{s'}].$$

$$(4) \quad J_s = z_s + \beta(1 - \delta) \sum_{s'} \pi_{s,s'}(J_{s'} - w_{s'}).$$

$$(5) \quad 0 = -k + \beta\rho(x_s) \sum_{s'} \pi_{s,s'}(J_{s'} - w_{s'}).$$

Equation (5) captures a central aspect of the model: Given the anticipated payoff from making a match, firms create vacancies up to the point where the payoff is canceled by the recruiting cost, k. As they create more vacancies, x_s rises, recruiting success, $\rho(x_s)$ falls, and the point of zero net payoff is achieved. This pins down the key variable, x_s, the state-contingent vacancy/unemployment ratio.

Conditional on the state-contingent wage, w_s, equation (4) determines the value the employer derives from the match. Equation (5) then determines the amount of recruiting effort and therefore the tightness of the labor market, x_s, for each state. Finally, equations (2) and (3) determine the two state-contingent values for workers. These do not enter the solution directly but are needed to verify that the wage lies within the bargaining set.

Newly formed and continuing matches result in the same wage-bargaining problem in this simple setup. The worker's reservation wage, $\underline{w}_s$, equates the unemployment value, U_s, to the employment value, $V_s + \underline{w}_s$, so

$$(6) \qquad \underline{w}_s = U_s - V_s.$$

The employer's reservation wage is the entire anticipated profit from the match,

$$(7) \qquad \bar{w}_s = J_s.$$

These values determine the boundaries of the bargaining set:

$$(8) \qquad B_s = [\underline{w}_s, \bar{w}_s].$$

Any wage in the bargaining set will result in the efficient formation or retention of a match, as both worker and employer will benefit from the match, in the sense of receiving a match value at least as large as the non-match values represented by the reservation wages.

D. *Equilibrium Wages*

Here I depart from the DMP model, which views wage determination as the outcome of a Nash bargain. The symmetric Nash bargain would be the average of the two reservation values:

$$(9) \qquad \frac{\underline{w}_s + \bar{w}_s}{2}.$$

Instead, I characterize wage determination in terms of a Nash (1953) demand game or auction (see also Kalyan Chatterjee and William Samuelson, 1983; Roger Myerson and Mark Satterthwaite, 1983). In the auction, worker and employer know one another's reservation values. The worker proposes a wage, w_L, and the firm, without knowing the worker's proposal, makes its own proposal, w_H. If $w_L \leq w_H$, the match is made or continues, and the wage is agreed to be $w = \kappa w_L + (1 - \kappa)w_H$ with $0 < \kappa < 1$. The auction has the property that any w in the bargaining set $[\underline{w}_s, \bar{w}_s]$ is a Nash equilibrium. Believing that the worker is bidding w_L,

the firm will bid w_L as well, provided that $w_L \leq \bar{w}_s$. Similarly, believing that the firm is bidding w_H, the worker will bid w_H as well, provided $\underline{w}_s \leq w_H$. Thus any $w = w_L = w_H \in B_s$ is a Nash equilibrium.

I use the demand-game auction as a metaphor for the unstructured bargaining that normally occurs in the labor market. I do not mean to suggest that the bargaining takes the particular structured form of the demand game. I am also aware that an auction setup leaves certain issues unsettled (see Abhinay Muthoo, 1999). In particular, the Nash equilibrium arises because the players believe that no bargain will occur unless they bid in a way that will achieve the bargain. They believe that they cannot make a deal later if the auction fails to make one. Muthoo (Ch. 8) studies the case where the two players in a bargaining problem make initial simultaneous bids but can revise the bids at a cost if the first round fails to reach a bargain. The later rounds follow the game studied by Ariel Rubinstein (1982) which converges to the Nash bargain. Because the Nash bargain leaves the volatility of employment unexplained, I rule out this setup. It is a topic for further work to develop a richer model of wage bargaining that retains the indeterminacy that is central to the view of this paper.

Nash proposed the celebrated equilibrium selection rule—the Nash bargain—adopted in the DMP model. I explore different equilibrium selection rules to pin down the wage within the bargaining set. I begin by considering rules that take the form of assigning a different wage in each state of the economy—the wage is w_s, a function of the state, s. Thus I exclude variables, often called sunspot variables, which might play a role even though they have no direct role in the substance of the bargaining problem. I also exclude the history of the economy. My exclusions are similar in spirit to the ones that define Markov-perfect equilibrium in dynamic games, though this setup lacks the state variable that usually is a central element of a dynamic game.

A wage rule w_s is an equilibrium in the economy if it results in a solution to equations (2) through (5) with $w_s \in B_s$. There is a rich space of equilibria, including the symmetric Nash bargain.

Because wages are frequently regarded as less flexible than a full spot market might imply, my next step is to consider the class of constant wages, where $w_s = w$ for all states s. For this purpose, define $\tilde{J}_s$ as the solution to the linear system,

$$(10) \qquad \tilde{J}_s = z_s + \beta(1 - \delta) \sum_{s'} \pi_{s,s'} \tilde{J}_{s'}.$$

$\tilde{J}_s$ is the value an employer would attach to a new hire who never receives any wage—it is the present value of the revenue generated by a worker hired when the economy is in state s. Then

PROPOSITION: A constant wage w is an equilibrium of the model if

$$(11) \qquad \lambda \leq w \leq \min_s [1 - \beta(1 - \delta)]\tilde{J}_s$$

that is, the wage lies between the flow value of being unemployed, λ, and the annuity value, $[1 - \beta(1 - \delta)]\tilde{J}_s$ of the lowest-expected-profit state.

