Macroeconomics
Business Cycle Analysis

Real Business Cycles
(F. Kydland- E. Prescott)

Main theoretical features of the RBC view:

- *equilibrium approach* to business cycle fluctuations, defined as a set of properties concerning *comovements* and *persistence* of main macroeconomic quantities

- unifying theory of *growth* and *fluctuations*

- focus on the *propagation mechanisms* (especially over time) of shock based on *intertemporal substitution* effects

- *technological shocks* as main driving force of business cycle fluctuations

Analytical features:

- reference model:

    neoclassical model of growth (Solow) with *uncertainty* in the rate of technological progress

    $\Rightarrow$ fluctuations are the aggregate result of the behavioral rules of rationally optimizing agents, subject to resource constraints in a stochastic environment

- empirical methodology:

    calibration, instead of traditional econometric testing

Basic RBC model

Structure of the economy:

- *preferences*: large number of infinitely-lived agents maximizing an expected utility function

$$U_t = E_t \left[\sum_{j=0}^{\infty} \beta^j u(\underbrace{c_{t+j}}_{\text{consumption}}, \underbrace{l_{t+j}}_{\text{leisure}}) \right] \qquad 0 < \beta < 1 \qquad (U)$$

$u(c, l) \Rightarrow$ preference for smooth paths of c and l (and intertemporal substitutability of c and l in the face of changes in the real wage and in the real interest rate)

- time *endowment*: amount of time normalized to 1 to divide between work n and leisure l

$$n_{t+j} + l_{t+j} = 1 \qquad (*)$$

- *technology* and *capital accumulation*: constant-return-to-scale production function

$$y_{t+j} = z_{t+j} \, f(k_{t+j}, n_{t+j}) \qquad (F)$$

z_{t+j}: total productivity shock;

$$k_{t+j+1} = i_{t+j} + (1 - \delta) \, k_{t+j} \qquad (K)$$

i_{t+j}: investment; δ: rate of physical capital depreciation;

- total *resource constraint* (no government spending, closed economy):

$$y_{t+j} = c_{t+j} + i_{t+j} \qquad (Y)$$

- combining (F), (K) e (Y)

$$c_{t+j} + k_{t+j+1} = z_{t+j} \, f(k_{t+j}, n_{t+j}) + (1 - \delta)k_{t+j} \qquad (**)$$

Problem:

find the sequences $\{c_{t+j}\}_0^\infty$, $\{l_{t+j}\}_0^\infty$, $\{n_{t+j}\}_0^\infty$ e $\{k_{t+j}\}_0^\infty$ that maximize expected utility given the time endowment and resource constraint (*) e (**) and the stochastic process generating technological shocks

$$\max \quad L \;=\; \sum_{j=0}^\infty \beta^j u(c_{t+j}, l_{t+j}) + \sum_{j=0}^\infty \beta^j \, \omega_{t+j}(1 - n_{t+j} - l_{t+j})$$

$$+ \sum_{j=0}^\infty \beta^j \, \lambda_{t+j} \, \left[z_{t+j} \, f(k_{t+j}, n_{t+j}) + (1 - \delta)k_{t+j} - c_{t+j} - k_{t+j+1} \right]$$

with $\{\omega_{t+j}\}_0^\infty$ and $\{\lambda_{t+j}\}_0^\infty$ sequences of Lagrange multipliers associated with the costraints (and interpreted as shadow prices of one additional unit of time and capital, respectively)

Solution:
system of first-order conditions for c_t, l_t, n_t and k_{t+1} with constraints (*) and (**)

$$u_c(c_t, l_t) = \lambda_t$$
$$u_l(c_t, l_t) = \omega_t$$
$$\lambda_t \, z_t \, f_n(k_t, n_t) = \omega_t$$
$$\beta \, E_t \left(\lambda_{t+1} \left[z_{t+1} \, f_k(k_{t+1}, n_{t+1}) + 1 - \delta \right] \right) = \lambda_t$$

3

Special case (with closed-form solution):

$$u(c, l) = \theta \log c + (1 - \theta) \log l$$
$$y_t = z_t \, k_t^{1-\alpha} n_t^{\alpha}$$
$$\delta = 1$$

