Answers to problems

1. (*Expectations and price dynamics*)

   (a) Equating demand and supply at t we get:

$$a_0 - a_1 p_t = b_0 + b_1 p^e_{t-1,t} + u_t$$
$$\Rightarrow p_t = \frac{a_0 - b_0}{a_1} - \frac{b_1}{a_1} p^e_{t-1,t} - \frac{1}{a_1} u_t$$

   Equilibrium price at t depends negatively on the expectation formulated at $t-1$: a high value of $p^e_{t-1,t}$ increases the supply of the good in period t, thereby negatively affecting price.

   (b) If $p^e_{t-1,t} = E_{t-1}p_t$, the equilibrium price becomes:

$$p_t = \frac{a_0 - b_0}{a_1} - \frac{b_1}{a_1} E_{t-1}p_t - \frac{1}{a_1} u_t$$

   Taking the expected value as of $t-1$ and solving for $E_{t-1}p_t$:

$$E_{t-1}p_t = \frac{a_0 - b_0}{a_1 + b_1}$$

   Substituting $E_{t-1}p_t$ in the above equation we get the equilibrium price with rational expectations:

$$p_t = \frac{a_0 - b_0}{a_1 + b_1} - \frac{1}{a_1} u_t$$

   The equilibrium price fluctuates randomly (in an unpredictable way) around a constant value given by $\frac{a_0 - b_0}{a_1 + b_1}$, and its level in t does not depend on its past behavior ($p_{t-1}, p_{t-2}, \ldots$ do not enter the equation for p_t). Moreover, any realization of the supply shock u_t affects the price only in the current period t, and does not trigger any further price adjustment in subsequent periods.

   (c) With adaptive expectations, applying the procedure used in Section 1, $p^e_{t-1,t}$ can be expressed in terms of past values of the price level:

$$p^e_{t-1,t} = \lambda \sum_{i=0}^{\infty} (1-\lambda)^i \, p_{t-1-i}$$

yielding the following equilibrium price level:

$$p_t = \frac{a_0 - b_0}{a_1} - \frac{b_1}{a_1} \lambda \sum_{i=0}^{\infty} (1-\lambda)^i \, p_{t-1-i} - \frac{1}{a_1} u_t$$

Now, differently from the case of rational expectations, the price depends on its past history and any realization of u_t starts a process of gradual adjustment of the price level (and its expected value) to the long-run stationary equilibrium value, given by $\frac{a_0 - b_0}{a_1 + b_1}$, the same as under rational expectations (check this by setting $p_t = p_{t-i} = \bar{p}$ for all i and $u_t = 0$ in the above equation, solving for $\bar{p}$, the long-run stationary equilibrium price that prevails when the adjustment has been completed).

2. (*Cagan's hyperinflation model*) With rational expectations and adding a shock to money demand, the equilibrium condition becomes:

$$m_t - p_t = -\alpha(E_t p_{t+1} - p_t) + u_t$$

   (a) Using the procedure described in Section 1, we get the equilibrium price level:

$$p_t = \frac{1}{1+\alpha} \sum_{i=0}^{\infty} \left(\frac{\alpha}{1+\alpha} \right)^i E_t \, m_{t+i} - \frac{1}{1+\alpha} u_t$$

   From the stochastic process generating money supply we get the expected values of m in all future periods:

$$\begin{aligned} E_t m_{t+1} &= E_t \left(\rho \, m_t + \varepsilon_{t+1} \right) = \rho \, m_t \\ E_t m_{t+2} &= E_t \left(\rho \, m_{t+1} + \varepsilon_{t+2} \right) = E_t \left(\rho^2 \, m_t + \rho \, \varepsilon_{t+1} + \varepsilon_{t+2} \right) = \rho^2 \, m_t \\ &\ldots \qquad\qquad \ldots \\ E_t m_{t+i} &= \rho^i \, m_t \end{aligned}$$

   and, substituting into the equation for p_t:

$$\begin{aligned} p_t &= \frac{1}{1+\alpha} \sum_{i=0}^{\infty} \left(\frac{\alpha}{1+\alpha} \right)^i \rho^i \, m_t - \frac{1}{1+\alpha} u_t \\ \Rightarrow p_t &= \frac{1}{1+\alpha(1-\rho)} m_t - \frac{1}{1+\alpha} u_t \end{aligned}$$