PROOF:
First, I verify that the constant wage does not fall short of the reservation wage $\underline{w}_s$ in any state. For this purpose, let $Y_s = w - (U_s - V_s)$, the excess of the constant wage over the reservation wage. Subtracting equation (3) from equation (2) yields

(12)

$$Y_s = w - \lambda + \beta(1 - \phi(x_s) - \delta) \sum_{s'} \pi_{s,s'} Y_{s'}.$$

In matrix notation, this equation is

$$(13) \qquad\qquad (\mathbf{I} - \mathbf{A})\mathbf{Y} = \mathbf{b}.$$

Because the characteristic values of $\mathbf{A}$ all have modulus less than one, the equation has the solution

$$(14) \qquad \mathbf{Y} = (\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \cdots)\mathbf{b}.$$

Because $w \geq \lambda$, $b \geq 0$, so all elements of $\mathbf{A}$ are positive. Hence $\mathbf{Y} \geq 0$ and so $w \geq \underline{w}_s$.

To show that $w \leq J_s$ for all s, let

$$(15) \qquad Z_s = \tilde{J}_s - J_s$$

$$= \beta(1 - \delta) \sum_{s'} \pi_{s,s'} (Z_{s'} + w).$$

The solution is

$$(16) \qquad Z_s = \frac{\beta(1 - \delta)}{1 - \beta(1 - \delta)} \, w.$$

Thus

$$(17) \qquad J_s = \tilde{J}_s - \frac{\beta(1 - \delta)}{1 - \beta(1 - \delta)} \, w.$$

By hypothesis, $w \leq [1 - \beta(1 - \delta)]\tilde{J}_s$. Hence

$$(18)$$

$$w \leq [1 - \beta(1 - \delta)]\left[J_s + \frac{\beta(1 - \delta)}{1 - \beta(1 - \delta)} \, w \right].$$

or $w \leq J_s$, as required.

A constant wage rule may be interpreted as a wage norm or social consensus. A related concept is a focal point. Much of the discussion of wage norms considers resistance to wage reductions—George Akerlof et al. (1996) discuss this type of a wage norm and Truman Bewley (1999) provides evidence about the operation of a modern labor market constrained by social forces. Those authors focus on the avoidance of downward wage adjustments, but many of their ideas point toward the absence of immediate upward wage adjustments as well. My specification is limited in a way not previously considered in the literature on wage rigidity—I do not permit the norm to lie outside the bargaining set. The earlier work implied inefficient outcomes, especially the loss of a job under conditions where both worker and employer could have been better off with a wage adjustment. The wage norm I consider interferes neither with the formation of efficient matches once the parties are in touch with one another nor with the preservation of jobs with positive surplus. Inefficient separations cannot occur. As a result, the model provides a full answer to the indictment of sticky wage models in Barro (1977) for invoking unexplained inefficiencies in economic arrangements.

The idea that the wage is constrained to lie in the bargaining set of the employment relationship but may be insensitive to current conditions apart from that constraint has an extensive history in the literature on employment theory (see James Malcomson, 1999, for many citations). The new feature of my model is the effect of wage stickiness on the pre-match recruiting efforts of employers and thus the implications of stickiness for unemployment. Because the variations in unemployment and vacancies respond to expectations formed when workers are hired, the essential stickiness in the model is in those expectations. If only post-employment wages were sticky, and wages paid in the first period of employment fluctuated to offset anticipated later wages, the model would deliver much smaller fluctuations in labor-market conditions. In Hall (2005) I formulate a related model in which the expected present value of wages over the life of a job is the sticky variable.

E. *Wage Rules in a Nonstationary Environment*

A realistic environment for wage determination is nonstationary. The stochastic upward trend in productivity rules out a constant wage rule: eventually the bottom of the bargaining set will rise above any constant wage and that wage can no longer be an equilibrium. To extend the idea of a wage norm to a nonstationary environment, suppose that productivity evolves as the product of two components:

$$(19) \qquad z_t = z_t^P z_{s_t}^M.$$

The component z_t^P is a slow-moving trend known to the public. The component z_{st}^M is a mean-reverting process similar to the single component studied earlier—it depends on a discrete state s_t as before. The analog of the constant wage rule in this economy is

$$(20) \qquad w_t = w z_t^P.$$

The Bellman equations for the nonstationary economy can be written in terms of values in the form $z_t^P U_{s_t}$, $z_t^P V_{s_t}$, $z_t^P J_{s_t}$, and $z_t^P w_{s_t}$. For example, the equation for the job value is

$$(21) \quad z_t^P J_s = z_t^P z_{s_t}^M + \tilde{\beta}_t (1 - \delta)$$

$$\times \sum_{s_{t+1}} \pi_{s_t, s_{t+1}} z_{t+1}^P (J_{s_{t+1}} - w_{s_{t+1}}).$$

I rewrite as

$$(22) \quad J_s = z_{s_t}^M + \left(\frac{z_{t+1}^P}{z_t^P} \right) \tilde{\beta}_t (1 - \delta)$$

$$\times \sum_{s_{t+1}} \pi_{s_t, s_{t+1}} (J_{s_{t+1}} - w_{s_{t+1}}).$$

I let $\beta = (z_{t+1}^P / z_t^P) \tilde{\beta}_t$, the inflation and growth adjusted discount, which I assume to be constant. Then equation (22) and its counterparts for U and V are the same as equations (2) through (4). I assume that λ and k also share the upward trend of z_t^P and reinterpret these parameters as the constant detrended values.

The effects of the mean-reverting component of productivity, $z_{s_t}^M$, on unemployment in the nonstationary economy are the same as the effects of the single productivity shift, z_s, in the earlier stationary economy. In particular, the sticky wage rule $w_{s_t} = w$ has exactly the same allocational consequences in the nonstationary economy as in the earlier stationary economy.