$\Rightarrow$ **solution**:

$$\frac{\theta}{c_t} = \lambda_t$$
$$\frac{1 - \theta}{l_t} = \omega_t$$
$$\alpha \, \lambda_t \, z_t \, k_t^{1-\alpha} \, n_t^{\alpha-1} = \omega_t$$
$$\beta \, (1 - \alpha) \, E_t \left(\lambda_{t+1} \, z_{t+1} \, n_{t+1}^{\alpha} \, k_{t+1}^{-\alpha} \right) = \lambda_t$$

to be solved for:

$$c_t = c(k_t, z_t)$$
$$n_t = n(k_t, z_t)$$
$$k_{t+1} = k(k_t, z_t)$$

NB: with log utility, labor supply does not change as wage w changes:

$$\max \theta \, \log c + (1 - \theta) \, \log l$$
$$\text{sub} \quad c = w \, n = w \, (1 - l)$$

f.o.c.: $\quad -\dfrac{\theta \, w}{w(1 - l)} + \dfrac{1 - \theta}{l} = 0 \quad \Rightarrow \quad l = 1 - \theta$

labor supply independent of w

$\Rightarrow$ *conjectures* (guesses) on the analytical form of the solutions:

$$
\begin{aligned}
n_t &= \bar{n} \\
c_t &= \pi_C \, z_t \, k_t^{1-\alpha} \\
k_{t+1} &= \pi_K \, z_t \, k_t^{1-\alpha}
\end{aligned}
$$

with π_C and π_K undetermined coefficients, related by the total resources constraint (with $n_t = \bar{n}$):

$$
\underbrace{z_t \, \bar{n}^\alpha \, k_t^{1-\alpha}}_{y_t} = \underbrace{\pi_C \, z_t \, k_t^{1-\alpha}}_{c_t} + \underbrace{\pi_K \, z_t \, k_t^{1-\alpha}}_{k_{t+1}}
$$

$$
\Rightarrow \pi_C + \pi_K = \bar{n}^\alpha
$$

Combining the f.o.c. for c and k and using the undetermined form of the solutions for n_t and c_t:

$$
\begin{aligned}
\frac{\theta}{\pi_C \, z_t \, k_t^{1-\alpha}} &= \beta \, (1-\alpha) \, E_t \left(\frac{\theta}{\pi_C \, z_{t+1} \, k_{t+1}^{1-\alpha}} \, z_{t+1} \, \bar{n}^\alpha \, k_{t+1}^{-\alpha} \right) \\
\Rightarrow \frac{\theta}{\pi_C \, z_t \, k_t^{1-\alpha}} &= \beta \, (1-\alpha) \, \bar{n}^\alpha \, E_t \left(\frac{\theta}{\pi_C \, k_{t+1}} \right)
\end{aligned}
$$

using the solution for k_{t+1}:

$$
\begin{aligned}
\frac{\theta}{\pi_C \, z_t \, k_t^{1-\alpha}} &= \beta \, (1-\alpha) \, \bar{n}^\alpha \, E_t \left(\frac{\theta}{\pi_C \, \left(\pi_K \, z_t \, k_t^{1-\alpha} \right)} \right) \\
\Rightarrow \pi_K &= \beta \, (1-\alpha) \, \bar{n}^\alpha \\
\Rightarrow \pi_C &= \left[1 - \beta \, (1-\alpha) \right] \bar{n}^\alpha
\end{aligned}
$$

$\Rightarrow$ solutions for c_t and k_{t+1}:

$$
\begin{aligned}
c_t &= \left[1 - \beta \, (1-\alpha) \right] z_t \, \bar{n}^\alpha \, k_t^{1-\alpha} \\
k_{t+1} &= \beta \, (1-\alpha) \, z_t \, \bar{n}^\alpha \, k_t^{1-\alpha}
\end{aligned}
$$

To check that the conjecture $n_t = \bar{n}$ is correct, by combining the f.o.c. for l and n and using the solution for c_t:

$$
\frac{1-\theta}{1-\bar{n}} = \alpha \, \frac{\theta}{c_t} \, z_t \, k_t^{1-\alpha} \, \bar{n}^{\alpha-1}
$$

$$
\Rightarrow \bar{n} = \frac{\alpha \, \theta}{\alpha \, \theta + (1-\theta) \left[1 - \beta \, (1-\alpha) \right]} \quad \text{constant}
$$

