

# Macroeconomics

## Business Cycle Analysis

### Lecture notes (5.2) on: *New Keynesian Macroeconomics (2): a framework for policy evaluation*

(F. Bagliano, 2024)

Over the last two decades, complete macroeconomic models incorporating the existence of imperfect competition on goods (and also labor) markets and nominal rigidities in the price (and also wage) setting process have been developed and have gradually become a standard tool in understanding the impact of various disturbances on the economy and in analysing the effects of monetary policy. In essence, they combine the basic equilibrium approach of the Real Business Cycle (RBC) literature with some "keynesian" ingredients such as monopolistic competition and nominal rigidities. The resulting class of models, referred to as *dynamic stochastic general equilibrium (DSGE)* models, has become widely used for policy evaluation. In these notes, a basic and highly stylized version of such "*new-keynesian DSGE*" model is presented. The more recent literature has extended the model to explicitly include, for example, a financial sector (with additional frictions and informational imperfections) and a labor market.

#### 1. Basic structure

The key elements of the NK-DSGE model are the following:

- adopting the *methodological approach of RBC models*, equilibrium conditions for macroeconomic variables are derived from the optimizing behavior of consumers and firms, under the assumptions of rational expectations and

simultaneous clearing in all markets. Differently from first-generation new-keynesian models, this approach rests on sound microeconomic foundations, emphasising optimal intertemporal choices by agents (the *Dynamic* dimension) and the interrelationships among markets (the *General Equilibrium* dimension) and between policy actions and agents' behavior. Random disturbances from various origins are explicitly added (the *Stochastic* dimension) and are the sources of aggregate fluctuations.

- firms act as *monopolistic competitors*, producing differentiated products and facing downward-sloping demand curves; they set prices as a result of intertemporal profit maximization.
- *nominal rigidities* are introduced and constitute the main source of non-neutrality of monetary policy. Firms are subject to constraints in the frequency of price adjustment; the possibility that prices will remain fixed for some time makes the firms' price-setting decision inherently forward-looking.
- in specifying the central bank's behavior, emphasis is given to the endogenous response of monetary policy to developments in the macroeconomy (e.g. output and inflation), i.e. to the *monetary policy rule*.

The basic model is made up of three main blocks, describing *demand*, *supply* and *monetary policy*. The main equations have explicit microfoundations, being derived from behavioral assumptions about households, firms and the policymaker. A sketch of the most important links among the model's blocks, determining the three endogenous variables (output  $Y$ , inflation  $\pi$  and the nominal interest rate  $i$ ), is depicted in Figure 1:

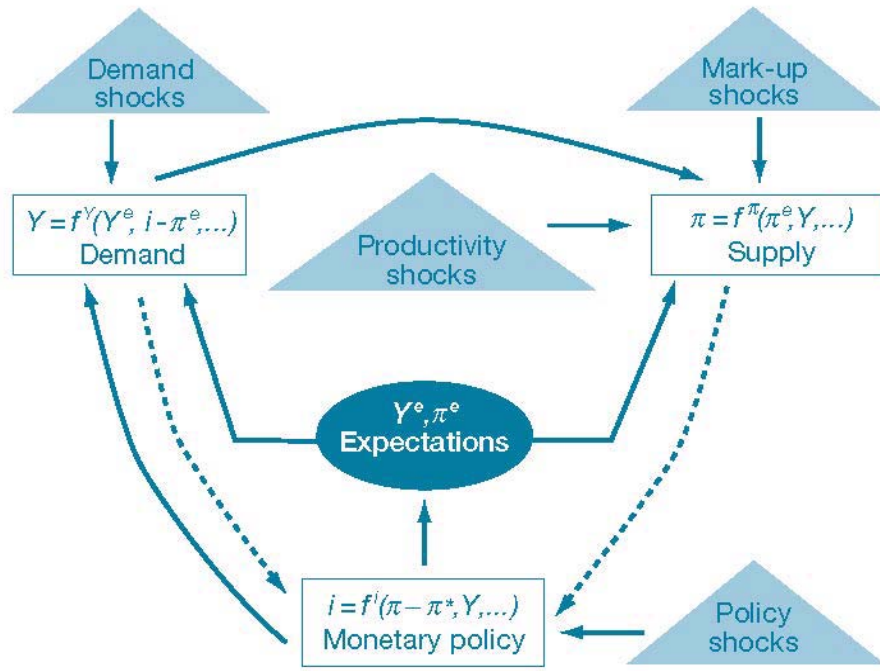
- the *demand* block determines current aggregate output ( $Y$ ) as a function of two main variables: the real expected interest rate (i.e. the nominal interest rate  $i$  less expected inflation  $\pi^e$ ), and expected future real activity ( $Y^e$ ). A higher expected real interest rate induces agents to save more and consume or invest less, whereas a higher expected level of economic activity positively affects current spending;
- the *supply* block describes how firms set prices and, in the aggregate, how the inflation rate evolves over time. When goods demand is high, firms face higher marginal production costs and therefore increase prices: this generates a positive link between current output and inflation. Moreover,