So far in the paper I have not specified the units for measuring the variables involving economic values. The stationary model would make little sense unless the units had stable purchasing power, but in the nonstationary model, the drift component z_t^P could be nominal, in which case part of its drift would arise from drift in the overall price level. In this case, there is a connection between the model of this paper and the idea of a Phillips curve. The Phillips curve describes short-run deviations around a nominal path that is interpreted as reflecting inertia in wage and price determination.

Milton Friedman (1968) and Edmund Phelps (1967) launched a rich literature on nominal inertia. They pointed out that the wage determination process adapts to persistent inflation and productivity growth. A key implication is that the unemployment effects of wage movements would be insulated from these longer-term trend-like movements. Friedman put the point the other way around: an attempt to keep unemployment low would result, in the longer run, in ever-increasing inflation. Experience in many countries in the ensuing three decades generally confirmed this proposition. The wage process summarized in equation (20) captures the Friedman-Phelps hypothesis.

The huge literature on wage determination in the Phillips-curve and related frameworks has distinguished backward-looking or adaptive behavior from forward-looking behavior. My approach sidesteps this issue by associating the component of wages that represents shifts of the Phillips curve with the trend variable z_t^P and the component that represents movements along the Phillips curve with the random variable $z_{s_t}^M$.

III. Parameters

To estimate the elasticity of the matching function, $\phi(x)$, I use the aggregate data from JOLTS shown in Table 2. I calculate x as the ratio of vacancies to unemployment and the job-filling rate as the job-finding rate divided by x, and estimate the elasticity as the change in the log of the job-finding rate divided by the change in the log of the vacancy/unemployment ratio, x. The resulting estimate is 0.765.

I assume that productivity takes on five discrete values z_s uniformly spaced in the interval $[1 - \gamma, 1 + \gamma]$. I assume that the transition probabilities are zero except as follows: $\pi_{1,2} = \pi_{4,5} = 2(1 - \theta)$, $\pi_{2,3} = \pi_{3,4} = 3(1 - \theta)$, with the upper triangle of the transition matrix symmetrical to the lower triangle and the diagonal elements equal to one minus the sums of the nondiagonal elements. The resulting serial correlation of z is θ.

The model operates at a monthly frequency. I calibrate as follows: According to JOLTS, the average value of the vacancy/unemployment ratio, x, during the period from December 2000 to December 2002 was 0.539. I solve the model with Nash wage bargaining—equations (2) through (5) and (9)—for the recruiting cost k

TABLE 2—CALCULATIONS FROM JOLTS DATA

	December 2000	December 2002
New hires	4.070 million	3.187 million
Unemployed	5.264 million	8.209 million
Vacancies	4.036 million	2.558 million
Job-finding rate, ϕ	0.773 per month	0.388 per month
Job-filling rate, ρ	1.008 per month	1.246 per month
Unemployment rate, u	3.6 percent	5.7 percent
Vacancy rate, v	2.8 percent	1.8 percent
x	0.767 vacancies per unemployed worker	0.312 vacancies per unemployed worker
α, elasticity of job finding with respect to x	0.765	
ω, efficiency of matching	0.947	

TABLE 3—PARAMETERS

Parameter	Interpretation	Value	Source
δ	Separation rate	0.034	JOLTS
λ	Flow value while searching (leisure or unemployment compensation)	0.4	Corresponds to a flow value while searching that is about 40 percent of the flow wage
k	Flow cost of a vacancy	0.986	Matches vacancy/unemployment ratio in median state to average, 2000–2002
β	Discount factor	0.995	Corresponds to 5-percent annual rate
θ	Serial correlation of mean-reverting component of productivity	0.9899	Serial correlation of U.S. unemployment, 1948–2003
γ	Dispersion parameter for mean-reverting component of productivity	0.00565	Matches standard deviation of unemployment to U.S. level of 1.54 percent

and all of the endogenous variables except x_3, which I set to 0.539. The resulting estimate of k is 0.986, measured in units of output per worker produced in the median state ($z_3 = 1$ in my normalization). Then I set the fixed wage to the Nash-bargain wage for the median state and solve the fixed-wage model—equations (2) through (5). I set the serial correlation of productivity equal to the historical serial correlation of the U.S. unemployment rate. I set the dispersion parameter γ so that the fixed-wage model matches the observed standard deviation of unemployment. Table 3 shows the results of these calculations.

The solved values of the variables in the median state for the fixed-wage model are shown in Table 4. The worker's career is worth about 230 units of monthly productivity. In the median state, the worker assigns almost exactly the same value to unemployment and to

TABLE 4—VALUES OF ENDOGENOUS VARIABLES IN THE MEDIAN STATE

Variable	Interpretation	Value
U	Value while searching	229.34
V	Value while working	229.28
J	Value of worker to the firm	1.8698
w	Wage	0.96572