(b) The parameter ρ does affect the reaction of p_t to m_t. ρ measures the degree of persistence over time of the money supply shock ε. In the extreme case $\rho \to 1$, the process generating m_t tends to a *random walk* and any disturbance ε_t changes money supply *permanently*; the new permanent level of m is immediately incorporated in all expected values $E_t m_{t+i}$ determining a relatively large change of p_t (the coefficient of m_t tends to 1). In the opposite extreme case in which $\rho \to 0$ money supply is a *white noise* process and the shock ε does not cause any persistent change of m; in this case the effect on p_t is relatively small and the coefficient of m_t tends to $\frac{1}{1+\alpha} < 1$.

(c) The forecast error in t may be obtained by computing the expected value

$$E_{t-1}p_t = \frac{1}{1+\alpha(1-\rho)} E_{t-1}m_t$$

and subtracting it from p_t, getting:

$$p_t - E_{t-1}p_t = \frac{1}{1+\alpha(1-\rho)} \underbrace{(m_t - E_{t-1}m_t)}_{\varepsilon_t} - \frac{1}{1+\alpha} u_t$$

from which it can be checked that $E_{t-1}(p_t - E_{t-1}p_t) = 0$. In terms of the "revisions in expectations" on future values of m, as in (1.20) of Section 1, we have: $E_t m_{t+i} - E_{t-1}m_{t+i} = \rho^i \varepsilon_t$. Substituting this term into (1.20) we get the same expression for the forecast error obtained above.

3. (*Lucas critique)*

(a) Equating aggregate demand and supply we get the following expression for the price level in t:

$$p_t = \frac{1}{1+\gamma} m_t + \frac{\gamma}{1+\gamma} E_{t-1}p_t - \frac{1}{1+\alpha} u_t$$

and its expected value at time $t-1$:

$$E_{t-1}p_t = \frac{1}{1+\gamma} E_{t-1}m_t + \frac{\gamma}{1+\gamma} E_{t-1}p_t \Rightarrow E_{t-1}p_t = E_{t-1}m_t$$

Substituting $E_{t-1}m_t$ for $E_{t-1}p_t$ into the price equation above we get the price "surprise" $p_t - E_{t-1}p_t$, which affects output:

$$p_t - E_{t-1}p_t = \frac{1}{1+\gamma}(m_t - E_{t-1}m_t) - \frac{1}{1+\gamma}u_t$$

Therefore only the unanticipated component of money supply ($m_t - E_{t-1}m_t = \varepsilon_t$, from the assumed monetary rule) causes unexpected variations in the price level, that in turn affect output.

(b) Using the fact that, from the monetary rule, $E_{t-1}m_t = \bar{m} + p_{t-1}$, the price surprise may me expressed as:

$$p_t - E_{t-1}p_t = p_t - (\bar{m} + p_{t-1}) = \pi_t - \bar{m}$$

Finally, from the aggregate supply function, the following relation between output and the inflation rate is derived:

$$y_t = \gamma\,\pi_t - \gamma\,\bar{m} + u_t$$

The constant term in this equation depends on a parameter of the monetary rule ($\bar{m}$): any change in this perfectly anticipated component of the monetary rule affects the relation between output and inflation, graphically shifting its "position" in the plane (π, y) in accordance with the Lucas critique.

