

# Skill Differentials and Inequality

UniTo, Labor Economics Part II  
Christoph Albert

Lecture 2

# Agenda

## Cross-sectional and Intergenerational Inequality

Some key facts about cross-sectional inequality

## The Canonical Model

The factor proportions model

Technological change and the skill premium

Tinbergen's race between skill demand and supply

Estimating the canonical model: Katz-Murphy '92

Education vs. "skill"

Other inputs

## Other Factors and Summary

# Cross-sectional and Intergenerational Inequality

*Inequality* has become a central topic in economics:

- ▶ Wage and income inequality has increased in the U.S. and many other countries
- ▶ Motivates research in labor, trade, development, macro...

We will cover:

1. *Cross-sectional inequality*
  - ▶ **skill-biased technological change**
  - ▶ task-biased technological change
2. *Intergenerational inequality*
  - ▶ parent-child evidence

## Cross-sectional vs. intergenerational inequality

**Intergenerational** or **social mobility**. From Solon (1999):

*Imagine two societies, society A and society B. The distribution of earnings [and] the degree of cross-sectional inequality is the same in both societies. At first glance, the two societies appear to be equally unequal. But now suppose that, in society A, one's relative position in the earnings distribution is exactly inherited from one's parents. If your parents were in the 90th percentile of earnings in their generation, it is certain that you place in the 90th percentile in your own generation. [...] In contrast, in society B, one's relative position in the earnings distribution is completely independent of the position of one's parents. [...] Unlike society A, society B displays complete intergenerational mobility. Although societies A and B have the same measured inequality within a generation, the two societies are tremendously different in the character of their inequality.*

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## Decomposing cross-section inequality

A useful starting point to think about cross-sectional inequality is the **Mincer regression** ( $\rightarrow$  Human Capital)

$$\begin{aligned}\log w_i &= \beta_0 + \beta_1 \text{Schooling}_i + \beta_2 \text{Exp}_i + \beta_3 \text{Exp}_i^2 + \varepsilon_i \\ &= \beta_0 + X_i \beta + \varepsilon_i\end{aligned}$$

where  $X_i = (\text{Schooling}_i, \text{Exp}_i, \text{Exp}_i^2)$ .

Can therefore **decompose** inequality (i.e. variance) into

$$\text{Var}(\log w_i) = \beta \text{Cov}(X_i) \beta' + \text{Var}(\varepsilon_i)$$

1. Inequality in “skills”  $\text{Cov}(X_i)$
2. Skill prices  $\beta$
3. Residual inequality  $\text{Var}(\varepsilon_i)$

Factors (2) and (3) appear more important

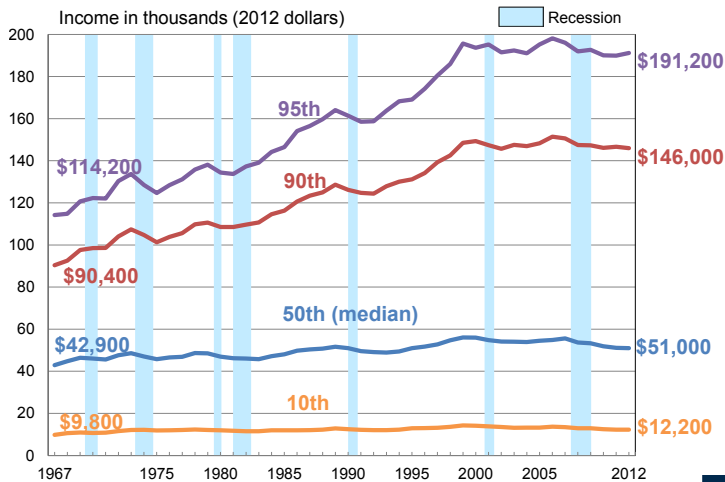
# Key facts on cross-sectional inequality

Some key descriptive facts:

- ▶ Wage inequality (e.g. 90-10 wage ratio) has been growing
- ▶ Bottom of distribution stagnating
- ▶ Labor markets “polarizing”: employment growing in bottom and top of distribution, decreasing in the middle
- ▶ Top incomes (e.g. top 1%) and wealth inequality increasing
- ▶ Labor share decreasing

... in the US and many other developed countries

# U.S. Real HH Incomes at Selected Percentiles, 1967-2012



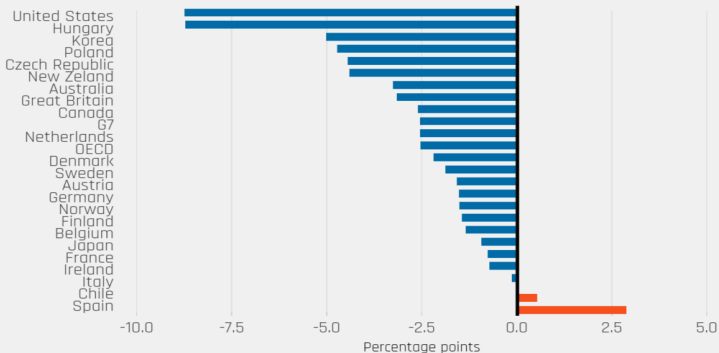
U.S. Census Bureau 2013



# Ratio of Median vs. Mean Wages, 1993-2013

## The United States is the outlier in the growth over time in the ratio of median and mean wages among OECD economies

Change in the ratio of median and average wages in the economies of select member nations of the Organisation for Economic Co-operation and Development, 1995-2013



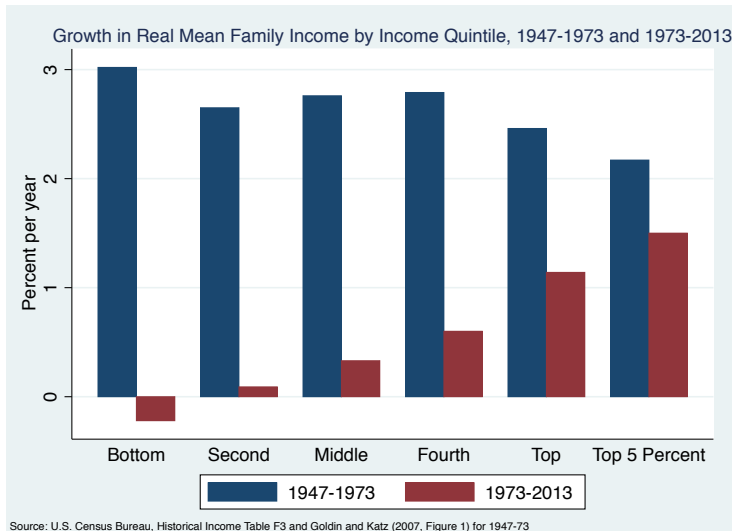
Source: Organisation for Economic Co-operation and Development, "Earnings Database" [n.d.], Cyrille Schwellnus, Andreas Kappeler, and Pierre-Alain Pionnier, "Decoupling of wages from productivity: Macro-level facts" (2017).