employment—the worker's reservation wage is close to zero. This implies that the wages to be earned in the future are sufficiently high that the worker—if pushed to the wall—would be willing to work for the first month for free. The employer values the relationship at a little below two units of monthly productivity. The wage of 0.96 units is 96 percent of the total value created from work. The remaining 4 percent compensates the employer for the initial cost of recruiting.
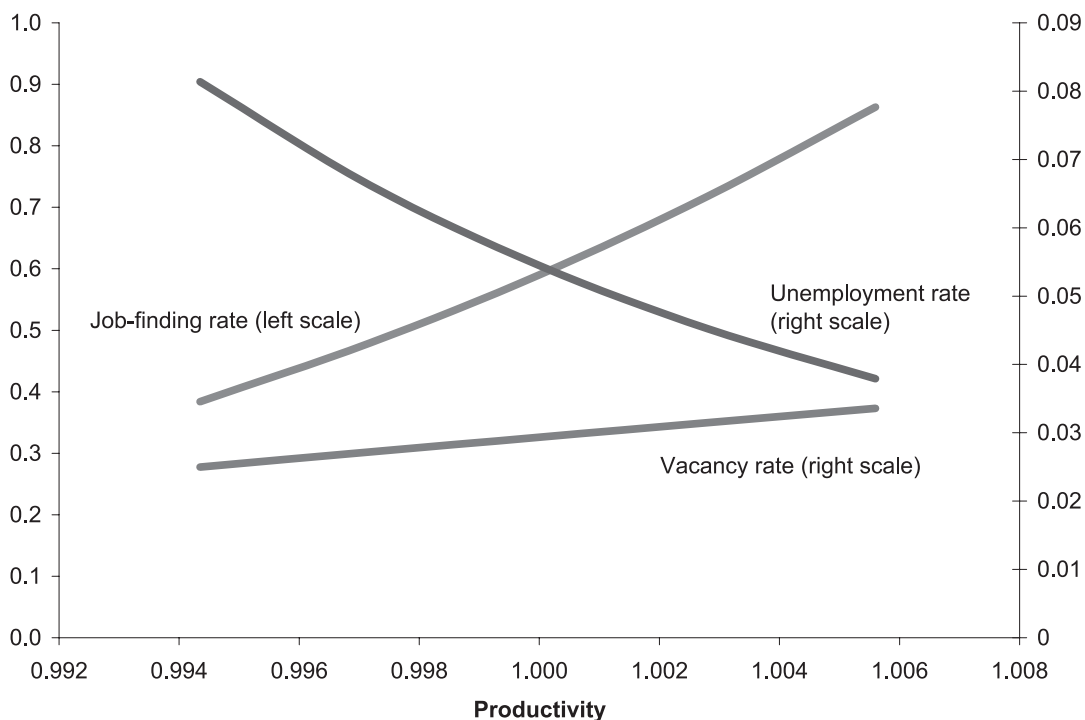
FIGURE 2. JOB FINDING, VACANCY, AND UNEMPLOYMENT RATES, FIXED WAGE

IV. Properties of the Model

In the model, the unemployment rate is a state variable. Unemployment is not a function of the current state, as are all of the other variables, but depends on the history of the economy. But, because the job-finding rate is so high, unemployment is a fast-moving state variable, and it departs only slightly from the value

$$(23) \qquad u_s^* = \frac{\delta}{\delta + \phi(x_s)}.$$

See Hall (2005) for a further discussion of this point and a comparison between the actual unemployment rate and the rate inferred from this formula. For the moment, I will treat the unemployment rate as a jump variable along with all the other variables, which are true jump variables. In a later section, I will show the full dynamic response of unemployment.

Figure 2 shows the basics of the model.

When productivity is high, toward the right of the figure, unemployment is low, vacancies are high, and the job-finding rate is high. The labor market is tight—it resembles conditions in the U.S. labor market in 2000. The higher productivity level, with the wage held fixed, results in higher profit per worker. Employers put more resources into recruiting because they receive a higher fraction of the surplus. Consequently, the job-finding rate is higher and the unemployment rate is lower.

The curves in Figure 2 display properties that are central to the view of the labor market embodied in the model. In the following discussion, I will use figures associated with the median state; the figures for the other states are similar. If productivity falls, unemployment rises substantially. The rise occurs because jobs become hard to find.

The high sensitivity of labor-market conditions to productivity when the wage is fixed arises for the following reason: the value that an employer achieves from a success in recruiting
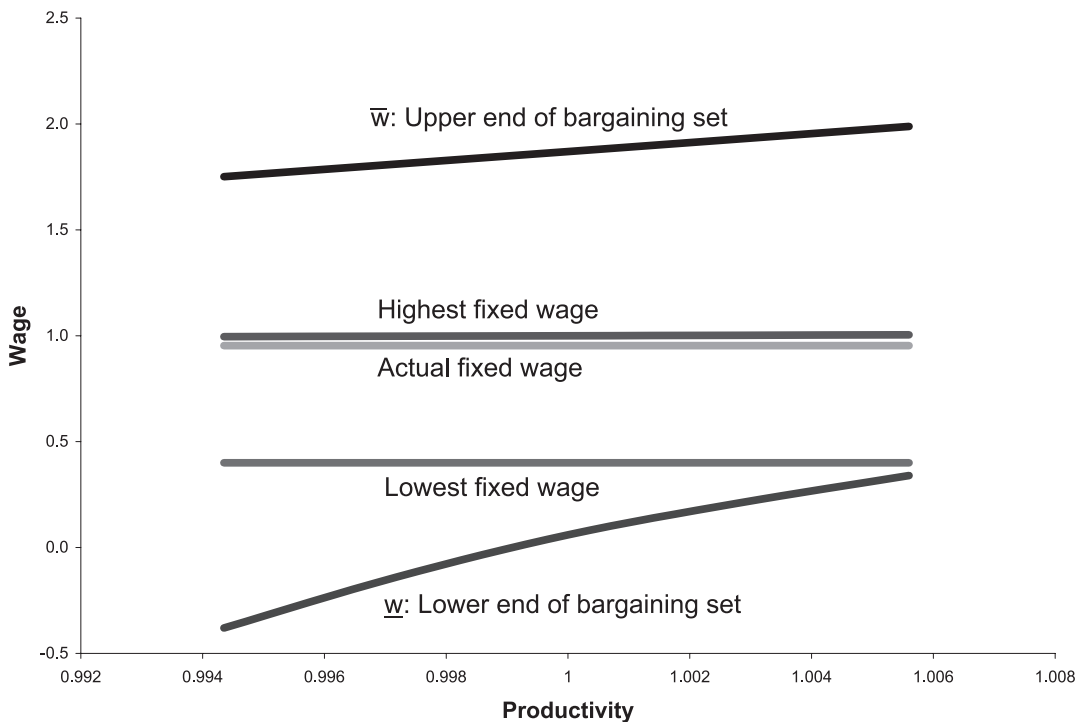
FIGURE 3. WAGE ELEMENTS