4. (*Intertemporal substitution and the Lucas critique*)

(a) Equating supply and demand we get

$$p_t = \frac{1}{1+\gamma}\,m_t + \frac{\gamma}{1+\gamma}\,E_t p_{t+1}$$

To solve for the price level we use the method of undetermined coefficients starting from the following "guess" solution

$$p_t = \pi_1\,m_t$$

from which we derive (using the monetary rule):

$$E_t p_{t+1} = \pi_1\,E_t m_{t+1} = \pi_1\,\rho\,m_t$$

Substituting into the above expression for p_t:

$$\pi_1\, m_t = \frac{1}{1+\gamma}\, m_t + \frac{\gamma}{1+\gamma}\pi_1\, \rho\, m_t$$

and equating coefficients on m_t:

$$\pi_1 = \frac{1}{1+\gamma(1-\rho)}$$

so that

$$p_t = \frac{1}{1+\gamma(1-\rho)} m_t$$

and

$$E_t p_{t+1} = \frac{1}{1+\gamma(1-\rho)}\rho\, m_t$$

yielding the following price surprise

$$p_t - E_t p_{t+1} = \frac{1}{1+\gamma(1-\rho)}(1-\rho)\, m_t$$

and level of output

$$y_t = \frac{\gamma}{1+\gamma(1-\rho)}(1-\rho)\, m_t$$

The response of output to the monetary shock ε_t (that is included in m_t) depends on the parameter of the monetary rule ρ, capturing the persistence of the shocks in the process generating money supply. Consider two extreme cases and suppose that a positive realization of ε_t occurs: (i) if $\rho = 0$ the monetary shock has a non-persistent nature (white noise), and only p_t but not $E_t p_{t+1}$ is affected; therefore, the producer has an incentive to exploit the current (relatively high) price and increases output in t. (ii) if $\rho = 1$ the shock has a permanent nature (the process for m is a random walk); the shock affects p_t and $E_t p_{t+1}$ in the same way (both have a unitary reaction) and no incentive to substitute production intertemporally arises; consequently the producer will not alter output and the monetary shock has no real effects.

(b) Using the equation for the equilibrium price level above, the output equation can be expressed as a relation between output and the price level (a "traditional" aggregate supply equation)

$$y_t = \gamma (1-\rho) p_t$$

The "slope" of this curve depends on the policy parameters ρ. In the classic representation (with p on the vertical and y on the horizontal axis) the slope of the curve tends to ∞ (a vertical line) when $\rho \to 1$ (the permanent nature of the shock does not induce any output change), and tends to the positive value $\frac{1}{\gamma}$ when $\rho \to 0$ (the purely temporary nature of the shock induces producers to change output in the current period and a monetary shock, shifting the aggregate demand function along a positively sloped aggregate supply, will have real effects). The dependence of the real effects of monetary shocks on a parameter of the monetary rule (perfectly known by agents) implies that if such parameter changes, also the slope of the aggregate supply function (capturing producers' behavior) changes, in accordance with the "Lucas critique".

5. (*New Classical Macroeconomics*)

(a) Equating aggregate demand and supply we get the following expression for the price level in t:

$$p_t = \frac{\alpha}{\alpha+\gamma} m_t + \frac{\gamma}{\alpha+\gamma} E_{t-1}p_t + \frac{1}{\alpha+\gamma}(v_t - u_t)$$

and its expected value at time $t-1$:

$$\begin{aligned} E_{t-1}p_t &= \frac{\alpha}{\alpha+\gamma} E_{t-1}m_t + \frac{\gamma}{\alpha+\gamma} E_{t-1}p_t \\ &\Rightarrow E_{t-1}p_t = E_{t-1}m_t \end{aligned}$$

Subtracting $E_{t-1}p_t$ from p_t we get the price surprise:

$$\begin{aligned} p_t - E_{t-1}p_t &= \frac{\alpha}{\alpha+\gamma} \underbrace{(m_t - E_{t-1}m_t)}_{\delta_1 u_t - \delta_2 v_t} + \frac{1}{\alpha+\gamma}(v_t - u_t) \\ &= \frac{1-\alpha\delta_2}{\alpha+\gamma} v_t - \frac{1-\alpha\delta_1}{\alpha+\gamma} u_t \end{aligned}$$

and the equilibrium output level:

$$y_t = \frac{\gamma}{\alpha+\gamma}\,[(1-\alpha\delta_2)\,v_t - (1-\alpha\delta_1)\,u_t] + \,u_t$$