Note: Three-year averages starting and ending in indicated years. OECD and G7 refer to unweighted averages for the relevant countries included in the Figure. Sample years vary for some countries.

## Evidence on U.S. wage and income inequality

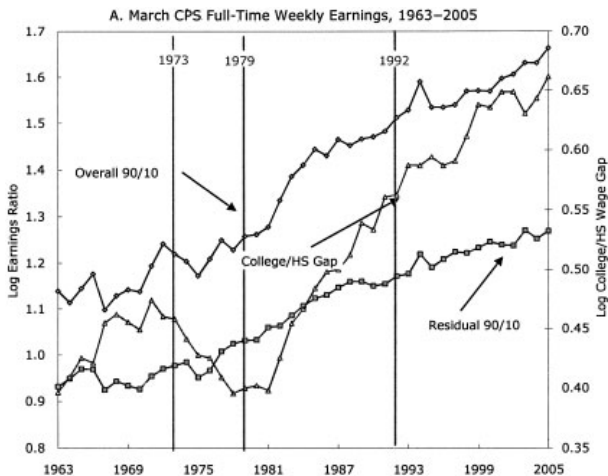
- ▶ Goldin and Katz (2007), “Long-Run Changes in the Wage Structure: Narrowing, Widening, Polarizing”, Brookings Papers on Economic Activity
- ▶ Goldin and Katz (2007), “The Race Between Education and Technology: The Evolution of US Educational Wage Differentials, 1890 to 2005”, NBER Working Paper No. 12984
- ▶ Autor, Katz, and Kearney (2008), “Trends in U.S. Wage Inequality: Revising the Revisionists.” The Review of Economics and Statistics
- ▶ Acemoglu and Autor (2011), “Skills, Tasks and Technologies: Implications for Employment and Earnings”, Handbook of Labor Economics

# Family Income by Quintile, 1947-73 and 1973-2013



# Autor, Katz, and Kearney (2008)

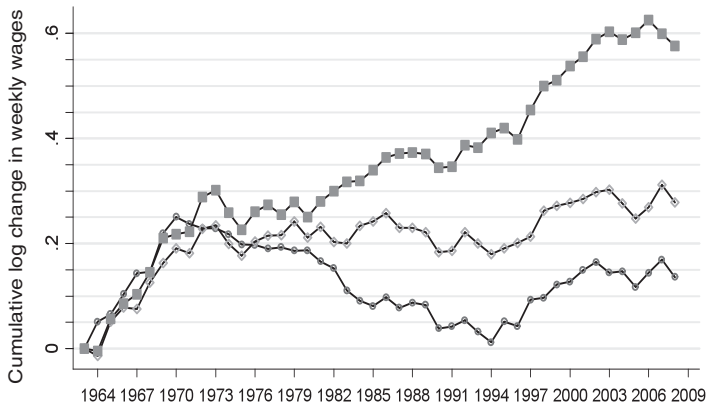
FIGURE 2.—THREE MEASURES OF WAGE INEQUALITY: COLLEGE/HIGH SCHOOL PREMIUM, MALE 90/10 OVERALL INEQUALITY, AND MALE 90/10 RESIDUAL INEQUALITY



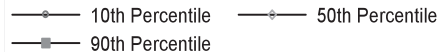
# Acemoglu and Autor (2011): 90-50-10 earnings

Cumulative log change in real weekly earnings at the 90th, 50th and 10th wage percentiles

1963-2008: full-time full-year males



(b)

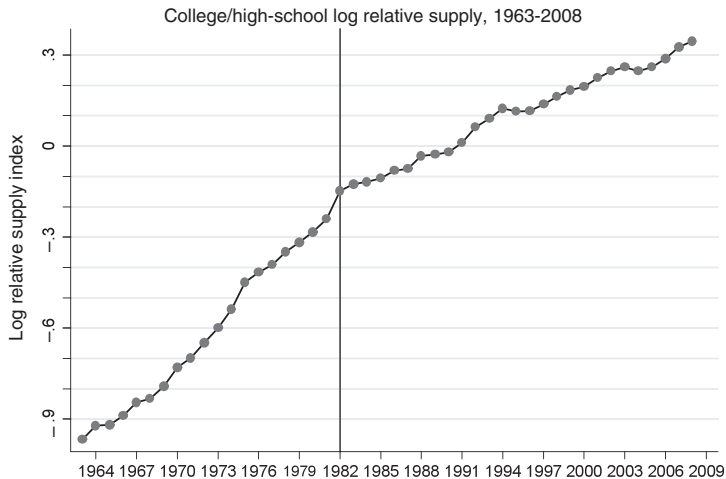


# U.S. Real Hourly Wages by Education Level



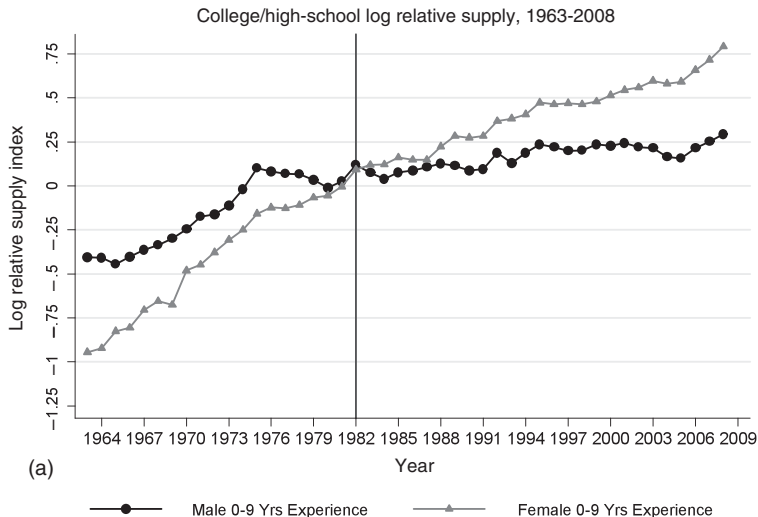
Autor (2019)