is J. Recruiting cost exhausts this value in equilibrium. The response of recruiting effort—and therefore of conditions in the labor market—depends on the change in J induced by a change in productivity. J is the present value to the firm of the profit margin generated by a worker in the course of the job and, with exogenous separation, does not depend on any other variables in the model. In the fixed-wage model, when productivity rises from state 3 to state 4, J rises with a slope of 21 units of J for each unit of z. The result is a large increase in recruiting effort. By contrast, with a symmetric Nash wage bargain, as in the DMP model, almost all of this increased profit goes into wages, because a higher z raises both $\underline{w}$ and $\bar{w}$, so the slope is only 1.4 units of J per unit of z. The productivity change has little effect on the employer's job value and thus little effect on recruiting effort.

The sensitivity of recruiting effort to productivity depends on the distribution of rents between workers and employers. If every employer makes take-it-or-leave-it offers to its

workers and captures all the rent, workers are indifferent between unemployment and employment and their wage is λ. Employers have large incentives to recruit workers at all times, but the elasticity of the value is unity and the response of recruiting effort to price changes is not very elastic. Thus the high amplification of price or productivity shocks that occurs in the model depends on the assumption that the typical worker shares a significant fraction of the joint surplus from the employment relationship.

Figure 3 shows the factors relating to the wage across the productivity states. The horizontal line in the middle is the actual fixed wage. The curves at the top and bottom are the upper and lower limits of the bargaining set for wages in each period, based on the expectation that the fixed wage will be paid in all subsequent periods. The actual wage lies at the middle of the bargaining set for the median productivity state.

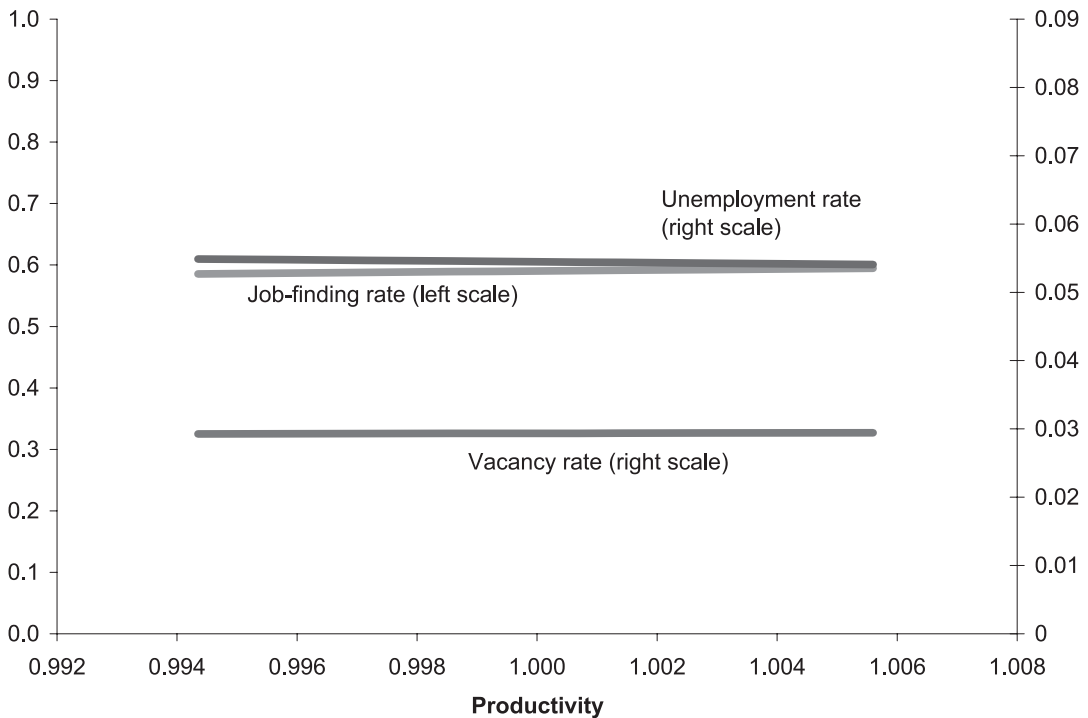The line just above the actual wage is the highest possible wage in that state, as defined in

FIGURE 4. JOB FINDING, VACANCY, AND UNEMPLOYMENT RATES, NASH-BARGAIN WAGE

equation (10). The horizontal line beneath the actual wage is the flow value of unemployment compensation and leisure. The actual wage lies inside these bounds as well.

Notice that the actual wage lies close to its highest permissible value and far above its lowest value, the leisure value λ. The model would be strained if the actual wage were well below the maximal level. In that case, the labor market would be exceptionally tight because the employer's payoff from a hire would be high. Employers could reasonably be expected to deal with the tight market by recruiting techniques that lie outside the model, such as advertising wages above the wage norm. The simple model provides a reasonable account of the market without those techniques, in the DMP tradition. Of course, a richer model would consider many possible recruiting techniques as part of a more detailed characterization of recruiting. That model would also consider a more active role for workers in the job-matching process.