higher expected future inflation puts a pressure on current prices, determining a rise in current inflation;

- finally, *monetary policy* is conducted by setting the nominal interest rate as a function of output and of deviation of the inflation rate from a target level ( $\pi^*$ ): the nominal rate is raised when output is relatively high and when inflation exceeds its target.

The solid and dashed lines in the figure capture the set of direct and feedback relationships among the main macroeconomic variables, with a central role for expected future output and inflation in determining agents' current choices. The stochastic nature of the economy is captured by a variety of disturbances that affect aggregate demand ("demand shocks"), the pricing decisions of firms ("mark-up" and "productivity shocks") and the setting of the nominal interest rate ("policy shocks", capturing deviations from the rule followed by the monetary authority).

## The Basic Structure of DSGE Models



**Figure 1.** The basic structure of DSGE models. Source: Sbordone *et al.* (2010).

## 2. A simple baseline New-Keynesian macro model

The basic structure outlined above can be formalized using a simple three-equation macroeconomic model capturing the main elements of the dynamic, general equilibrium framework with new-keynesian features. The model specifies the aggregate demand, supply and monetary policy blocks and rests on several simplifying and convenient assumptions in order to highlight the basic mechanisms at work.

### 2.1. Aggregate demand

The aggregate demand relation is obtained from the solution of an intertemporal utility maximization problem by a representative consumer. As in the standard RBC model, the first-order condition for the optimal allocation of resources between consumption and saving has the following typical form (known as *Euler equation*):

$$u'(C_t) = \frac{1}{1 + \rho} E_t \{ [1 + (i_t - \pi_{t+1})] u'(C_{t+1}) \}$$

where  $u'(C)$  is the marginal utility of consumption,  $i_t$  is the nominal interest rate on a riskless one-period bond,  $\pi_{t+1}$  is the inflation rate between  $t$  and  $t + 1$ , and  $\rho$  is the rate of time preference with which consumers discount future utility (corresponding, in an economic environment without long-run growth, to the steady state real interest rate). Along an optimal consumption path, the agent is indifferent between consuming one unit at time  $t$ , yielding marginal utility  $u'(C_t)$ , and saving for consumption in the next period  $t + 1$ , when she will be able to consume the proceeds of a one-period investment in the bond, yielding a real rate of return  $i_t - \pi_{t+1}$ .

A log-linearization procedure, together with the assumption of a constant relative risk aversion (CRRA) form for the utility function,<sup>1</sup> yields the following form for the above first-order condition (with  $c \equiv \log(C)$ , and  $\sigma > 0$  capturing the degree of consumers' risk aversion):

$$E_t c_{t+1} - c_t = \frac{1}{\sigma} [(i_t - E_t \pi_{t+1}) - \rho]$$

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<sup>1</sup>The CRRA utility function is:

$$u(C) = \frac{C^{1-\sigma} - 1}{1 - \sigma}$$

where  $\sigma > 0$  is the coefficient of (relative) risk aversion. The larger  $\sigma$ , the more risk averse is the consumer. When  $\sigma = 1$  the utility function becomes logarithmic:  $u(C) = \log(C)$ .

