HANDOUT #2 The Matching Model of Unemployment

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September 23, 2019

Contents

1 THE BASELINE MATCHING MODEL OF UNEMPLOYMENT

The Matching Function The number of matches per unit of time is mL, (most of the time $L = 1$ is the size of the labor force)

$$mL = m(uL, vL)$$

with $m_u > 0$ and $m_v > 0$ $m_{uu} < 0$ and $m_{vv} < 0$ u is a measure of job seekers' effort (read unemployment rate) while v is a measure of firm's recruiting effort (read vacancy rate, the number of vacancies normalized by the labor force). There are CRS to matching

$$\Delta m = m(\Delta u, \Delta v)$$

so that the size of the market is irrelevant (a nice feature). CRS is also "empirically plausible".

Transition Rates

- Let $q()$ be the rates at which vacancies are filled

$$q() = \frac{m(u,v)}{v} = m(\frac{u}{v}, 1)$$

Let $\theta = \frac{v}{u}$ so that

$$q(\theta) = m(\frac{u}{v}; 1) \qquad q'(\theta) < 0$$

If Cobb-Douglas $m(u,v) = v^{1-\alpha}u^{\alpha}$ and $q(\theta) = \frac{v^{1-\alpha}u^{\alpha}}{v} = (\frac{v}{u})^{-\alpha} = \theta^{-\alpha}$

- The rates at which workers find jobs is

$$\alpha(\theta) = \frac{m(u,v)}{u} = \frac{v}{u}\frac{m(u,v)}{v} = \theta q(\theta) \qquad\qquad \alpha'(\theta) > 0$$

  Since $\alpha'(\theta) = q(\theta) + q'(\theta)\theta = q(\theta)[1 + \frac{q'(\theta)\theta}{q(\theta)}] = q(\theta)[1 - \eta(\theta)] > 0$ where $\eta(\theta) = \epsilon_{q,\theta} \left|\frac{q'(\theta)\theta}{q(\theta)}\right| < 1$ with CRS in matching the elasticity of the matching function with respect to θ is less than one in absolute value. With a Cobb Douglas matching function the elasticity is also independent of θ and is equal to α. $\alpha(\theta) = \theta^{(1-\alpha)}$ and $\eta(\theta) = \alpha$

- There are trading externality (transition rates depends on the relative numbers of agents
- $q(\theta)dt$ is the arrival rate of offer, a Poisson Process with arrival rate $q(\theta)$
- There is stochastic rationing (can not be eliminated by price adjustment): $(1 - q(\theta)dt) > 0$ and $(1 - q(\theta)dt) > 0$.

Unemployment Flows
Labor force is constant and normalized to 1

$$u + n = 1$$

Steady state unemployment: inflows=outflows.

In the simple model unemployment inflows are equal to total job destruction. Jobs are destroy at an exogenous rate equal to λ, where λ is the arrival rate (Poisson) of idiosyncratic shocks. Unemployment outflows are $\theta q(\theta)u$, the aggregate number of matches formed in a given instant. There is no on the job search. Unemployment is constant if

$$uq(\theta)u = \lambda(1 - u)$$

which implies

$$u = \frac{\lambda}{\lambda + \theta q(\theta)}, \tag{UV-KEY1}$$

where θ is one of the key endogenous variables and λ is an exogenous parameter. This is a negative relationship between u and θ. Alternatively, in the $u - v$ space the unemployment relationship is $\lambda(1 - u) = x(u, v)$. This is called **Beveridge curve**. Totally differentiating yields

$$-\lambda du = x_u du + x_v dv$$

which implies

$$\frac{dv}{du} = -\frac{\lambda + x_u}{x_v} < 0$$

1.0.1 Firm Behavior

Firms and workers are risk neutral, and discount the future by $r > 0$. At the firm level, the arrival rate of offer follow a Poisson process with rate $q(\theta)$. Firms are one firm-one job. The value of a vacancy reads

$$rV = -pc + q(\theta)[J - V]$$

where pc are search costs Free entry in the job market implies

$$V = 0$$

so that

$$J = \frac{pc}{q(\theta)} \tag{1}$$

which implies that the value of a job is equal to the search costs. The value of a filled job is

$$rJ = p - w + \lambda[V - J]$$

which implies

$$J = \frac{p - w}{r + \lambda}$$

where $p - w$ are operational profits and $r + \lambda$ is the firm's effective discount rate. Substituting this into equation 1 yields