Deviations of output from its natural value are determined only by the shocks v e u, unpredictable by producers in the private sector. In this model, the impact on output of the demand and supply disturbances depends on the parameters of the monetary rule (δ_1 and δ_2). In fact, monetary authorities exploit a larger information set than private agents, since they set money supply at t after observing the realizations of v_t and u_t. (Note: the same result may be obtained by means of the method of undetermined coefficients described in Section 3, starting from a guess solution for the price level of the form: $p_t = \pi_1\, m_{t-1} + \pi_2\, u_t + \pi_3\, v_t$).

(b) If the monetary authorities aim at stabilizing output around its perfect information level $y_t = u_t$, the optimal values δ_1^* and δ_2^* for the policy parameters are found by solving the following problem:

$$\begin{aligned} \min_{\delta_1,\delta_2}\; var\,(p_t - E_{t-1}p_t) \quad &= \quad \left(\frac{1-\alpha\delta_2}{\alpha+\gamma}\right)^2 \sigma_v^2 + \left(\frac{1-\alpha\delta_1}{\alpha+\gamma}\right)^2 \sigma_u^2 \\ &\Rightarrow \qquad \delta_1^* = \frac{1}{\alpha} \quad , \quad \delta_2^* = \frac{1}{\alpha} \end{aligned}$$

The optimal monetary policy rule, which perfectly stabilizes output around the target value is:

$$m_t = m_{t-1} + \frac{1}{\alpha}\,u_t - \frac{1}{\alpha}\,v_t$$

Stabilization monetary policy is effective here since it exploits superior information with respect to the economy's private sector.

6. *(New Classical Macroeconomics)*

(a) Equating aggregate demand and supply and using the monetary rule to substitute for m_t we get thw following expression for the price level:

$$p_t = \frac{1}{\alpha+\beta+\gamma}\;(\alpha\delta_1 u_{t-1} - \alpha\delta_2 v_{t-1} + \beta\, E_t p_{t+1} + \gamma\, E_{t-1}p_t + v_t - u_t) \qquad (*)$$

Using the method of undetermined coefficients, guess a solution for the price level of the form:

$$p_t = \pi_1\, u_t + \pi_2\, v_t + \pi_3\, u_{t-1} + \pi_4\, v_{t-1}$$

with the expected values:

$$\begin{aligned} E_{t-1}p_t &= \pi_3\, u_{t-1} + \pi_4\, v_{t-1} \\ E_t p_{t+1} &= \pi_3\, u_t + \pi_4\, v_t \end{aligned}$$

which, by substitution into (*), lead to the following equation:

$$\begin{aligned} p_t = \frac{1}{\alpha+\beta+\gamma} \quad & (\alpha\delta_1 u_{t-1} - \alpha\delta_2 v_{t-1} + \beta\,\pi_3\, u_t + \beta\,\pi_4\, v_t + \\ & +\gamma\,\pi_3\, u_{t-1} + \gamma\,\pi_4\, v_{t-1} + v_t - u_t) \qquad (**) \end{aligned}$$