When the expected real interest rate ( $i_t - E_t \pi_{t+1}$ ) is higher than the time preference discount rate  $\rho$ , the consumer has a greater incentive to save more in  $t$  (therefore reducing  $c_t$ ) and to increase her consumption level expected for  $t + 1$ . The coefficient  $\frac{1}{\sigma}$  measures the strength of the agent's willingness to transfer resources over time in reaction to changes in the expected real interest rate (so called *intertemporal elasticity of substitution*).

Considering a simplified economy in which consumption is the only component of aggregate demand (i.d.  $c_t = y_t$ , where  $y_t$  is aggregate output) and rearranging the above equation we get:

$$y_t = -\frac{1}{\sigma} [(i_t - E_t \pi_{t+1}) - \rho] + E_t y_{t+1} \quad (2.1)$$

Current output is negatively affected by the expected real interest rate and positively determined by future expected output. The first relation is a hallmark of the traditional *IS* relation (albeit with a different rationale) whereas the second is a consequence of the forward-looking nature of the consumer's problem. The result is the so-called *dynamic IS* equation, determining current aggregate output in the model.

## 2.2. Aggregate supply

The aggregate supply block is centered around the specification of inflation dynamics. Monopolistic competition and nominal rigidities are introduced as essential features in the model. The productive sector of the economy is composed of a large number of firms supplying differentiated products and setting prices under conditions of monopolistic competition. Price setting occurs in a *staggered* fashion: in each period only a fraction of firms adjust their prices, whereas the others keep prices fixed and adjust output to meet demand.

A convenient way of formalizing this kind of nominal rigidity is due to Calvo (1983): in each period a firm has a fixed probability  $1 - \theta$  of adjusting its price which is independent of its own history of price changes (i.e. it is independent of how long a firm has kept its price fixed). This assumption tries to capture a feature of price dynamics at the individual firm level, that is the occurrence of discrete price adjustments at irregularly spaced intervals of time. In this setup the (log of the) aggregate price level  $p_t$  evolves over time as a weighted average of the price set by firms which are allowed to adjust and the price of firms that do not adjust:

$$p_t = (1 - \theta) p_t^* + \theta p_{t-1} \quad (2.2)$$

where  $p_t^*$  is the price chosen optimally by the adjusting firms, whereas  $p_{t-1}$  is the average price of firms that do not adjust (given the assumption that firms are randomly attributed the ability to adjust prices). Thus, a single parameter,  $\theta$ , measures the degree of nominal price rigidity in the model.

In any given period  $t$ , firms that are (randomly) allowed to adjust prices choose the optimal price  $p_t^*$  taking into account the constraint on the frequency of price adjustment, that is the possibility that the chosen price will be fixed for several periods. To understand their price-setting behavior, start from the extreme case of perfect price flexibility (i.e. absence of any nominal rigidity: all firms are allowed to adjust prices in any period, therefore  $\theta = 0$ ): in this case monopolistic competitive firms set prices at time  $t$  as a constant<sup>2</sup> mark-up  $\mu$  over current (log) nominal marginal costs  $mc_t$ :

$$p_t^* = mc_t + \mu \quad (2.3)$$

Instead, when nominal rigidities are present, firms rationally consider future market conditions in setting current prices. In fact, they set  $p_t^*$  as a mark-up on a weighted average of current and expected future marginal costs. The weight on the marginal costs in any future period  $t+k$  depends on the discounted probability that the firm will still have its price fixed at level  $p_t^*$  at that time. Applying the appropriate weights to each future expected nominal marginal cost  $mc_{t+k}$  we obtain the following expression for the optimal price  $p_t^*$ :

$$p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t mc_{t+k} \quad (2.4)$$

where  $\beta = \frac{1}{1+\rho}$  is the time discount factor.<sup>3</sup> (2.4) can be equivalently written as:

$$p_t^* = \beta\theta E_t p_{t+1}^* + (1 - \beta\theta)(mc_t + \mu) \quad (2.5)$$

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<sup>2</sup>The constancy of the mark-up rests on the assumption that the demand elasticity faced by firms is constant.