# Potential explanation: Changes in relative skill supplies?



Acemoglu and Autor (2011)

# Relative Supplies (by gender)



(a)

Acemoglu and Autor (2011)



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Some key facts about cross-sectional inequality

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# The factor proportions model

Are trends in **relative wages** due to changes in **relative supplies**?

Consider an aggregate production function with high and low skilled workers  $N_{Ht}$  and  $N_{Lt}$ , **constant elasticity of substitution (CES)** and (“**extensive**” and “**factor-augmenting**”) **technological change**,

$$Y_t = [\alpha_t (A_{Lt} N_{Lt})^\rho + (1 - \alpha_t) (A_{Ht} N_{Ht})^\rho]^{1/\rho}$$

Why **imperfect substitutability**? Possible interpretations:

1. One good and high/low skilled workers are imperfect substitutes in production
2. Two goods (produced by either high- or low-skilled workers) and goods are imperfect substitutes in consumption

# The factor proportions model

Aggregate production function

$$Y_t = [\alpha_t (A_{Lt} N_{Lt})^\rho + (1 - \alpha_t) (A_{Ht} N_{Ht})^\rho]^{1/\rho}$$

where  $\rho \in (-\infty, 1)$  and  $\sigma = 1/(1 - \rho) \in [0, \infty]$  is the **elasticity of substitution**. This production function is an abstraction [▶ discussion](#)

Three special cases:

1.  $\sigma \rightarrow 0$  (or  $\rho \rightarrow -\infty$ ): Skilled and unskilled workers are perfect complements ('Leontief' with fixed proportions)
2.  $\sigma \rightarrow \infty$  (or  $\rho \rightarrow 1$ ): Skilled and unskilled workers are perfect substitutes. Changes in supplies do not affect the relative wage  $w_H/w_L$  of high and low skilled
3.  $\sigma \rightarrow 1$  (or  $\rho \rightarrow 0$ ): Production function is Cobb Douglas, with fixed shares paid to each factor

# The factor proportions model

Aggregate production function

$$Y_t = [\alpha_t (A_{Lt} N_{Lt})^\rho + (1 - \alpha_t) (A_{Ht} N_{Ht})^\rho]^{1/\rho}$$

where  $\sigma = 1/(1 - \rho) \in [0, \infty]$  is the **elasticity of substitution**.

Key distinction:

- ▶  $\sigma < 1$ : Gross complements. A reduction in supply of one input demand for the other
- ▶  $\sigma > 1$ : Gross substitutes. A reduction in supply of one input demand for the other

## The factor proportions model

Aggregate production function

$$Y_t = [\alpha_t (A_{Lt} N_{Lt})^\rho + (1 - \alpha_t) (A_{Ht} N_{Ht})^\rho]^{1/\rho}$$

Given competitive markets, factors are paid their marginal products,

$$w_{Lt} = \frac{\partial Y_t}{\partial N_{Lt}} = \alpha_t A_{Lt}^\rho N_{Lt}^{\rho-1} [\alpha_t A_{Lt}^\rho N_{Lt}^\rho + (1 - \alpha_t) A_{Ht}^\rho N_{Ht}^\rho]^{(1-\rho)/\rho}$$

$$w_{Ht} = \frac{\partial Y_t}{\partial N_{Ht}} = \dots$$

and the relative demand for skills is determined by

$$\log\left(\frac{w_{Ht}}{w_{Lt}}\right) = \log\left(\frac{\alpha_t}{1 - \alpha_t}\right) + \frac{\sigma - 1}{\sigma} \log\left(\frac{A_{Ht}}{A_{Lt}}\right) - \frac{1}{\sigma} \log\left(\frac{N_{Ht}}{N_{Lt}}\right)$$

- ▶ relative demand curve for  $N_H/N_L$  is downward sloping ( $\sigma \geq 0$ )
- ▶ an increase in relative supplies  $N_H/N_L$  lowers relative wages with inverse elasticity  $-\frac{1}{\sigma}$  (for a given 'skill bias'  $A_H/A_L$ .)

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# The factor proportions model: Skill-biased technical change

Relative demand for skills

$$\log \left( \frac{w_{Ht}}{w_{Lt}} \right) = \log \left( \frac{\alpha_t}{1 - \alpha_t} \right) + \frac{\sigma - 1}{\sigma} \log \left( \frac{A_{Ht}}{A_{Lt}} \right) - \frac{1}{\sigma} \log \left( \frac{N_{Ht}}{N_{Lt}} \right)$$

Effect of **skill-based technological change** (increase in  $A_H/A_L$ ) depends on sign of  $\frac{\sigma-1}{\sigma}$ :

- ▶  $\sigma < 1$ : Gross complements. Skill premium decreases
- ▶  $\sigma > 1$ : Gross substitutes. Skill premium increases



## Summary of key relationships: Skill-biased technical change

An exogenous increase in  $A_H$ , holding  $A_L$  and  $N_H/N_L$  constant

1.  $w_H/w_L$  rises if  $\sigma > 1$ , falls if  $\sigma < 1$ , and is unchanged if  $\sigma = 1$
2. Average wages rise if  $\sigma > 0$ . Why not if  $\sigma = 0$ ?
3. Wages of  $N_L$  workers rise if  $\sigma < \infty$ . Why not if  $\sigma = \infty$ ?
4. Both  $w_H$  and  $w_L$  rise if  $\sigma \geq 1$ . Why not if  $\sigma < 1$ ?