5

Dynamic properties of k and c:

$$\begin{aligned} \log k_{t+1} &= \phi_0 + (1 - \alpha) \log k_t + \log z_t \\ \log c_t &= \phi_1 + (1 - \alpha) \log k_t + \log z_t \end{aligned}$$

assumption of $AR(1)$ stochastic process for z:

$$\log z_t = \rho \log z_{t-1} + \varepsilon_t \qquad 0 < \rho < 1$$

From:

$$\begin{aligned} \log k_{t+1} &= \phi_0 + (1 - \alpha) \log k_t + \log z_t \\ \rho \log k_t &= \rho\,\phi_0 + \rho\,(1 - \alpha) \log k_{t-1} + \rho \log z_{t-1} \end{aligned}$$

taking $\log k_{t+1} - \rho \log k_t$, we get the $AR(2)$ processes for k:

$$\log k_{t+1} = (1 - \rho)\phi_0 + (1 - \alpha + \rho) \log k_t - \rho\,(1 - \alpha) \log k_{t-1} + \varepsilon_t$$
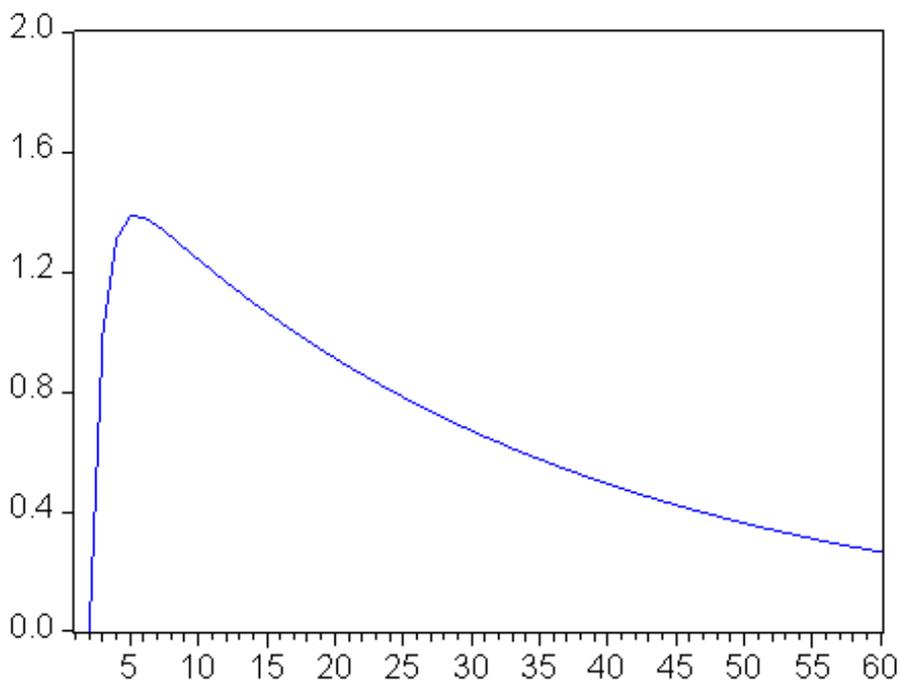
and c:

$$\log c_t = [\alpha(1 - \rho)\phi_1 + (1 - \alpha)(1 - \rho)\phi_0] + (1 - \alpha + \rho) \log c_{t-1} - \rho\,(1 - \alpha) \log c_{t-2} + \varepsilon_t$$

Example of dynamic response (*impulse response*) of log k to a unit realization of ε with:

$$\log k_{t+1} = (1 - \alpha + \rho) \log k_t - \rho (1 - \alpha) \log k_{t-1} + \varepsilon_t$$

for $\alpha = 0.66$, $\rho = 0.98$ (constant omitted)

Main results from calibration of a "standard" RBC model
(R. King-S. Rebelo, "Resuscitating business cycles", *Handbook of Macroeconomics*, 2000)

Main parameter values used in calibration: $\alpha = 0.66$, $\delta = 0.025$ (quarterly), $\rho = 0.98$.

reported in the second columns of Tables 1 and 3. In particular, investment is about three times more volatile than output in both the actual economy (where the ratio of standard deviations is 5.30/1.81=2.93) and the model economy (where the ratio of standard deviations is 2.95). Consumption is substantially smoother than output in both the model and actual economies. In our basic model, however, consumption is only about one-third as volatile as output while it is over two thirds as volatile as output in the U.S. economy. We return to discussion of this feature of the economy in section 6 below.