<sup>3</sup>To derive the above expression (2.4) we note that the probability that the price set at  $t$ ,  $p_t^*$ , will still be in force after  $k$  periods is  $\theta^k$ . This probability is "discounted" to period  $t$  using the discount factor  $\beta^k$  (where  $\beta \equiv \frac{1}{1+\rho}$ ). Therefore, the discounted probability attached to any future (period  $t+k$ ) marginal cost is  $(\beta\theta)^k$ . To obtain the coefficients used in (2.4) we need to rescale each discounted probability by the sum of all discounted probabilities. This (infinite) sum is given by:

$$1 + \beta\theta + (\beta\theta)^2 + (\beta\theta)^3 + \dots = \sum_{k=0}^{\infty} (\beta\theta)^k = \frac{1}{1 - \beta\theta}$$

since  $0 < \beta\theta < 1$ . Therefore the weights attached to expected marginal costs for any future

where the optimal price expected for the following period  $t+1$  captures the effect of future expected marginal costs on the current optimal price. In both formulations it is clear that firms' optimal price setting behaviour is *forward-looking*, taking into account the future expected dynamics of marginal costs.

In order to derive the dynamics of the inflation rate  $\pi_t \equiv p_t - p_{t-1}$  we need to combine (2.2) and (2.4). After some algebraic manipulations (outlined in the *Appendix*), we can obtain the inflation rate in  $t$  as:

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} (mcr_t + \mu) \quad (2.6)$$

where  $mcr_t \equiv mc_t - p_t$  is the firms' *real* marginal cost. The last term in (2.6),  $mcr_t + \mu$ , can be expressed as the deviation of the current level of real marginal cost from the level that would prevail if prices were fully flexible (i.e. in the absence of any nominal rigidity). In this situation, real marginal cost is (using the fact that, with perfect flexibility, prices are set at the level given by (2.3)):

$$\begin{aligned} \overline{mcr}_t &= mc_t - p_t^* = mc_t - (mc_t + \mu) \\ &= -\mu \end{aligned} \quad (2.7)$$

and therefore the deviation  $\widehat{mcr}_t$  is

$$\begin{aligned} \widehat{mcr}_t &\equiv mcr_t - \overline{mcr}_t = mcr_t - (-\mu) \\ &= mcr_t + \mu \end{aligned} \quad (2.8)$$

The equation for the current inflation rate (2.6) can then be written as

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} \widehat{mcr}_t \quad (2.9)$$

In this model inflation is the aggregate result of price-setting decisions by firms, which adjust prices on the basis of current and expected future marginal costs. Therefore, the current level of inflation depends on expected inflation (which captures the expected future dynamics of real marginal costs) and on the current deviation of real marginal costs from its level with fully flexible prices.

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period  $t+k$  are:

$$\frac{(\beta\theta)^k}{1-\beta\theta} = (1-\beta\theta)(\beta\theta)^k$$

Those weights are used in (2.4) above.



To obtain an expression for inflation dynamics closer to the traditional "Phillips curve" formulation (i.e. a relationship between the inflation rate and a measure of aggregate economic activity), conventional assumptions on the aggregate production function factor supply elasticities can be used to derive a positive link between  $\widehat{mcr}_t$  and deviations of aggregate output ( $y_t$ ) from the level that would prevail under perfect price flexibility ( $\bar{y}_t$ ):

$$\widehat{mcr}_t = \varphi (y_t - \bar{y}_t) \quad (2.10)$$

with  $\varphi > 0$ . Real marginal costs will rise with the level of economic activity if some factors of production are available in fixed quantity or if higher real wages are needed to induce workers to supply additional hours. We can use that relation to write inflation dynamics in the form of the so-called *new Keynesian Phillips curve* (NKPC) as:

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} \varphi (y_t - \bar{y}_t) \\ &= \beta E_t \pi_{t+1} + \kappa (y_t - \bar{y}_t) \end{aligned} \quad (2.11)$$