*General takeaway: It's hard for factor-augmenting technical change to lower wages (though of course  $w_H/w_L$  may rise) if factors are gross substitutes.*

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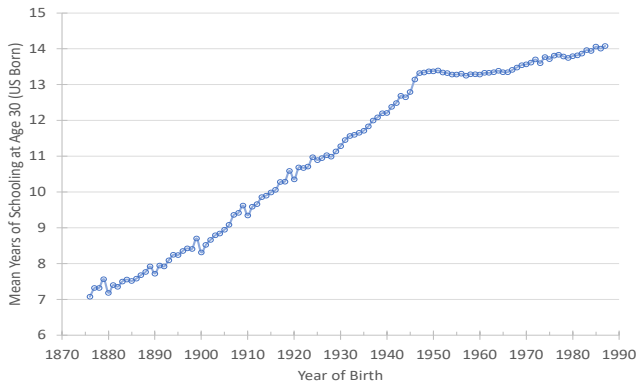
## The long term skill bias of technical change

As  $N_H/N_L$  increases, the skill premium ( $w_H/w_L$ ) falls:

- ▶ In every advanced country the supply of educated workers has risen dramatically in the past seven decades
- ▶ Yet relative wages of better educated workers have remained consistently above those of less educated
- ▶ U.S. college-educated share rose from 6.4% to 29.7% from 1940 to 2000, and to approximately 35% by 2017. High school dropout share fell from 68% to  $< 9\%$  of workforce
- ▶ Yet, the skill premium in 2000 or 2018 (measured in a variety of ways) was at or above that of in 1940 or 1915

To rationalize these pattern within the canonical model, relative demand for skilled workers *must* have risen practically everywhere (→ skill-biased technological change)

# Years of Completed Schooling by Birth Cohort and Sex, 1876 - 1988

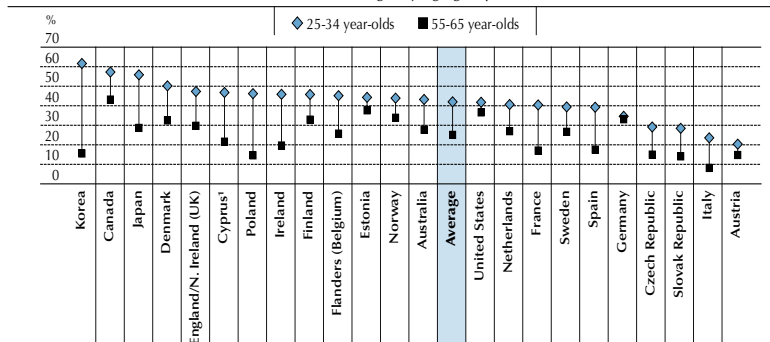


*Sources and Notes:* US Census IPUMS data from 1940 to 2000 and CPS MORG data from 2005 to 2018. The figure updates Goldin and Katz (2007, figure 7). See the on-line appendix for details.

# Tertiary Education Completion in OECD Countries

## Population with tertiary education

Percentage, by age group



1. See notes at the end of this chapter.

Countries are ranked in descending order of the percentage of 25-34 year-olds with tertiary education.

Source: Survey of Adult Skills (PIAAC) (2012), Table B2.2 in Annex B.

OECD Skills Outlook 2013

# Tinbergen's race between skill demand and supply

Jan Tinbergen, 1975

*“The two preponderant forces at work are technological development, which made for a relative increase in demand and hence in the income ratio... and increased access to schooling, which made for a relative decrease.”*

Translation:

- ▶ Long term trend increases towards greater relative demand *and* greater supply of skilled workers
- ▶ Bursts of supply and/or technologically-induced demand accelerations/decelerations that cause demand to temporarily move out more rapidly than supply or vice versa in some eras.

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## Bringing the factor proportions model to the data

Katz and Murphy (1992) estimated this canonical model.

- ▶ Switch from theoretical to estimating equation by adding unobserved heterogeneity  $\varepsilon_t$ , ignore  $\log\left(\frac{\alpha_t}{1-\alpha_t}\right)$ , and replace SBTC or other determinants of changing skill demand with linear time trend:  $\frac{\sigma-1}{\sigma} \log\left(\frac{A_{Ht}}{A_{Lt}}\right) = \gamma_0 + \gamma_1 t$

Estimating equation:

$$\log\left(\frac{w_{Ht}}{w_{Lt}}\right) = \gamma_0 + \gamma_1 t + \gamma_2 \log\left(\frac{N_{Ht}}{N_{Lt}}\right) + \varepsilon_t$$

- ▶  $\gamma_1$  estimates  $\left(\frac{\sigma-1}{\sigma}\right) \log\left(\frac{A_{Ht}}{A_{Lt}}\right)$
- ▶  $\gamma_2$  estimates  $-\frac{1}{\sigma}$



# The factor proportions model: Findings

Consensus that in U.S.  $\sigma \approx 1.5$

Katz and Murphy (1992), results in Appendix [▶ Katz and Murphy \(1992\)](#)

▶  $\gamma_1 = 0.033, \gamma_2 = -0.731 \rightarrow \sigma = 1.4$

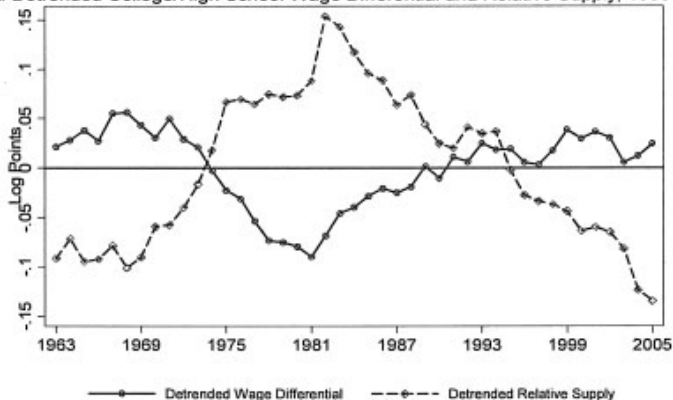
Autor and Acemoglu (2011), results on next slides