A. Comparison to the Same Model with Nash Wage Bargain

A model in the DMP family can be created by replacing the wage determination process developed above with a symmetric Nash wage bargain, as in equation (9). Figure 4 displays its properties in the same format as Figure 2. The figure reveals the finding about the DMP model stressed by Shimer (2003) and Veracierto (2002)—the small shifts in productivity that suffice to explain movements in the labor market of typical magnitude in the fixed-wage model cause almost no visible movements with the Nash-bargain wage.

B. Dynamic Response

I derive the model's dynamic response to a productivity shock by comparing the expected unemployment rates of two economies. The first starts with the level of unemployment associated
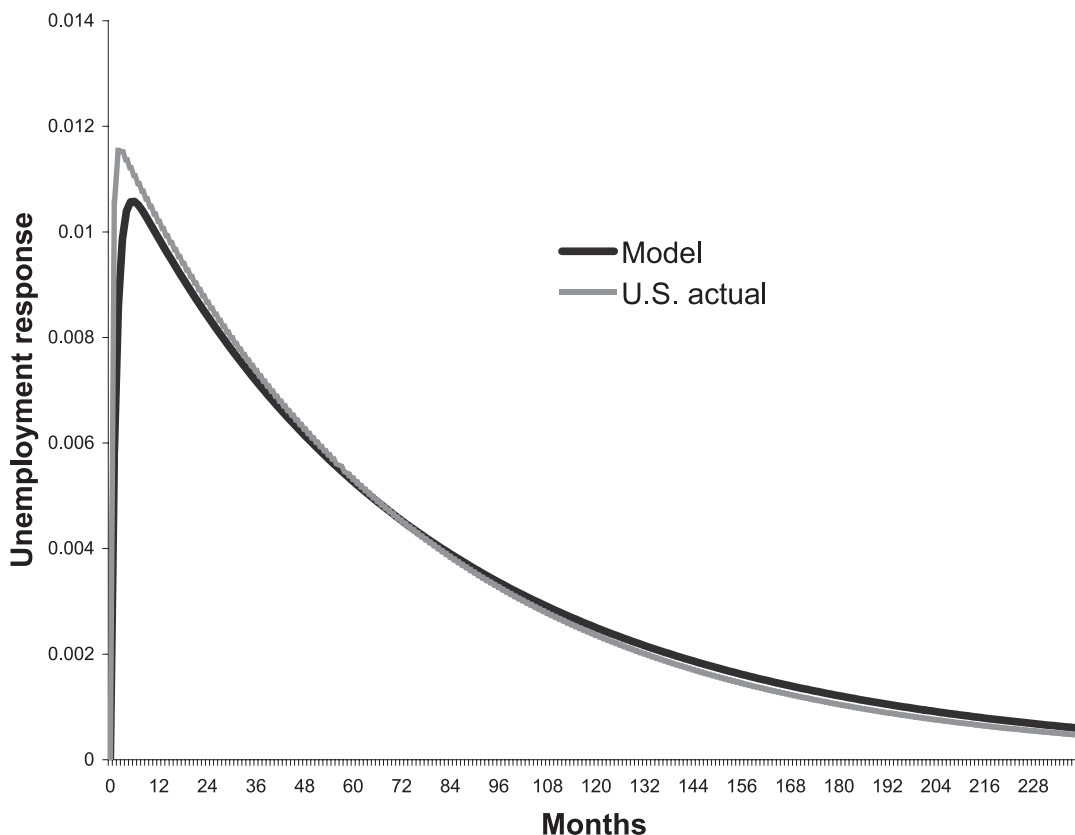
FIGURE 5. RESPONSE OF UNEMPLOYMENT TO NEGATIVE IMPULSE, MODEL AND ACTUAL

with the median state but is in the state below the median state. The second starts with the same level of unemployment and is in the median state. The difference is the response over time to the shock, which is the transition between the median and lower states that occurred at time zero.

Figure 5 shows the dynamic response to a negative productivity shock in the sticky-wage model, along with the univariate moving-average representation of actual U.S. unemployment. Unemployment in the model matches the actual response reasonably closely. Both rise quickly in the early months following a shock and then decline gradually over a period of several years.

Figure 6 shows the response of the job-finding rate, ϕ, to the same impulse. As soon as productivity drops, the labor market slackens

and the job-finding rate falls by 11 percentage points from its normal level of about 60 percent per month. With a constant inflow to unemployment and a diminished outflow, unemployment builds rapidly to a maximum effect of about one percentage point. Because the productivity shock is mean-reverting, the expected job-finding rate rises continuously after the shock. At six months, improved job finding and higher unemployment combine to equate the outflow from unemployment to the exogenous inflow, and unemployment reaches its maximum. From that point forward, further improvements in job finding bring the unemployment rate back down to its unconditional mean. The vacancy rate (not shown) moves in the same way as the job-finding rate.