$$w = p - \frac{(\lambda + r)pc}{q(\theta)} \qquad \text{(LABOR DEMAND)}$$

which is a sort of labor demand and downward sloping in a $w - \theta$ space

Worker Behavior

Firms and workers are risk neutral, and discount the future by $r > 0$. At the firm level, the arrival rate of offer follow a Poisson process with rate $q(\theta)$. An unemployed worker enjoys an income z and its value of search reads

$$rU = b + \theta q(\theta)[W - U] \tag{2}$$

where W is the P.D.V. value of having a job while U is the P.D.V. value of unemployment. The value of a job is

$$rW = w + \lambda[W - U] \tag{3}$$

Wage Determination-NASH BARGAINING There is a pure economic rent. Matched firms and workers have local monopoly power. In the baseline model wages are obtained through the outcome of a generalized Nash-bargaining between the **firm and the worker after they have met**. The axiomatic approach to Nash Bargaining (see Obsborne and Rubinstein) suggests that wages are obtained as the solution to

$$w = \arg\max (W - U)^{\beta}(J - V)^{(1-\beta)}$$

where $0 \le \beta \le 1$ is the worker's bargaining strength. To obtain the wage first take the log of the Nash-maximand $\Omega = (W - U)^{\beta}(J - V)^{(1-\beta)}$ and take the derivative with respect to w.. So that $\frac{\partial}{\partial w} \ln \Omega = 0$ or

$$\frac{\beta}{W - U}\frac{\partial W}{\partial w} + \frac{1 - \beta}{J - V}\frac{\partial J}{\partial w} = 0$$

Since $(\lambda + r)\frac{\partial W}{\partial w} = 1$ and $(\lambda + r)\frac{\partial J}{\partial w} = -1$, we have

$$(W - U)(1 - \beta) = \beta J \tag{4}$$

or

$$W - U = \beta(J - V + W - U)$$

where $S = J - V + W - U$ is the total surplus.

To get the wage proceed as follows. From (2) and (3)

$$\begin{aligned} (\lambda + r)W &= w + \lambda U \\ rU &= b + \theta q(\theta)[W - U] \end{aligned}$$

subtracting the previous two equations one obtains

$$(\lambda + r)[W - U] = w - b - \theta q(\theta)[W - U]$$

but by virtue of (4) $W - U = \frac{\beta}{1-\beta}J = \frac{\beta}{1-\beta}\frac{cp}{q(\theta)}$. Substituting above yields

$$(\lambda + r)[W - U] = w - b - \frac{\beta cp}{1 - \beta}\theta$$

Since

$$(\lambda + r)J = p - w$$

we can write equation 4 so that

$$(1-\beta)[w - b - \frac{\beta cp}{1-\beta}\theta] = \beta[p - w]$$

and simplifying

$$w - (1-\beta)b - \beta cp\theta = \beta p$$

yields the final expression for the wage

$$w = b(1-\beta) + \beta p(1 + c\theta) \qquad \text{(Wage Equation)}$$

where $pc\theta$ is the average hiring cost per unemployed

1.0.2 EQUILIBRIUM

Definition 1 ***The Search equilibrium with exogenous job destruction is a triple*** (u, θ, w) *satisfying*

- ***Definition 2*** – *Free entry* ($V = 0$)
  - *Wage Bargaining* $(1-\beta)[W - U] = \beta J$
  - *Balance Flow* ($m(u, v) = \lambda(1-u)$)

The reduced form is obtained by the three equations

$$\begin{aligned} w &= p - \frac{(\lambda + r)pc}{q(\theta)} \\ w &= b(1-\beta) + \beta p(1 + c\theta) \\ u &= \frac{\lambda}{\lambda + \theta q(\theta)} \end{aligned}$$

The system is recursive. The wage equation and the free entry gives w and θ. The Beveridge curve gives u given θ

2 JOB DESTRUCTION WITH $\beta = 0$

Assume that firms have all the bargaining power. The value of the labor product is now px where x is drawn from a continuous distribution function $F(x)$ with support Ω, and where x^u is the upper support of Ω, and λ is the arrival rate of shocks. Two assumptions are important: i) there is complete ex-post irreversibility, and ii) free disposability is always an option. The P.D.V. of a job is now index by x and is $J(x)$. The destruction rule is

$$\text{Destroy } \forall x : J(x) < 0$$

If we are able to prove that $J(x)$ is monotonic and continuous, the destruction follows the reservation property and there exist a reservation productivity R such that the firm destroys is $x > R$, where R solves

$$J(R) = 0.$$

If $\beta = 0$ the P.D.V. of the firm $J(x)$ reads

$$rJ(x) = px - b + \lambda[\int_{z \ni \Omega} MAX[J(z), 0]dF(z) - J(x)]. \tag{5}$$