▶  $\sigma \approx 1.6$

# The factor proportions model: Findings

FIGURE 4.—COLLEGE/HIGH SCHOOL RELATIVE SUPPLY AND WAGE DIFFERENTIAL, 1963–2005 (MARCH CPS)

A. Detrended College/High School Wage Differential and Relative Supply, 1963–2005



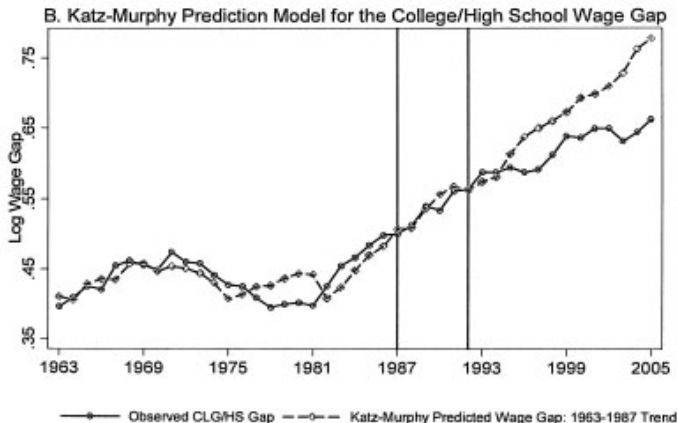
# The factor proportions model: Findings

TABLE 2.—REGRESSION MODELS FOR THE COLLEGE/HIGH SCHOOL LOG WAGE GAP, 1963–2005

	(1) 1963–1987	(2)	(3)	(4)	(5) 1963–2005	(6)	(7)	(8)
CLG/HS relative supply	-0.636 (0.130)	-0.411 (0.046)	-0.619 (0.066)	-0.599 (0.112)	-0.609 (0.102)	-0.728 (0.155)	-0.403 (0.067)	
Log real minimum wage						-0.049 (0.051)	-0.117 (0.047)	-0.144 (0.065)
Male prime-age unemp. rate						0.004 (0.004)	-0.001 (0.004)	-0.018 (0.003)
Time	0.026 (0.005)	0.018 (0.001)	0.026 (0.002)	0.028 (0.006)	0.021 (0.006)	0.028 (0.007)	0.017 (0.002)	0.006 (0.001)
Time <sup>2</sup> /100				-0.011 (0.006)	0.030 (0.015)	0.017 (0.017)		
Time <sup>3</sup> /1000					-0.006 (0.002)	-0.005 (0.002)		
Time × post-1992			-0.008 (0.002)					
Constant	-0.159 (0.119)	0.043 (0.037)	-0.146 (0.057)	-0.143 (0.108)	-0.124 (0.098)	-0.160 (0.191)	0.266 (0.112)	0.689 (0.120)
Observations	25	43	43	43	43	43	43	43
R-squared	0.563	0.934	0.953	0.940	0.952	0.955	0.944	0.891

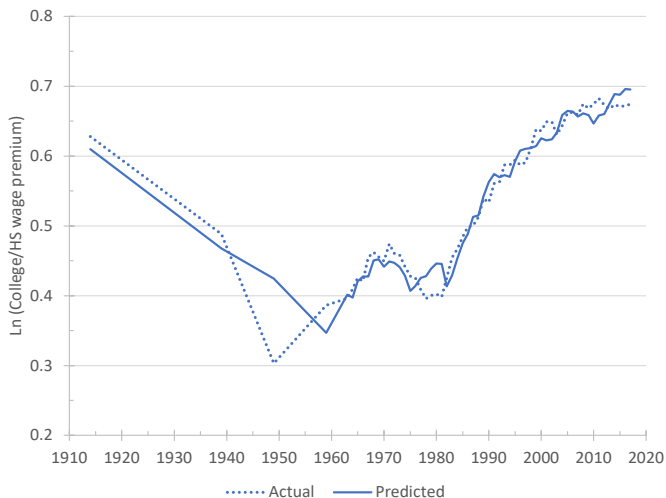
Standard errors in parentheses. Each column presents an OLS regression of the fixed-weighted college/high school wage differential on the indicated variables. The U.S. federal minimum wage is deflated by the personal consumption expenditure deflator. Source for labor supply and earnings measures is the March CPS, earnings years 1963–2005.

# The factor proportions model: Findings



Composition-adjusted college/high school relative wages are calculated using March FTFY earners data, sorted into sex-education-experience groups of two sexes, five education categories, and four potential experience categories. Mean log wages for broader groups in each year represent weighted averages of the relevant (composition-adjusted) cell means using a fixed set of weights that are equal to the mean share of total hours worked by each group over 1963 to 2005 from the March CPS. See table 1 notes for additional details.

# Katz-Murphy Model Fit to a Century of Data, 1914-2017



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## Education vs. “skill”

Are [degrees](#) or [years of schooling](#) good proxies for “[skill](#)”?

[Carneiro and Lee \(2011\)](#) on compositional effects and SBTC:

- ▶ study whether increases in college enrollment led to a decline in the “quality” of college graduates
- ▶ factor proportions model can account for trend in college and age premia, but need to account for changes in quality

[Broecke, Quintini and Vandeweyer \(2019\)](#), in an update of [Leuven, Oosterbeek and van Ophem \(2004\)](#):

- ▶ ask whether cross-country differences in skill supply and demand can explain differences in wage premia
- ▶ background: surprisingly weak cross-country link between average years of schooling and average cognitive skill scores (*IALS* or *PIAAC*)
- ▶ find a negative relation between relative skill prices and relative supplies

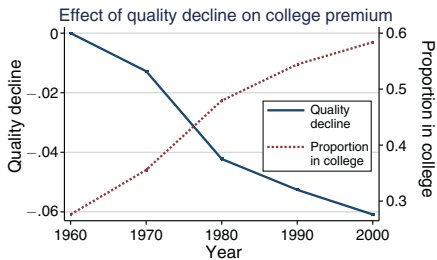
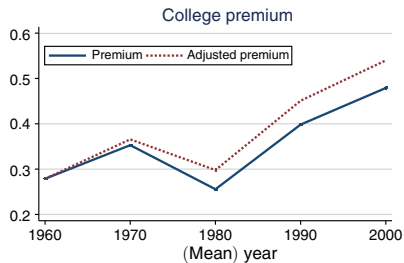
## Carneiro and Lee (2011) on 'quality-adjusted' college premia