The persistence of slack conditions after a negative shock comes essentially entirely from
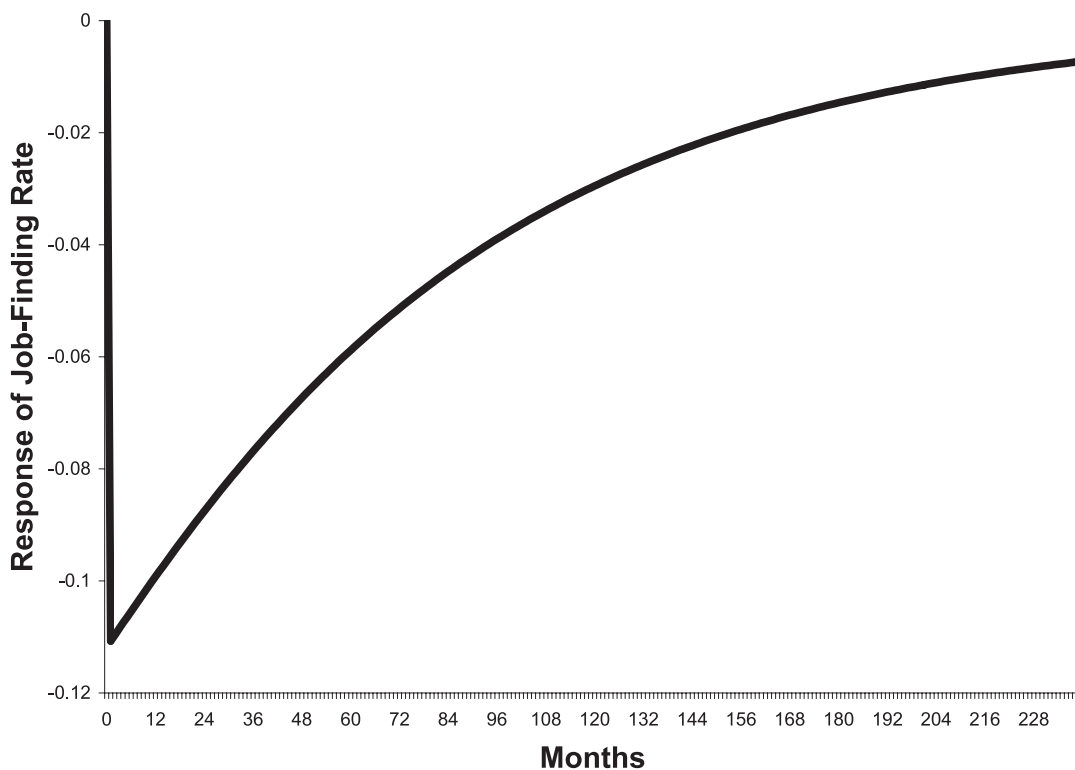
FIGURE 6. RESPONSE OF JOB-FINDING RATE TO NEGATIVE IMPULSE IN THE MODEL

the persistence of low productivity and hardly at all from the time required for workers to find new jobs. Michael Pries (2004) considers a complementary explanation of the persistence of unemployment—the workers who find new jobs after an adverse shock leave those jobs relatively soon and experience multiple spells of unemployment before finding stable jobs.

C. *Comparison to Shimer's Results*

Shimer (2005) compares the response of the vacancy/unemployment ratio (x in my notation) to productivity shocks (z in my notation). In Shimer's version of the DMP model with Nash wage bargaining, the elasticity of that response is 1.7 (his Table 3—1.7 is the ratio of the standard deviation of the log vacancy/unemployment ratio to the standard deviation of productivity). In my version of the same model, the response is 1.8, the slope of the log unemploy-

ment/vacancy ratio with respect to log productivity in Figure 4. The similarity of the two figures demonstrates that our basic calibrations are similar.

Shimer concludes that the response of the DMP model with Nash bargaining is well below the value needed to understand the volatility of unemployment and vacancies. In filtered quarterly data for the United States for the years 1951 through 2003, he shows that the regression coefficient of the log vacancy/unemployment ratio on log productivity is 7.2 (the correlation coefficient of 0.391 multiplied by the ratio of the standard deviations of the two variables). The response in my fixed-wage model is much stronger—the elasticity is 94 (the slope from Figure 2). Part of the difference arises from noise in Shimer's measure of productivity. Presumably another explanation is that not all wages are literally fixed. I conclude that the fixed-wage model is easily capable of explaining

the observed high volatility of labor-market allocations, even if it does not apply everywhere in the market.

V. Other Equilibrium Wage Rules

I commented earlier on the richness of the set of equilibrium wage rules that make the wage depend only on the productivity state, s, and on the trend, z_t^P. I have focused on the subset of constant wages. One example of a nonconstant wage is the partially smoothed wage,

$$(24) \qquad w_s^P = \alpha w + (1 - \alpha)w_s^N.$$

Here w_s^N is the state-contingent Nash-bargain wage and $0 \leq \alpha \leq 1$ indexes the amount of smoothing. If w is an equilibrium wage, then w_s^P is also an equilibrium wage. With partial smoothing, the effects of productivity shocks on employment will be smaller than in the case of a constant wage. The volatility of employment will be controlled by the smoothing parameter α.

Interesting examples of equilibrium wage rules arise outside the class of purely productivity-state-dependent wages. One possibility is an adaptive wage,

$$(25) \qquad w_t^A = (1 - \alpha)w_{t-1}^A + \alpha w_s^N.$$

The wage becomes a new state variable of the model. It must be constrained to remain in the current bargaining set, though this is unlikely to have any practical effect. In Hall (2003) I rationalize an adaptive wage in terms of the aggregation of individual wage decisions, each perturbed by a match-specific random component. If the random component results in a bargaining set for the match that does not contain the aggregate wage norm, the wage is reset to the nearest boundary of the bargaining set. The average wage in one period becomes the norm in the next period. The match-level adjustment of the wage to keep it inside the bargaining set was studied earlier by Jonathan Thomas and Tim Worrall (1988). The reason for the rule in their model is to keep wage volatility as low as possible but to retain efficient matches.

With an adaptive wage, employment is less persistent than productivity. The model with an adaptive wage delivers stationary unemploy-

ment even with nonstationary productivity. Thus the adaptive mechanism provides an alternative way to characterize the role of nonstationary elements of the environment. But the adaptive mechanism is essentially arbitrary and could take many other forms.