This new Keynesian version has some specific features, also with respect to the traditional Phillips curve formulation:

- its form is explicitly derived from a fully-specified microeconomic price-setting problem faced by firms with market power, in the presence of constraints on the frequency of price adjustments. The consequence is that the inflation rate becomes a *forward-looking* variable, since firms will take into account the expected evolution of their marginal costs in setting optimal current prices, as captured by the term  $E_t \pi_{t+1}$ . (A measure of expected inflation was also a determinant of current inflation in the traditional, expectations-augmented, Phillips curve formulation: however, this expectations term was usually specified as  $E_{t-1} \pi_t$ , so that current realization of economic variables at time  $t$  had no effect in determining revisions in expected inflation over the future horizon, as in the NKPC).
- the reaction of inflation to current output developments, represented by the deviation  $y_t - \bar{y}_t$ , captured by the coefficient  $\kappa$ , depends on some basic parameters in the model. In particular:
  - it is positively affected by  $\varphi$ , the elasticity of real marginal costs to output. Since real marginal costs are the basic determinant of inflation

in this setting, if costs react more to output movements, also prices will be affected in the same direction.

- it is negatively affected by  $\theta$ , the degree of nominal rigidity in the model. If  $\theta$  increases, opportunities of adjusting prices occur less frequently and firms will weight current marginal costs less in setting prices: therefore inflation will be less sensitive to current output fluctuations.
- the measure of activity that enters inflation dynamics, the *output gap*  $y_t - \bar{y}_t$  is the deviation of current output from its equilibrium level in the absence of nominal rigidities. That level,  $\bar{y}_t$ , can change over time as a result of real shocks (e.g. technology) but is invariant to monetary policy.

### 2.3. Monetary policy

To close the model, monetary policy is specified as a simple rule that the central bank follows in setting the short-term nominal interest rate  $i_t$  to hit a target level. The rule specifies the response of the interest rate to economic conditions, captured by the level of the inflation rate and the output gap. In its simplest form, such rule can be written as:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y (y_t - \bar{y}_t) \tag{2.12}$$

where  $\rho$  (the rate of households' time preference) is interpreted here as the real interest rate prevailing in a steady state (no growth) equilibrium. The policy parameters capture the reaction of the central bank to the inflation rate ( $\phi_\pi > 1$ , so that when an increase in inflation occurs, the resulting positive response of the policy nominal rate implies an increase of the *real* short-term interest rate) and to the output gap ( $\phi_y > 0$ ).

### 3. Dynamic properties of the model: monetary policy shocks

The simple three-equation version of the new-Keynesian macroeconomic framework outlined above can be used to simulate the reaction of the main aggregate variables to some shocks hitting the economy. The model is formed by the dynamic IS equation (2.1), the new-Keynesian Phillips curve (2.11) and the monetary policy rule (2.12), reported here:

$$\begin{aligned}y_t &= -\frac{1}{\sigma} [(i_t - E_t \pi_{t+1}) - \rho] + E_t y_{t+1} \\ \pi_t &= \beta E_t \pi_{t+1} + \kappa (y_t - \bar{y}_t) \\ \dot{i}_t &= \rho + \phi_\pi \pi_t + \phi_y (y_t - \bar{y}_t) + v_t\end{aligned}$$