Carneiro and Lee (2011) note that secular changes in college-going might also create differences in the cohort *quality* of education

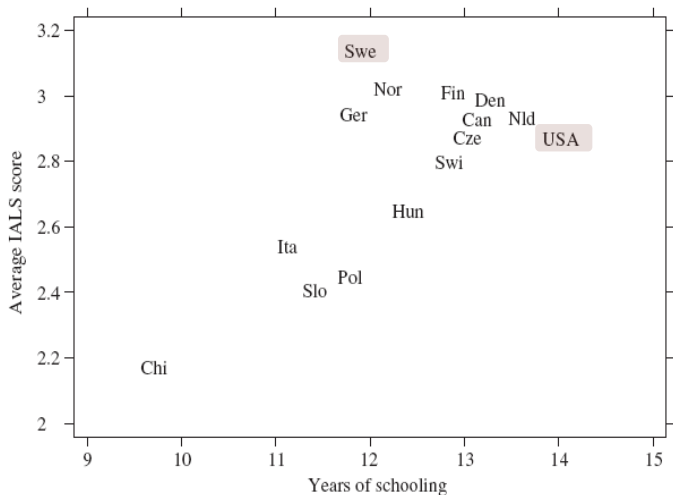
- ▶ 'Lower-quality' individuals go on to college
- ▶ Alternatively, quality of education might deteriorate when there is a large influx of students (or both)
- ▶ How this might affect the wage structure – specifically, the measured college/high-school gap?



# Carneiro and Lee (2011) on 'quality-adjusted' college premia



# Does More School Equal More Skill?



Leuven, Oosterbeek and van Ophem 2004

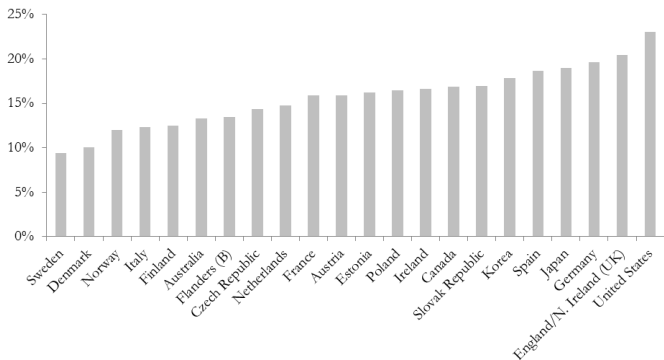
Fig. 1. *Cross Sectional Relation Between Years of Schooling and Skill*

# International Adult Literacy Survey

- *Prose literacy* – the knowledge and skills needed to understand and use information from texts including editorials, news stories, poems and fiction,
- *Document literacy* – the knowledge and skills required to locate and use information contained in various formats, including job applications, payroll forms, transportation schedules, maps, tables and graphics; and
- *Quantitative literacy or numeracy* – the knowledge and skills required to apply arithmetic operations, either alone or sequentially, to numbers embedded in printed materials, such as balancing a checkbook, figuring out a tip, completing an order form or determining the interest on a loan from an advertisement.

# Estimated Wage Returns to PIAAC (2012) Skills

Figure 2. The return to skill, United States and other PIAAC countries

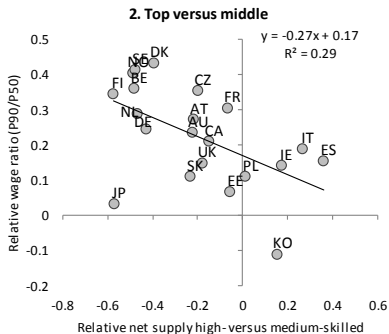
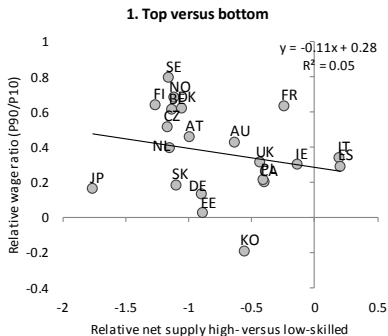


Notes: The figure shows the coefficient on skill from a regression of log hourly wages (including bonuses) for wage and salary earners (in PPP corrected USD) on standardized numeracy scores and a quartic of experience.

Broecke, Quintini and Vandeweyer (2019), PIAAC 2012

# Broecke update of LOvO analysis for 90/10 and 90/50 gaps

Relative wages and skills in the US vs other countries:



Broecke, Quintini and Vandeweyer (2019), PIAAC 2012

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## Wait, there are more inputs!

We can extend the logic of this 'factor proportions model' to other input factors.

Indeed, similar type of (nested) CES models serve as a standard framework in many different contexts:

- ▶ Skill  $\times$  experience cells ([Card and Lemieux, 2001](#))
- ▶ Capital-labor and capital-skill complementarity ([Krusell, Ohanian, Rios-Rull, and Violante, 2000](#) ; [Lewis 2011](#))
- ▶ Some immigration studies model natives and immigrants as distinct (imperfectly substitutable) inputs

# College Premium: Young vs. Experienced Workers in U.S.

FIGURE 6.—COMPOSITION-ADJUSTED LOG RELATIVE COLLEGE/HIGH SCHOOL WAGE AND SUPPLY BY POTENTIAL EXPERIENCE AND AGE GROUPS, 1963–2003 (MARCH CPS)



► Similar evidence for other countries



## Card and Lemieux (2001)

Card and Lemieux (2001) consider a nested, two-level CES model:

Upper level: Identical to simple Katz-Murphy model

- ▶ Output a function of  $N_{Ht}, N_{Lt}, A_{Ht}, A_{Lt}$

Lower level: Supplies of each education group are themselves CES aggregates of the labor supply of different age groups

- ▶ Aggregate education supplies  $N_{Ht}, N_{Lt}$  depend on age-group specific supplies  $N_{Hjt}, N_{Ljt}$

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## Other Factors and Summary

## Limitations and other factors

The factor proportions model w/ SBTC does a surprisingly good job in explaining changes in skill premia, and also has some bite in explaining cross-country differences.