VI. Concluding Remarks

Strong evidence supports the following view of fluctuations in employment and unemployment: When the labor market is tight and unemployment is low, employers devote substantial resources to recruiting workers. Job-finding rates for the unemployed are high. By contrast, when the market is slack and unemployment is high, employers recruit less aggressively and job-finding rates are low. Data on help-wanted advertising, vacancies, and unemployment confirm these relations. Further, transitions from strong markets with low unemployment and high vacancies to weak markets with high unemployment and low vacancies seem to occur without large measurable changes in driving forces. Rather, small shocks stimulate large responses of unemployment.

I have offered a model of fluctuations in the labor market that mimics all of these properties. In the model, the labor market becomes slack when recent events have lowered the benefit to the employer from hiring. These events, such as a small decline in productivity or a small rise in input prices, substantially reduce the payoff to hiring if the wage is sticky. Stickiness is plausible, because it occurs only within the range where the wage does not block efficient bargains from being struck and maintained. The outcome of the bargain between worker and employer is fundamentally indeterminate and wage stickiness is an equilibrium selection mechanism. The stickiness can be interpreted in terms of a wage norm that provides the equilibrium selection function.

The wage-stickiness model developed in this paper, based on a wage norm as an equilibrium selection mechanism, achieves a strict standard of predictive power in one respect—that the wage never falls outside the bargaining set—but is permissive with respect to wage-determination mechanisms that keep the wage inside the bargaining set. Application of the model in practice

needs to be guided by evidence about actual wage determination, because theory is unrestrictive apart from the role of the bargaining set.

REFERENCES

▶**Abraham, Katharine G. and Katz, Lawrence F.** "Cyclical Unemployment: Sectoral Shifts or Aggregate Disturbances?" *Journal of Political Economy*, 1986, *94*(3), pp. 507–22.

Akerlof, George A.; Dickens, William T. and Perry, George L. "The Macroeconomics of Low Inflation." *Brookings Papers on Economic Activity*, 1996, *0*(1), pp. 1–59.

▶**Barro, Robert J.** "Long-Term Contracting, Sticky Prices, and Monetary Policy." *Journal of Monetary Economics*, 1977, *3*(3), pp. 305–16.

Bewley, Truman F. *Why wages don't fall during a recession*. Cambridge, MA: Harvard University Press, 1999.

Blanchard, Olivier J. and Diamond, Peter A. "The Cyclical Behavior of the Gross Flows of U.S. Workers." *Brookings Papers on Economic Activity*, 1990, *0*(2), pp. 85–143.

▶**Calvo, Guillermo A.** "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics*, 1983, *12*(3), pp. 383–98.

Chatterjee, Kalyan and Samuelson, William F. "Bargaining under Incomplete Information." *Operations Research*, 1983, *31*(5), pp. 835–51.

▶**Diamond, Peter A.** "Aggregate Demand Management in Search Equilibrium." *Journal of Political Economy*, 1982, *90*(5), pp. 881–94.

Friedman, Milton. "The Role of Monetary Policy." *American Economic Review*, 1968, *58*(1), pp. 1–17.

Hall, Robert E. "Wage Determination and Employment Fluctuations." National Bureau of Economic Research, Inc., NBER Working Papers: No. 9967, 2003.

Hall, Robert E. "Employment Efficiency and Sticky Wages: Evidence from Flows in the Labor Market." *Review of Economics and Statistics*, 2005, forthcoming.

Howitt, Peter. "The Keynesian Recovery." *Canadian Journal of Economics,* 1986, *19*(4), pp. 626–41.

Malcomson, James M. "Individual Employment Contracts," in Orley Ashenfelter and David Card, eds., *Handbook of labor economics,* Vol. 3B. New York: North-Holland, 1999, pp. 2291–372.

Mortensen, Dale T. "Property Rights and Efficiency in Mating, Racing, and Related Games." *American Economic Review*, 1982, *72*(5), pp. 968–79.

Muthoo, Abhinay. *Bargaining theory with applications*. Cambridge: Cambridge University Press, 1999.

▶**Myerson, Roger B. and Satterthwaite, Mark A.** "Efficient Mechanisms for Bilateral Trading." *Journal of Economic Theory*, 1983, *29*(2), pp. 265–81.

Nash, John. "Two-Person Cooperative Games." *Econometrica*, 1953, *21*(1), pp. 128–40.

▶**Phelps, Edmund S.** "Phillips Curves, Expectations of Inflation and Optimal Unemployment over Time." *Economica*, 1967, *34*(135), pp. 254–81.

Pissarides, Christopher A. "Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages." *American Economic Review*, 1985, *75*(4), pp. 676–90.

Pissarides, Christopher A. *Equilibrium unemployment theory,* 2nd ed. Cambridge, MA: MIT Press, 2000.

▶**Pries, Michael.** "Persistence of Employment Fluctuations: A Model of Recurring Job Loss," *Review of Economic Studies*, 2004, *71*(1), pp. 193–215.

▶**Rubinstein, Ariel.** "Perfect Equilibrium in a Bargaining Model." *Econometrica*, 1982, *50*(1), pp. 97–109.

▶**Shimer, Robert.** "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review*, 2005, *95*(1), pp. 25–49.

Thomas, Jonathan and Worrall, Tim. "Self-Enforcing Wage Contracts." *Review of Economic Studies*, 1988, *55*(4), pp. 541–54.

Veracierto, Marcelo. "On the Cyclical Behavior of Employment, Unemployment and Labor Force Participation." Federal Reserve Bank of Chicago, Working Paper Series: WP-02–12, 2003.