where the integral can be interpreted as the expected value of a new job conditional on having a positive value. It is immediate to show that

$$J'(x) = \frac{p}{r+\lambda} > 0 \tag{6}$$

or that the derivative is constant and independent of x, so that J is indeed monotonic and satisfies the *reservation rule*. Basically, we are now confident that

$$\exists R : J(x) > 0 \qquad \forall x > R$$

This allows us to solve the maximization problem in the equation (5) so that the maximization inside the integral can be written as

$$\int_{z \ni \Omega} MAX[J(z), 0] dF(z) = \int_{x_l}^{R} 0 dF(z) + \int_{R}^{x^u} J(z) dF(z)$$

and $J(x)$ becomes

$$(r+\lambda)J(x) = px - b + \lambda \int_{R}^{x^u} J(z) dF(z).$$

The expected value of the job can be further simplified with an integration by parts[1]

$$\int_{R}^{x^u} J(z) dF(z) = |J(z)F(z)|_{R}^{x^u} - \int_{R}^{x^u} J'(z)F(z) dz.$$

which by virtue of equation (6) becomes

$$\int_{R}^{x^u} J(z) dF(z) = J(x^u) - \frac{p}{r+\lambda} \int_{R}^{x^u} F(z) dz.$$

and substitute into $J(x)$ becomes

$$(r+\lambda)J(x) = px - b + \lambda\{J(x^u) - \frac{p}{r+\lambda} \int_{R}^{x^u} F(z) dz\}.$$

while the reservation productivity solves

$$pR - b + \lambda\{J(x^u) - \frac{p}{r+\lambda} \int_{R}^{x^u} F(z) dz\} = 0 \tag{7}$$

To obtain an expression for $J(x^u)$, we can write

$$(r+\lambda)J(x^u) = px^u - pR$$

so that

$$J(x^u) = p\frac{x^u - R}{\lambda + r} = \frac{p}{r+\lambda} \int_{R}^{x^u} dz$$

Substituting this expression into equation 7 we obtain the final expression for the reservation productivity

$$pR - b = -\frac{\lambda p}{r+\lambda} \int_{R}^{x^u} (1 - F(z)) dz \tag{8}$$

This expression uniquely solves for R.

[1] Recall $\int u dv = uv - \int v du$. Here $v = F(z)$ and $u = J(z)$

To insert this into the model, assume that firms create jobs at the upper support of the distribution x^u, since the technology is ex-ante perfectly flexible. The value of vacancy reads

$$rV = -pc + q(\theta)[J(x^u) - V]$$

with free entry this becomes

$$J(x^u) = \frac{pc}{q(\theta)}$$

Using the fact that $J(x^u) = p\frac{x^u - R}{\lambda + r}$ we obtain

$$\frac{x^u - R}{\lambda + r} = \frac{c}{q(\theta)} \tag{KEY2: JC}$$

which uniquely solves for θ

To complete the model we need an expression for unemployment. Jobs are destroyed when hit by a reservation productivity that falls below the reservation rule R, while the are created when an unemployed worker is matched with a vacancy. This implies that

$$\lambda G(R)(1-u) = \theta q(\theta) u$$

so that

$$u = \frac{\lambda F(R)}{\lambda F(R) + \theta q(\theta)}$$

2.0.1 EQUILIBRIUM

Definition 3 • ***Definition 4 The Search equilibrium with endogenous job destruction is a triple*** (u, R, θ) *satisfying*

- *Free entry* $(V = 0)$
- *Optimal Job Destruction* $J(R) = 0$
- *Balance Flow* $(\lambda G(R)(1-u) = \theta q(\theta)$

The reduced form is obtained by the three equations

$$\begin{aligned} pR - b &= -\frac{\lambda p}{r+\lambda}\int_R^{x^u}(1 - F(z))dz \\ \frac{x^u - R}{\lambda + r} &= \frac{c}{q(\theta)} \\ u &= \frac{\lambda F(R)}{\lambda F(R) + \theta q(\theta)} \end{aligned}$$

The model is still fully recursive. The reservation productivity yields a unique R, free-entry determines θ given R while the Beveridge curve gives u given R and θ.