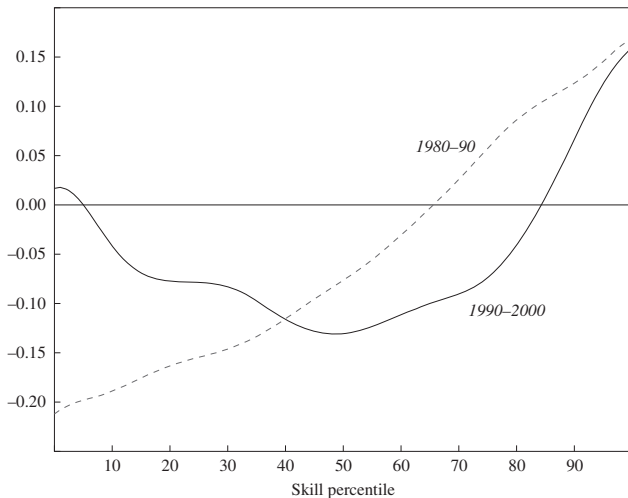
But:

- ▶ Few data points
- ▶ Endogeneity of skill supply
- ▶ A single measure of skill cannot explain job **polarization** (*next slide*)

# Goldin and Katz (2007): Job Polarization

**Figure 8. Changes in Share of Employment by Percentile of the Occupational Skill Distribution, 1980–90 and 1990–2000\***

Log change in employment  
share  $\times 100$



## Limitations and other factors

Other potential factors affecting inequality:

- ▶ Leading alternative: erosion of **labor market institutions**, such as decline of trade unions or minimum wages.
  - ▶ Predicts cross-country variation in lower-tail inequality
- ▶ **External shocks**: Trade, immigration, offshoring, etc
  - ▶ Trade shown to have important distributional impacts
  - ▶ Distributional impacts of immigration are more controversial
- ▶ Task-biased technological change (→ next lecture)

# Appendix

# The Canonical Model

Three interpretations of the aggregate demand function:

1. Only one good, skilled and unskilled workers are imperfect substitutes in its production
2. Two-good economy:
  - ▶ Consumers have utility function  $[Y_L^\rho + Y_H^\rho]^{1/\rho}$  with elasticity of substitution  $\sigma = 1/(1 - \rho)$
  - ▶ Good  $Y_H$  is produced with  $Y_H = A_H N_H$
  - ▶ Good  $Y_L$  is produced with  $Y_L = A_L L_L$
3. A mixture of these two where different sectors produce goods that are imperfect substitutes, and high and low education workers are employed in all sectors

## with Skill-Replacing Technologies

$$Y = K^\alpha [(1 - b)[A_L N_L + B_L]^\rho + b[A_H N_H + B_H]^\rho]^{(1-\alpha)/\rho}$$

- ▶ In prior setup, only *factor augmenting* technologies
- ▶ Here,  $B_L$ ,  $B_H$  are directly *skill-replacing* technologies
- ▶ Intensive versus extensive technical changes
  - ▶  $b$  is 'extensive' technological change that shifts the allocation of tasks among factors
  - ▶  $A_L$ ,  $A_H$  terms are 'intensive' technical changes, augmenting without reallocating
- ▶  $K$  is capital: enters in Hicks-neutral form above, no bearing on skill premium
- ▶ Note that if  $\sigma \rightarrow 1$ , the  $b$  terms limit to the exponents in the Cobb-Douglas production function



# The Canonical Model

## Seriously, what is $\sigma$ ? [▶ back](#)

- ▶ Aggregate production function is an abstraction
- ▶ *Not* intended to correspond to production function of any given firm
- ▶ Combines substitution in production and consumption across consumers, industries, firms, plants within firms, etc.
- ▶ We would expect factors to be less substitutable at the firm level than at the aggregate level

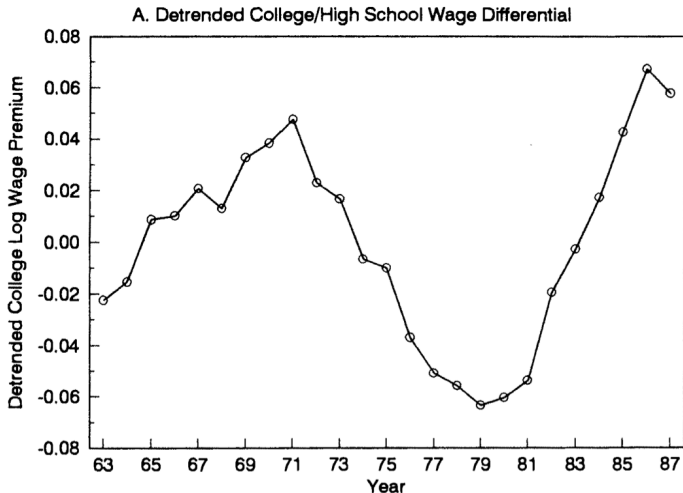
## Where do aggregate production functions come from?

- ▶ See Houthakker 1955 *ReStud*, Jones 2005 *QJE*

## What are plausible values of $\sigma$ ?

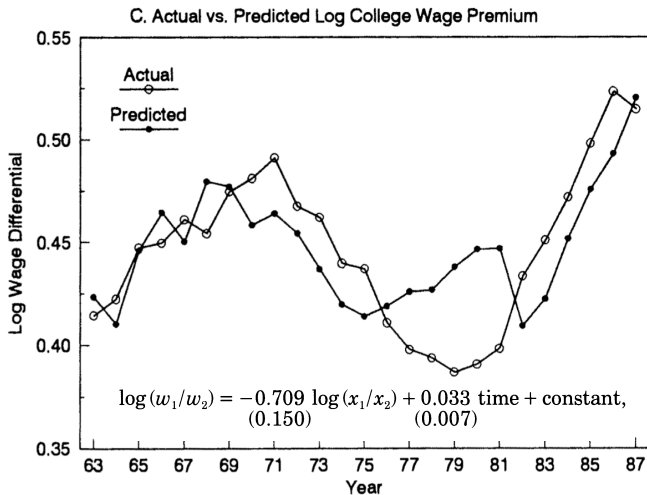
- ▶ Surprising degree of consensus:  $\sigma \in [1, 2]$