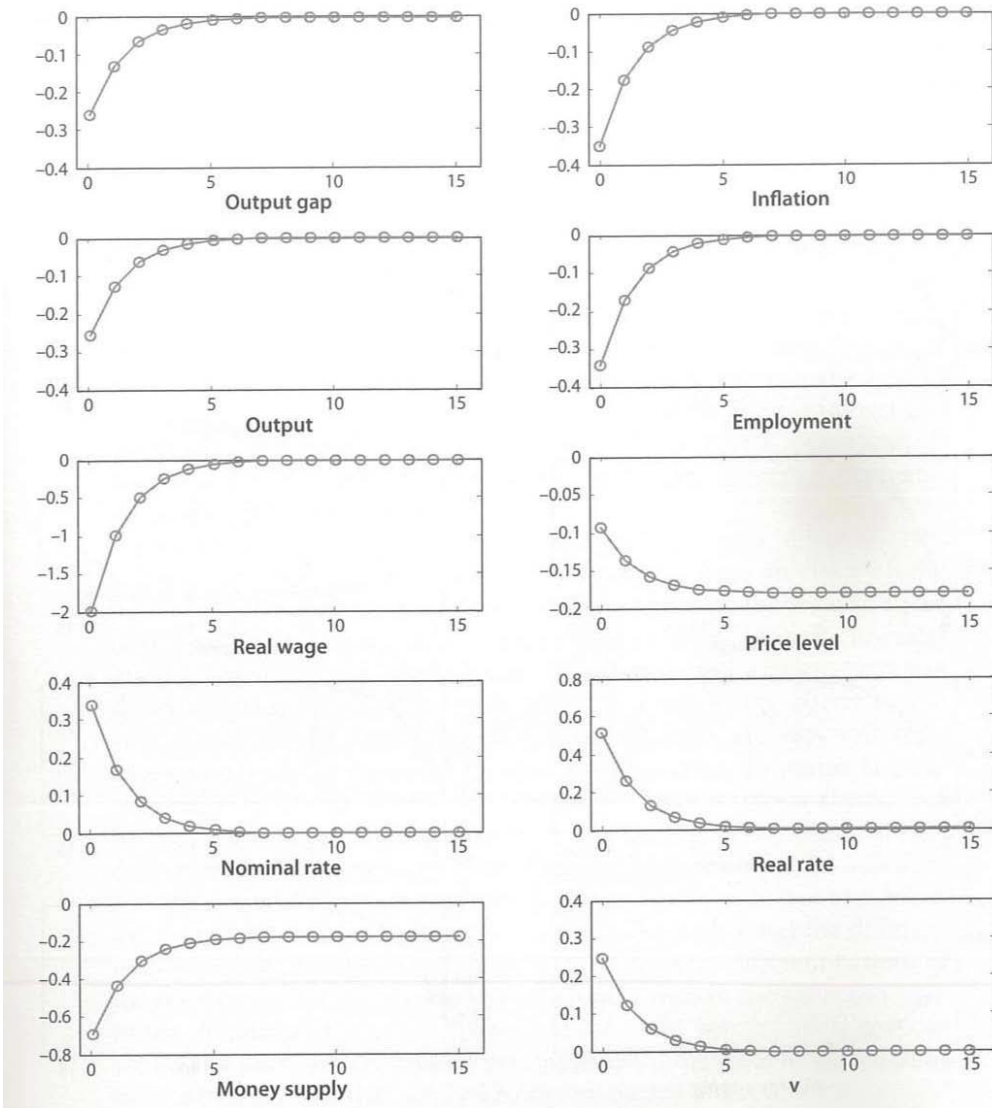
where a stochastic element  $v_t$  has been added to the interest rate setting rule to capture contractionary (if  $v_t > 0$ ) or expansionary (if  $v_t < 0$ ) monetary policy shocks, leading to a rise or a decrease in the nominal interest rate for given values of inflation and the output gap.

Simulating the dynamics of the model in the face of an interest rate disturbance  $v_t$  (with a standard numerical calibration for the parameters) allows us to follow the channels whereby monetary policy can affect aggregate variables (see Galí 2015). Let us consider the case of a *contractionary monetary policy* shock:  $v_t > 0$ . The main channel whereby monetary policy disturbances can affect the economy in the new Keynesian macro framework works through changes in the real interest rate. Such changes affect consumers' intertemporal choice, therefore having an impact on current output, together with the expected path for activity. Movements in the output gap affect current inflation, also in this case together with expectations about the future course of the inflation rate. Both changes in inflation and the output gap entail reactions of the nominal interest rate through the monetary policy rule both in the current period and in future periods, with a feedback effect on agents' expectations of output and inflation dynamics.