3 EFFICIENCY IN THE BASELINE MODE (Exogenous Destruction)

In the search equilibrium there are trading externalities, since agents utility depends on the relative number of traders in the market. An additional participant on the same side decreases an individual effort (*congestion effect*) while an additional participant of the other side increase welfare (*strategic complements*) The natural question to ask in whether the unemployment resulting from the decentralized equilibrium is constrained-efficient, in the sense that it maximizes net output of the aggregate economy. The measure of social output is the present discounted value of

$$\Omega = \int_0^\infty e^{-rt}[p(1-u) + zu - pc\theta u]dt$$

where output is the sum of three components: total production, total unemployment income minus average search cost. The social planner maximizes Ω subject to the matching frictions imposed by the matching technology. Basically, the problem is

$$\begin{aligned} & Max_{u,\theta} \int_0^\infty e^{-rt}[p(1-u) + zu - pc\theta u]dt \\ \text{s.t. } u\theta q(\theta) &= \lambda(1-u) \end{aligned}$$

To solve this problem from the Hamiltonian as

$$H = e^{-rt}[p(1-u) + zu - pc\theta u] + \phi[-u\theta q(\theta) + \lambda(1-u)]$$

whose first order conditions are

$$\begin{aligned} \frac{\partial H}{\partial \theta} &= 0. \\ \frac{\partial H}{\partial u} &= -\dot{\phi} \end{aligned}$$

(note that θ is a control variable and u is a state variable).The first order conditions in our model are respectively

$$-e^{-rt}pcu - \phi(1-\eta(\theta))q(\theta)u = 0$$
$$-e^{-rt}[p - z + pc\theta] - \phi(\lambda + \theta q(\theta)) = -\dot{\phi}$$

where $\eta(\theta)$ is the elasticity of the matching function with respect to θ. From the first equation we obtain

$$\phi = -\frac{pce^{-rt}}{q(\theta)[1-\eta(\theta)]}; \quad \dot{\phi} = \frac{pce^{-rt}}{q(\theta)[1-\eta(\theta)]}$$

which can be substituted into the first equation to obtain

$$-[p - z + pc\theta]e^{-rt} + \frac{pce^{-rt}}{q(\theta)[1-\eta(\theta)]}(\lambda + \theta q(\theta)) = -\frac{rpce^{-rt}}{q(\theta)[1-\eta(\theta)]}$$

which simplifies to

$$-q(\theta)(1-\eta(\theta)(p - z + c\theta) + c(\lambda + \theta q(\theta)) = -rc \tag{9}$$

and

$$(p - z)[1-\eta(\theta)] - pc\theta\eta(\theta) = \frac{(\lambda + r)}{q(\theta)}pc \tag{10}$$

How does equation 9 compare with the social optimum? To see this recall the firm's problem in the decentralized search equilibrium

$$\frac{(r+\lambda)pc}{q(\theta)} = p - w$$

where the wage rate is obtained via decentralized bargaining or as the solution to

$$w = (1-\beta)z + \beta[p + pc\theta]$$

so that the supply of jobs in a decentralized equilibrium is

$$(p-z)(1-\beta)-pc\theta\beta=\frac{(r+\lambda)pc}{q(\theta)} \tag{11}$$

Comparing the social outcome with decentralized outcome, it is clear that the decentralized bargaining maximize social net output if

$$\beta=\eta(\theta)$$

or if the sharing parameter is identical to the elasticity of the matching function.

4 POLICY INSTRUMENTS IN THE BASELINE MODE (Exogenous Destruction)

Wage Taxes

- Let w be the wage paid when there are no taxes, so that labor costs are equal to the net wage.
- τ is a wage subsidy so that the wage would become $w+\tau$
- there is a marginal income tax equal to t so that the net wage received by the worker is $(1-t)(w+\tau)$
- the total tax paid is then $T(w)=w-(1-t)(w+\tau)=tw-\tau(1-t)$

Wage taxes are linear and a smooth function of income. If the gross wage is w the net wage received by a worker is $(1-t)(w+\tau)$.One can think of a tax subsidy τ received by the worker and subsequently being taxed at t. The tax paid by the worker is tw minus the net subsidy received $\tau(1-t)$. Total tax is

$$T(w)=tw-\tau(1-t)$$

While t is the marginal tax rate, the tax subsidy makes τ makes the tax system progressive ($\tau>0$), purely proportional ($\tau=0$) or regressive ($\tau<0$). To see this not that

$$\frac{T(w)}{w}=t-\frac{\tau(1-t)}{w}$$

Employment Subsidy

employment is subsidized at rate a per job. The labor product become $p+a$

Hiring Subsidy

Firms receive a hiring subsidy once and for all when the worker is hired The value of the subsidy is pH

Firing Taxes

Firms pay a firing tax when a separation takes place. The value of the firing tax is pT Note that the firing tax is not a transfer to the worker (a pure severance payments has no effect in equilibrium)

Unemployment Compensation

The policy parameter is the after tax replacement rate, or the ratio of net unemployment benefit to the average net income from work. b is defined as

$$\begin{aligned} b &= \rho[w-T(w)] \\ b &= \rho[(1-t)w+\tau(1-t)] \end{aligned}$$

Obviously b is not taxed since it is already defined in net terms.