Equating coefficients on the same variables in (*) and (**) we get the following system of equations:

$$\begin{aligned} u_t \;:\quad \pi_1 &= \frac{\beta}{\alpha+\beta+\gamma}\pi_3 - \frac{1}{\alpha+\beta+\gamma} \\ v_t \;:\quad \pi_2 &= \frac{\beta}{\alpha+\beta+\gamma}\pi_4 + \frac{1}{\alpha+\beta+\gamma} \\ u_{t-1} \;:\quad \pi_3 &= \frac{\alpha\delta_1}{\alpha+\beta+\gamma} + \frac{\gamma}{\alpha+\beta+\gamma}\pi_3 \\ &\Rightarrow \quad \pi_3 = \frac{\alpha\delta_1}{\alpha+\beta} \\ v_{t-1} \;:\quad \pi_4 &= -\frac{\alpha\delta_2}{\alpha+\beta+\gamma} + \frac{\gamma}{\alpha+\beta+\gamma}\pi_4 \\ &\Rightarrow \quad \pi_4 = -\frac{\alpha\delta_2}{\alpha+\beta} \end{aligned}$$

and the values also of π_1 and π_2:

$$\begin{aligned} \pi_1 &= \frac{1}{\alpha+\beta+\gamma}\left(\frac{\alpha\beta\delta_1}{\alpha+\beta} - 1\right) \\ \pi_2 &= \frac{1}{\alpha+\beta+\gamma}\left(1 - \frac{\alpha\beta\delta_2}{\alpha+\beta}\right) \end{aligned}$$

Using the above coefficients the output level is:

$$y_t = \frac{\gamma}{\alpha+\beta+\gamma}\left[\left(\frac{\alpha\beta\delta_1}{\alpha+\beta}-1\right)u_t + \left(1-\frac{\alpha\beta\delta_2}{\alpha+\beta}\right)v_t\right] + u_t$$

Deviations of y from its natural value are determined only by the unpredictable demand and supply shocks. However, the parameters of the *feedback* monetary rule δ_1 and δ_2 enter the output equation, affecting the response of y to the shocks (as in Problem 5, in which the information asymmetry was in favor of private agents and not of the policymaker).

(b) If monetary authorities aim at stabilizing output around its perfect information value $y_t = u_t$, the optimal values δ_1^* and δ_2^* for the policy parameters are found by solving the following problem:

$$\min_{\delta_1,\delta_2} var\,(p_t - E_{t-1}p_t) = \left(\frac{\gamma}{\alpha+\beta+\gamma}\right)^2 \left[\left(\frac{\alpha\beta\delta_1}{\alpha+\beta}-1\right)^2 \sigma_u^2 + \right.$$
$$\left. + \left(1-\frac{\alpha\beta\delta_2}{\alpha+\beta}\right)^2 \sigma_v^2\right]$$
$$\Rightarrow \quad \delta_1^* = \frac{\alpha+\beta}{\alpha\beta} \quad , \quad \delta_2^* = \frac{\alpha+\beta}{\alpha\beta}$$

The optimal monetary policy rule, which perfectly stabilizes output around the target value is:

$$m_t = \frac{\alpha+\beta}{\alpha\beta}u_{t-1} - \frac{\alpha+\beta}{\alpha\beta}v_{t-1}$$

Even in this case, notwithstanding their informational *disadvantage*, monetary authorities are able to achieve effective output stabilization.

(c) In this setting, a purely feedback monetary policy is effective *because* private agents' inflation expectations incorporate information available at time t (whereas monetary authorities act on the basis of information dated $t-1$ only). For instance, when an unexpected increase in aggregate demand occurs ($v_t > 0$), both the price level and output tend to increase. Private agents observe the positive realization of v_t, and, knowing that monetary policy follows a feedback rule with (optimal) coefficients δ_1^* and δ_2^*, they come to expect a restrictive monetary policy reaction in $t+1$ (since m_{t+1} will depend negatively on v_t through the coefficient $-\delta_2^*$), with a negative pressure on prices in $t+1$. This expectation, formed using more information than the policymaker, reduces expected inflation $E_t p_{t+1} - p_t$, whereby moderating aggregate demand in t. If the policymaker optimally chooses the reaction of m to past realizations of the shocks, the effect on expected inflation will exactly offset the effect of current disturbances on aggregate demand, leading to perfect stabilization, even if monetary authorities have an informational disadvantage.Similar reasoning applies to the case of a supply shock u_t.