# Katz-Murphy (1992): Data to be Explained:



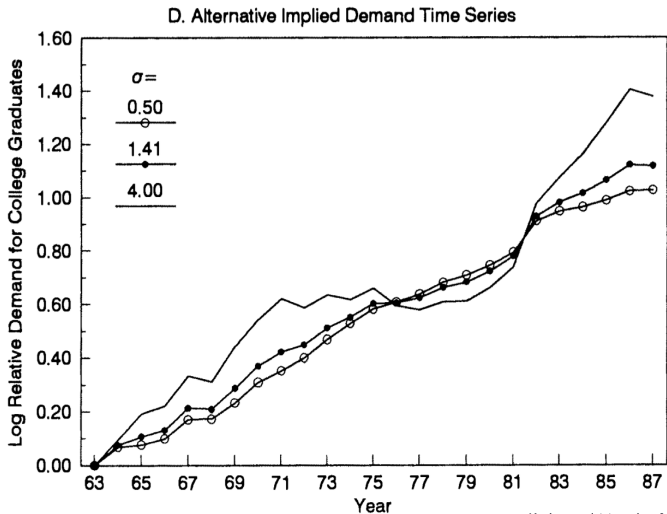
Katz and Murphy 1992

Model Fit:  $\hat{\sigma} = -(1/0.71) = 1.41$  [ $R^2 = 0.52$ ]



Katz and Murphy 1992

# Implied Demand Series: Alternative Values of $\sigma$



Katz and Murphy 1992

[▶ back](#)

# Krusell, Ohanian, Rios-Rull, and Violante (2000)

## Declining Log Relative Price of Equipment Capital:

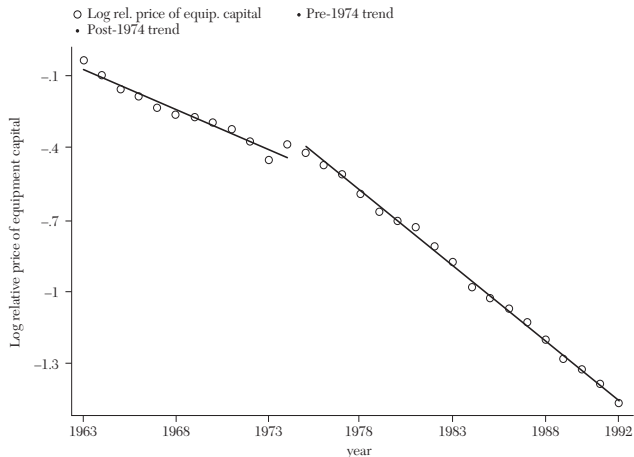


Figure 6. Behavior of the Log Relative Price of Equipment Capital, 1963–92

Krusell, Ohanian, Rios-Rull and Violante, 2000 (reprinted in Acemoglu 2002)

## KORRV 2000: An Alternative Explanation, Declining Log Relative Price of Equipment Capital, 1963-92

$$G(k_{st}, k_{et}, L_t, H_t) = k_{st}^\alpha \left[ \beta N_{Lt}^\delta + (1 - \beta) (\lambda k_{et}^\rho + (1 - \lambda) N_{Ht}^\rho)^{\delta/\rho} \right]^{(1-\alpha)/\delta}$$

- ▶  $k_{st}$  is structures capital,  $k_{et}$  is equipment capital
- ▶  $\alpha$  is structure share of output (note: Cobb-Douglas)
- ▶  $\beta$  is the extensive margin technological parameter
- ▶  $\sigma_e = 1/(1 - \rho)$  is elasticity btwn  $N_H$  labor and equipment capital  $k_e$
- ▶  $\sigma_u = 1/(1 - \delta)$  is elasticity btwn  $N_H + k_e$  aggregate and  $N_L$

**Key condition:**  $\sigma_u > \sigma_e$

- ▶ If  $\sigma$  between  $N_L$  and  $(N_H + k_e)$  is greater than  $\sigma$  between  $k_e$  and  $N_H$ , then  $k_e$  is a *relative* complement to  $N_H$
- ▶  $\sigma_u > \sigma_e$  implies **equipment-skill complementarity**

# Estimation of K-S Complementarity Model (KORRV 2000)

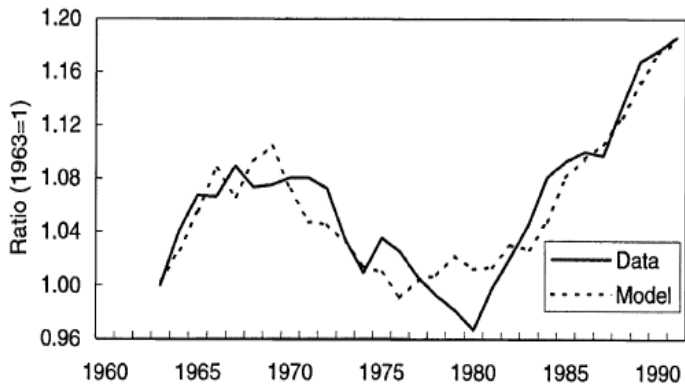
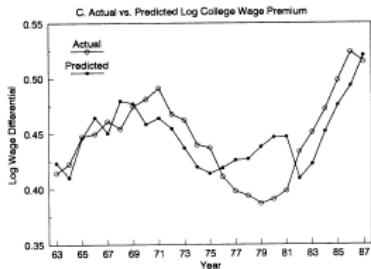


Figure 8. The skill premium: Skilled vs. unskilled wages per hour (normalized with 1963=1).

Krusell, Ohanian, Rios-Rull and Violante, 2000

# Does this Chart Look Familiar?



Katz and Murphy 1992

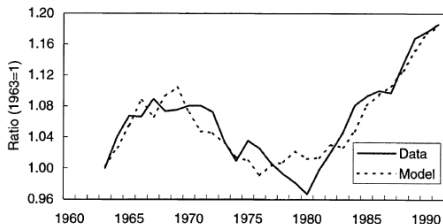


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Krusell, Ohanian, Rios-Rull and Violante, 2000

▶ back