VALUE FUNCTIONS WITH POLICY

The value of unemployment is given by

$$rU=z+\rho[(1-t)w+\tau t]+\theta q(\theta)[W-U]$$

The employed worker net worth in a job that pays w_j is

$$rW_j = (1-t)w_j + \tau(1-t) + \lambda[U - W_j]$$

The value of a vacancy is

$$rV = -pc + q(\theta)[J_j + pH - V]$$

where the firms, conditional upon job formation, gains a value of a job plus the one time hiring subsidy. The value of a filled job is

$$rJ_j = p + a - w_j + \lambda[-pF - J]$$

where $-pF$ is the firing tax to be paid in case of job separation. The free entry condition implies

$$J_j + pH = \frac{pc}{q(\theta)}$$

4.1 WAGE DETERMINATION

Wages are chosen to maximize the Nash product and can be renegotiated at any time. When there are hiring subsidies and firing taxes the application of the wage rule is more problematic.

There are two important issues to discuss: the two-tier regime and the role of taxation.

On impact, when they first meet, the initial wage is determined by

$$B_o = (W_j - U)^{\beta}(J_j + pH - V)^{1-\beta}$$

where $J + pH$ is the gain to the firm from wage agreement.

When the employment relationship is undergoing, hiring subsidy are no longer available but the firm has to pay the firing tax if the negotiation breaks does. This implies that the wage rule is now

$$B_1 = (W_j - U)^{\beta}[J_j - (V - pF)]^{1-\beta}$$

where $V - pF$ is the firm's threat point in the negotiation rule. There will be a two-tier wage regime where, w_o is the "outside" wage that solves the Nash product B_o while w_1 is the inside wage that solves B_1 Formally

$$\begin{aligned} w_o &= \arg\max B_o \\ w_i &= \arg\max B_1 \end{aligned}$$

The solution to w_o solves (by taking logs of B_o and taking the derivative with respect to w_o)

$$\frac{\beta}{W_j - U}[\frac{\partial W_j}{\partial w_{oj}} - \frac{\partial U_j}{\partial w_{oj}}] + \frac{1-\beta}{J_j + pH - V}[\frac{\partial J_j}{\partial w_{oj}} - \frac{\partial V_j}{\partial w_{oj}}] = 0$$

Similarly the solution to w_{ij} is

$$\frac{\beta}{W_j - U}[\frac{\partial W_j}{\partial w_{ij}} - \frac{\partial U_j}{\partial w_{ij}}] + \frac{1-\beta}{J_j + pF - V}[\frac{\partial J_j}{\partial w_{ij}} - \frac{\partial V_j}{\partial w_{ij}}] = 0$$

Since there are taxes, the derivative of the value functions do not cancel out. While $\frac{\partial U_j}{\partial w_{ij}} = \frac{\partial U_j}{\partial w_{oj}} = \frac{\partial V_j}{\partial w_{oj}} = \frac{\partial V_j}{\partial w_{ij}} = 0$ with respect to the specific wage j (note that is only the average wage that enters the value functions).

$$\begin{aligned} \frac{\partial W_j}{\partial w_{ij}} &= \frac{\partial W_j}{\partial w_{oj}} = \frac{1-t}{r+\lambda} \\ \frac{\partial J_j}{\partial w_{ij}} &= \frac{\partial J_j}{\partial w_{oj}} = \frac{1}{r+\lambda} \end{aligned}$$

So that the sharing rules become

$$\begin{aligned}(1-\beta)(W_j - U) &= \beta(1-t)(J_j + pH - V) \qquad \text{for } w_{oj} \\ (1-\beta)(W_j - U) &= \beta(1-t)(J_j + pF - V) \qquad \text{for } w_{ij}\end{aligned}$$

This implies that the share of the surplus going to the worker depends on t. To see this (add and subtract $\beta t[W-U]$) to the outside surplus(it's the same for the inside wage) to obtain

$$W_j - U = \beta(1-t)(J_j + pH - V) + \beta(W_j - U) + \beta t(W_j - U) - \beta t(W_j - U)$$

to obtain

$$\begin{aligned}W_j - U &= \frac{\beta(1-t)}{1-\beta t}[J_j + pH - V + W_j - U] \\ W_j - U &= \frac{\beta(1-t)}{1-\beta t} B_o\end{aligned}$$

so that the worker's share of the surplus is now $\frac{\beta(1-t)}{1-\beta t}$ which is a decreasing function of t. The marginal tax rate influences the division of the surplus from a job, whereas average tax rates and subsidies influence the outcome of the bargain only through their effect on the surplus shared. An increase in t reduces the worker's share of the surplus, but to see whether wages actually fall depend on what happens to the total surplus and to the alternative return to labor. The idea is that the marginal tax rate system imposes a joint loss to the firm that can be reduced by keeping wages low.