Table 3
Business Cycle Statistics for Basic RBC Model[35]

	Standard Deviation	Relative Standard Deviation	First Order Auto-correlation	Contemporaneous Correlation with Output
Y	1.39	1.00	0.72	1.00
C	0.61	0.44	0.79	0.94
I	4.09	2.95	0.71	0.99
N	0.67	0.48	0.71	0.97
Y/N	0.75	0.54	0.76	0.98
w	0.75	0.54	0.76	0.98
r	0.05	0.04	0.71	0.95
A	0.94	0.68	0.72	1.00

Note: All variables have been logged (with the exception of the real interest rate)and detrended with the HP filter.

Persistence and comovement with output. Business cycles are persistently high or low levels of economic activity: one measure of this persistence is the first-order serial correlation coefficient. Table 3 shows that the persistence generated by the basic model is generally high, but weaker than in the data (see Table 1). The relative standard deviations also provide a measure of the limited extent to which

[35]The moments in this table are population moments computed from the solution of the model. Prescott [1986] produced multiple simulations, each with the same number of observations available in the data, and reported the average HP-filtered moments across these simulations.

of 0.9. This high serial correlation is the reason why there is some predictability to the business cycle.

Table 1
Business Cycle Statistics for the U.S. Economy

	Standard Deviation	Relative Standard Deviation	First Order Auto-correlation	Contemporaneous Correlation with Output
Y	1.81	1.00	0.84	1.00
C	1.35	0.74	0.80	0.88
I	5.30	2.93	0.87	0.80
N	1.79	0.99	0.88	0.88
Y/N	1.02	0.56	0.74	0.55
w	0.68	0.38	0.66	0.12
r	0.30	0.16	0.60	-0.35
A	0.98	0.54	0.74	0.78

Note: All variables are in logarithms (with the exception of the real interest rate) and have been detrended with the HP filter. Data sources are described in Stock and Watson [1998], who created the real rate using VAR inflation expectations. Our notation in this table corresponds to that in the text, so that Y is per capita output, C is per capita consumption, I is per capita investment, N is per capita hours, w is the real wage (compensation per hour), r is the real interest rate, and A is total factor productivity.

In presenting these business cycle facts, we are focusing on a small number of empirical features that have been extensively discussed in recent work on real business cycles. For example, in the interest of brevity, we have not discussed the lead-lag relations between our variables. In choosing the series to study, we have also left out nominal variables, whose cyclical behavior is at the heart of many controversies over the nature of business cycles.[13] However, we do report the

---

[13]See Stock and Watson [1998, sections 3(d), 3(f), and 4.1] for a discussion of literature and empirical results.

Problems (some results are not consistent with the observed properties of macro-economic time series, especially concerning the labor market):

- "*employment variability puzzle*":

  data: employment is as much volatile as output and is strongly procyclical, whereas real wage is less volatile and only weakly procyclical

  RBC: the observed pattern is obtained only by assuming a very large wage elasticity of labor supply (which is not supported by empirical microeconomic evidence)

  but: by introducing *indivisibility* (non convexity) in labor supply decisions (i.e. workers can choose *whether* to work or not to work, but not *how many hours* to work per week), it is possible to reconcile a high volatile employment with a low microeconomic labor supply elasticity

- "*productivity puzzle*":

  data: labor productivity and employment are not highly correlated

  RBC: large correlation between productivity and employment (due to the technological nature of the shocks)

  but: (1) the productivity-employment correlation can be reduced by the introduction of shocks to labor supply (e.g. due to monetary disturbances in the presence of nominal rigidities)
  (2) in the presence of *labor hoarding* behavior by firms, the correlation between *effective* labor input and productivity could be higher than that measured using data on hours worked; in this case the measure of productivity shocks based on the Solow residual (SR):

  $$\log SR_t = \log Y_t - (1 - \alpha) \log K_t - \alpha \log N_t$$

  would overestimate the actual changes in total factor productivity.