Figure 2 reports the dynamic responses of a large set of macroeconomic variables obtained from simulating an extended version of the model outlined above when a contractionary monetary policy occurs ( $v_t > 0$ ). In the simulation, such shock has some positive persistence over time, as shown in the lower right panel. The key variable in the monetary policy transmission mechanism is the real interest rate ( $i_t - E_t \pi_{t+1}$ ), which increases on impact and gradually returns to its steady state value (represented by the zero horizontal axis in the figure). This behaviour is the result of an increase of the nominal interest rate ( $i_t$ ) and a decline of the

expected inflation rate ( $E_t \pi_{t+1}$ ). The responses of output ( $y_t$ ) and the output gap ( $y_t - \bar{y}_t$ ) are the same since a monetary policy shock does not affect the flexible-price equilibrium ( $\bar{y}_t$ ). As suggested by the dynamic IS equation, current output declines one-to-one with expected output ( $E_t y_{t+1}$ ) and is further depressed by the increase of the real interest rate, which induces consumers to increase savings and decrease current spending. Current inflation is pushed down by two effects: a decrease in expected inflation ( $E_t \pi_{t+1}$ ) due to the decline of expected future real marginal costs (represented in this simulation by the real wage), and a fall in current output. Over time, all real variables revert back gradually to their previous long-run equilibrium levels and only the price level is (negatively) permanently affected.

The model therefore provides a fairly complete picture of the macroeconomic effects of a contractionary monetary policy shock, taking into account a sluggish price adjustment dynamics and an important role for expectations in determining current spending and inflation.



**Figure 2.** Dynamic responses to a contractionary monetary policy shock.  
Source: Gali (2015)

### 3.1. Appendix: Derivation of (2.6)

To derive (2.6) we start from (2.2) and (2.5), reported here:

$$p_t = (1 - \theta) p_t^* + \theta p_{t-1} \quad (\text{A1})$$

$$p_t^* = \beta \theta E_t p_{t+1}^* + (1 - \beta \theta) (m c_t + \mu) \quad (\text{A2})$$

Leading (A1) forward by one period and taking expectations at  $t$  we obtain

$$E_t p_{t+1} = (1 - \theta) E_t p_{t+1}^* + \theta p_t$$

Multiplying the last expression by  $\beta \theta$  and subtracting the result from (A1) we get (after rearranging terms):

$$p_t - \beta \theta E_t p_{t+1} = \theta (p_{t-1} - \beta \theta p_t) + (1 - \theta) (p_t^* - \beta \theta E_t p_{t+1}^*)$$

and, substituting for  $p_t^*$  from (A2):

$$p_t - \beta \theta E_t p_{t+1} = \theta (p_{t-1} - \beta \theta p_t) + (1 - \theta) (1 - \beta \theta) (m c_t + \mu) \quad (\text{A3})$$

Using the relationship between nominal ( $m c_t$ ) and real ( $m c r_t$ ) marginal cost ( $m c_t \equiv m c r_t + p_t$ ) in (A3) we get:

$$p_t - \beta \theta E_t p_{t+1} = \theta (p_{t-1} - \beta \theta p_t) + (1 - \theta) (1 - \beta \theta) (m c r_t + p_t + \mu) \quad (\text{A4})$$

Collecting terms in  $p_t$  and rearranging we obtain:

$$p_t - p_{t-1} = \beta (E_t p_{t+1} - p_t) + \frac{(1 - \theta)(1 - \beta \theta)}{\theta} (m c r_t + \mu) \quad (\text{A5})$$

Finally, letting  $\pi_t \equiv p_t - p_{t-1}$  be the inflation rate in period  $t$  and  $E_t \pi_{t+1} \equiv E_t p_{t+1} - p_t$  be the expected inflation rate for period  $t + 1$  as of time  $t$ , we finally get:

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \theta)(1 - \beta \theta)}{\theta} (m c r_t + \mu) \quad (\text{A6})$$

which is (2.6) in the main text.

## References

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- [3] Sbordone A., A. Tambalotti, K. Rao, K. Walsh (2010) "Policy analysis using DSGE models: an introduction", *Economic Policy Review*, Federal Reserve Bank of New York, October