4.1.1 OUTSIDE AND INSIDE WAGE

To obtain the outside wage proceed as follows

$$(r+\lambda)[W_j - U] = (1-t)w_j - \rho[(1-t)w + \tau t] - \theta q(\theta)[W-U]$$

Imposing symmetry $w = w_j$ (all outside and inside wages are the same in equilibrium) and using the free entry condition

$$W - U = \frac{\beta(1-t)}{1-\beta}\frac{cp}{q(\theta)}$$

one obtains

$$(r+\lambda)[W-U] = (1-t)w + \tau(1-t) - z - \rho[(1-t)w + \tau t] - \frac{\theta\beta(1-t)cp}{1-\beta}$$

while

$$(r+\lambda)(J + pH) = p + a - w - \lambda pF + (r+\lambda)pH$$

So that the satisfies

$$(1-\beta)[(1-t)w + \tau(1-t) - z - \rho[(1-t)w + \tau t] - \frac{\theta\beta(1-t)cp}{1-\beta}] =$$
$$\beta(1-t)[p + a - w - \lambda pF + (r+\lambda)pH]$$

Dividing everything by $(1-t)$ and collecting terms

$$w[1-\rho(1-\beta)] = \frac{z(1-\beta)}{1-t} - (1-\beta)\tau + (1-\beta)\rho + \beta[p + a - \lambda pF + (r+\lambda)pH + \theta cp]$$

which simplifies to the final expression as

$$w_o = \frac{1-\beta}{1-\rho(1-\beta)}[\frac{z}{1-t} - \tau(1-\rho)] + \frac{\beta}{1-\rho(1-\beta)}[p(1+\theta c + (r+\lambda)pH - \lambda F) + a] \qquad \text{(outside wage)}$$

To get the inside wage proceed in a similar way using

$$(r+\lambda)(J+pF) = p + a - w - \lambda pF + (r+\lambda)pF$$

and substituting into the sharing rule one obtains

$$w_i = \frac{1-\beta}{1-\rho(1-\beta)}[\frac{z}{1-t} - \tau(1-\rho)] + \frac{\beta}{1-\rho(1-\beta)}[p(1+\theta c + rF) + a] \quad \text{(inside wage)}$$

Differences among wages

- if there are no hiring subsidies ($H = 0$) the wage increases after renegotiation
  - the inside wage does not depend on the hiring subsidy (which is already been paid)
  - if $H = F$ the two wages are the same
  - $w_o < w_i$ if $F > H$

Similarities

- the replacement rate increases wages since it increase the outside option
  - the employment subsidy increase wages
  - the net tax subsidy receive by the match is $\tau(1-\rho)$ so that they increase the wage, the employed worker gets the full subsidy while the unemployed gets only a fraction ρ
  - The marginal tax rate influences the wage to the extant that specific unemployed income z is not taxed
  - If $z = 0$ and $\tau = 0$ (proportional taxation) the tax system does not introduce any distortion
  - If $z = 0$ and $\tau > 0$ (progressive taxation) the tax system reduces the negotiated wage

4.2 EQUILIBRIUM WITH POLICY

Labor demand is obtained from

$$\frac{pc}{q(\theta)} - pH = \frac{p + a - w - \lambda pF}{r+\lambda}$$

so that the expression is

$$w = p + a + p[(r+\lambda)H - \lambda F] - \frac{(r+\lambda)cp}{q(\theta)} \quad (12)$$

and is a negative relationship between w and θ. The second relationship for obtain θ is the outside wage, since job formation is obtained on the basis of the expected profit from a new job. The outside wage and labor demand (equation 11) solve a unique θ while equilibrium unemployment is simply given by

$$u = \frac{\lambda}{\lambda + \theta q(\theta)}$$

Definition 5 ***The Search equilibrium with exogenous job destruction and a set of policy instruments*** (ρ, τ, t, F, H) ***is a couple*** (u, θ) *and two wage rules* $w_o w_i$ *satisfying*

Free entry ($V = 0$)

Outside Wage bargaining $(1-\beta)[W-U] = \beta(1-t)[J+pH]$

Inside Wage Bargaining $(1-\beta)[W-U] = \beta(1-t)[J+pF]$

Balance Flow $m(u,v) = \lambda(1-u)$

The effects of the various policies. To obtain the effects of the single policies it suffices to substitute the outside into equation 11 the outside wage to obtain

$$p + a + p[(r+\lambda)H - \lambda F] - \frac{(r+\lambda)cp}{q(\theta)} =$$

$$\frac{1-\beta}{1-\rho(1-\beta)}[\frac{z}{1-t} - \tau(1-\rho)] + \frac{\beta}{1-\rho(1-\beta)}[p(1+\theta c + (r+\lambda)pH - \lambda F) + a]$$

Employment subsidies increase market tightness. To see this simply totally differentiate the previous equation with respect to a to obtain

$$1 + \frac{(r+\lambda)cq'(\theta)}{q^2(\theta)}\frac{\partial\theta}{\partial a} = \frac{\beta}{1-\rho(1-\beta)}$$

which simplifies to

$$\frac{(1-\rho)(1-\beta)}{1-\rho(1-\beta)} = -\frac{(r+\lambda)cq'(\theta)}{q^2(\theta)}\frac{\partial\theta}{\partial a}$$

so that

$$\frac{\partial\theta}{\partial a} > 0 \rightarrow \frac{\partial u}{\partial a} < 0$$

Hiring Subsidies increase market tightness

Proceeding as above one obtains

$$\frac{(1-\rho)(1-\beta)}{1-\rho(1-\beta)} = -\frac{(r+\lambda)cq'(\theta)}{q^2(\theta)}\frac{\partial\theta}{\partial H}$$

so that

$$\frac{\partial\theta}{\partial H} > 0 \rightarrow \frac{\partial u}{\partial H} < 0$$

Marginal tax rates reduce market tightness

$$-\frac{(1-\beta)z}{1-\rho(1-\beta)}\frac{1}{(1-t)^2} = \frac{(r+\lambda)cq'(\theta)}{q^2(\theta)}\frac{\partial\theta}{\partial t}$$

so that

$$\frac{\partial\theta}{\partial t} < 0 \rightarrow \frac{\partial u}{\partial t} > 0$$

Note that in a model without home production $z = 0$ taxation is completely neutral.

Tax subsidies increase market tightness

$$\frac{(1-\rho)(1-\beta)}{1-\rho(1-\beta)} = -\frac{(r+\lambda)cq'(\theta)}{q^2(\theta)}\frac{\partial\theta}{\partial\tau}$$

so that

$$\frac{\partial\theta}{\partial\tau} > 0 \rightarrow \frac{\partial u}{\partial\tau} < 0$$

Firing Taxes reduce market tightness

$$\frac{(1-\rho)(1-\beta)}{1-\rho(1-\beta)} = \frac{(r+\lambda)cq'(\theta)}{q^2(\theta)}\frac{\partial\theta}{\partial F}$$

so that

$$\frac{\partial\theta}{\partial F} < 0 \rightarrow \frac{\partial u}{\partial F} > 0$$

Finally, replacement Rate reduce market tightness

$$\frac{\partial\theta}{\partial\rho} < 0 \rightarrow \frac{\partial u}{\partial\rho} > 00$$

5 FIRING TAXES IN A MODEL WITH TWO MARGINS

How robust are the result of the previous section in terms of modeling features? Most of the results are fairly general and can help understand some of the feature of European unemployment The replacement rate is much higher in Europe (there is substantial agreement on its effect) as are tax rates. Yet, there is much less agreement on the effect of taxation on unemployment. The neutrality result when $z = 0$ is taken fairly seriously in the literature, so that there is a strong view that reducing labor taxes would not reduce unemployment. The effect of firing taxes is also fairly controversial, and the results of the previous section is indeed a particular case. In most models, the effect of firing taxes on equilibrium unemployment is ambiguous, but this ambiguity can be seen only in a model with two margins, (.i.e. with a model with endogenous job destruction). We now formalize this effect by introducing job destruction in the model. Yet, to keep the notation as simple as possible we do not work explicitly with wages, and we set $\beta = 0$. Firms are hit by productivity shocks at rate λ and they draw from a distribution function $F(x)$

The value of a vacancies read

$$rV = -pc + q(\theta)[J(x_u) - V]$$

where $J(\epsilon_u)$ is the value of a job at the upper support of the distribution. The value of an existing job at idiosyncratic productivity x is

$$rJ(x) = px - b + \lambda[\int_{x_l}^{R} -TdF(z) + \int_{R}^{x_u} J(z)dF(z) - J(x)] \tag{13}$$

where the reservation rule for the firm is

$$J(R) = -T$$

The firm gets $-T$ if the new shock is below the reservation rule while it gets the expected value of a the job if the new shock is above R. From equation (), it is immediate to see that (by taking the difference between $J(x)$ and $J(R) = -T$ that

$$(r + \lambda)J(x) + (r + \lambda)T = p(x - R)$$

or that

$$J(x) = p\frac{x - R}{r + \lambda} - T$$

The integral in equation (), after an integration by parts become

$$\int_{R}^{x_u} J(z)dF(z) = J(x_u) + TF(R) - p\frac{1}{r + \lambda}\int_{R}^{x_u} F(z)dz \tag{14}$$

and using the fact that $J(x^u) = p\frac{x^u - R}{r+\lambda} - T$ we get

$$\int_{R}^{x_u} J(z)dF(z) = \frac{p}{r + \lambda}\int_{R}^{x_u} (1 - F(z))dz - (1 - F(R))T$$

Substituting back into equation (12) and substituting for $J(R) = -T$ we get an expression for the reservation productivity which reads

$$-(\lambda + r)T = pR - b - \lambda F(R)T + \frac{\lambda p}{r + \lambda}\int_{R}^{x_u} (1 - F(z))dz - \lambda(1 - F(R))T$$

which simplifies to

$$pR - b = -\frac{\lambda p}{r + \lambda}\int_{R}^{x_u} (1 - F(z))dz - rT \tag{15}$$

which is the final expression for the reservation productivity R the job destruction margin.

To get an expression for job creation, simply use $V = 0$ to obtain

$$\frac{cp}{q(\theta)} = p\frac{x_u - R}{r + \lambda} - T \tag{16}$$

Equilibrium unemployment is then

$$u = \frac{\lambda F(R)}{\lambda F(R) + \theta q(\theta)}$$

Definition 6 ***Definition 7*** *The equilibrium with firing taxes and endogenous reservation productivity is a triple (R, θ, u) that satisfies:*

i) optimal job destruction ($J(x) = -R$)

ii) optimal job creation ($V = 0$)

iii) steady state unemployment

What happens when T goes up?

First, one can see that R reduces the reservation productivity. To see this, take the derivative of T with respect to R

$$p\frac{\partial R}{\partial T} = \frac{\lambda p}{r + \lambda}(1 - F(R))\frac{\partial R}{\partial T} - r$$

which simplifies to

$$p\frac{\partial R}{\partial T}[\frac{r + \lambda F(R)}{r + \lambda}] = -r$$

which implies that

$$\frac{\partial R}{\partial T} < 0$$

so that the firm holds on to less productive jobs when firing taxes increase.

The effect of T on θ is

$$-\frac{cpq'(\theta)}{q(\theta)^2}\frac{\partial \theta}{\partial T} = -\frac{p}{r + \lambda}\frac{\partial R}{\partial T} - 1$$

which looks ambiguous. But substituting the value of $\frac{\partial R}{\partial T}$ one obtains

$$\begin{aligned} -\frac{cpq'(\theta)}{q(\theta)^2}\frac{\partial \theta}{\partial T} &= \frac{1}{r + \lambda}\frac{r(r + \lambda)}{r + \lambda F(R)} - 1 \\ -\frac{cpq'(\theta)}{q(\theta)^2}\frac{\partial \theta}{\partial T} &= \frac{-\lambda F(R)}{r + \lambda F(R)} \end{aligned}$$

so that

$$\frac{\partial \theta}{\partial T} < 0$$

which shows that fewer jobs come to the market. Basically, firing taxes reduce job destruction and job creation at give unemployment. The effect on unemployment is indeed ambiguous since

$$\begin{aligned} \frac{\partial u}{\partial T} &= \frac{\lambda f(R)\frac{\partial R}{\partial T}[\lambda F(R) + \theta q(\theta)] - [\lambda f(R)\frac{\partial R}{\partial T} + q(\theta)(1 - \eta(\theta))\frac{\partial \theta}{\partial T}]\lambda F(R)}{(\lambda F(R) + \theta q(\theta))^2} \\ \frac{\partial u}{\partial T} &= \frac{\lambda f(R)\frac{\partial R}{\partial T}\theta q(\theta) - q(\theta)(1 - \eta(\theta))\frac{\partial \theta}{\partial T}\lambda F(R)}{(\lambda F(R) + \theta q(\theta))^2} = ? \end{aligned}$$