

# Chapter 2

## Business Cycle Properties

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# Outline

1. Dynamics of the benchmark model
2. Calibration
3. Volatility in alternative structures
4. Nonlinear dynamics in the labor market

# DYNAMICS OF THE BENCHMARK MODEL

# Loglinearization of the Job Creation Condition

- ▶ Let  $x_t$  be the time-varying stochastic value of productivity.
- ▶ Use  $\hat{x}_t \equiv (x_t - x)/x$  as the proportional deviations of  $x_t$  around its deterministic steady state
- ▶ Similarly, use  $\hat{\theta}_t$  as proportional deviations of labor market tightness.

# Loglinearization of the Job Creation Condition

- ▶ Assumptions

- ▶ 1. Wage is fixed at some level  $\bar{w}$  within the bargaining set
- ▶ 2. Elasticity of the matching function  $\eta_L(\theta)$  is a constant  $\eta_L$

# Loglinearization of the Job Creation Condition

- ▶ Using equation 1.19, the log-linearization around the steady state of the job creation condition with a fixed wage is:

$$\hat{\theta}_t = \left(\frac{1}{\eta_L}\right) \left(\frac{x}{x - \bar{w}}\right) \left(\frac{r + s}{1 + r}\right) \mathbb{E}_t \left[ \sum_{i=0}^{\infty} \left(\frac{1 - s}{1 + r}\right)^i \hat{x}_t + 1 + i \right] \quad (1)$$

# Loglinearization of the Job Creation Condition

- ▶ First element of equation 2.1: The response of equilibrium labor market tightness to changes in labor productivity is decreasing in the elasticity of the matching function with respect to unemployment
- ▶ Second element: The response of market tightness is decreasing in the magnitude of the profit flow  $x - \bar{w}$
- ▶ Amplification of labor market tightness due to labor market congestion and the size of profit flow:

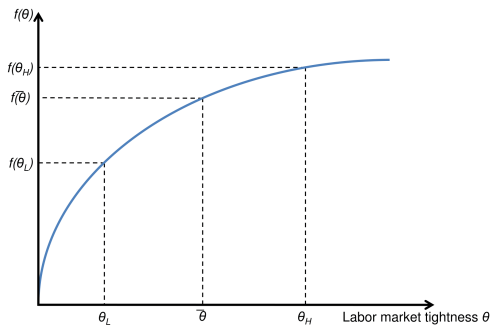
$$\frac{1}{\eta_L} \frac{x}{x - \bar{w}} \tag{2}$$

# Asymmetries in the cycle

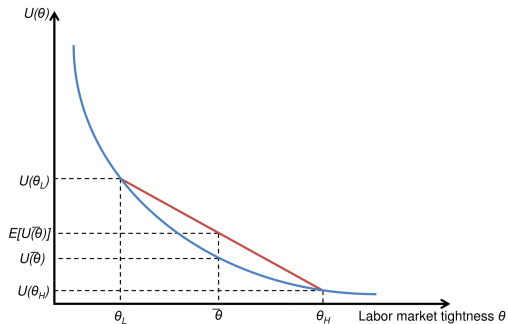
- ▶ Movements in labor market tightness affect the unemployment rate through the job finding rate  $f(\theta_t)$
- ▶ The concavity of the matching function means that increases in unemployment during recessions will be pronounced
- ▶ Declines in unemployment during expansions will be modest



# The Job Finding Rate as a Function of Labor Market Tightness



# Relation of Market Tightness to Unemployment



# Asymmetries in the cycle

- ▶ As economy fluctuates between high and low unemployment, the average rate of unemployment is strictly greater than steady-state unemployment
- ▶ The convexity of unemployment-labor market tightness is due to the congestion in the matching function
- ▶ The job finding rate decreases more as  $\theta$  drops from its steady state to  $\theta_L$  than it increases when  $\theta$  rises to  $\theta_H$

# CALIBRATION

# Calibrated Parameters

- ▶ Assumption: Use the model's stimulated moments, matching them to empirical moments in the calibration process

# Parameter Values in the Monthly US Calibration of the Benchmark Model

	Parameter	Value		Reference or Target:
<b>Technology:</b>				
persistence parameter	$\rho_x$	$0.95^{1/3}$	→	BLS labor productivity
standard deviation	$\sigma_x$	0.00625	→	BLS labor productivity
<b>Labor market:</b>				
job separation rate	$s$	0.035	→	JOLTS
matching curvature	$\nu_L$	1.25	→	DenHannRameyWatsonAER2000
vacancy cost	$\gamma$	0.26	→	Unemployment rate
worker bargaining weight	$\alpha_L$	0.50	→	Equal surplus sharing
nonemploymentnon-employment value	$z$	0.71	→	HallMilgrom2008

# Discounting and Productivity

- ▶ The time discounted rate  $r$  is set to 4% per annum, which is the average return on 3-month US Treasury Bill.
- ▶ Labor productivity  $x$  assumed to follow AR(1) process in logs:

$$\log x_t = \rho_x \log x_{t-1} + \sigma_x \epsilon_t \quad (3)$$

- ▶  $\rho_x \in (0, 1)$
- ▶  $\sigma_x > 0$

# Matching Function Parameters

- ▶ Matching function  $\mathcal{M}_L = \mathcal{V}_t \mathcal{U}_t / (\mathcal{V}_t^{\nu_L} + \mathcal{U}_t^{\nu_L})^{1/\nu_L}$ , where  $\nu_L > 0$ , and set  $\nu_L = 1.25$  (den Haan et al, 2000)
- ▶ Meeting rates  $f(\theta_t)$  and  $q(\theta_t)$  are bounded between 0 and 1



# Flow Value of Unemployment $z$

- ▶ The value of  $z$  involves two components:
  1. a value of leisure and nonmarket activities  $l$
  2. the value of unemployment benefits  $b$
- ▶ Allowing for a leisure component  $l$  in the flow value of unemployment  $z$  permits a calibration of  $z$  above the replacement value of unemployment benefits.

## Job Separation rate $s$

- ▶ Job separation rate ( $s$ ) set to 3.5%, based on the Bureau of Labor Statistic's Job Openings and Labor Turnover Survey (JOLTS).

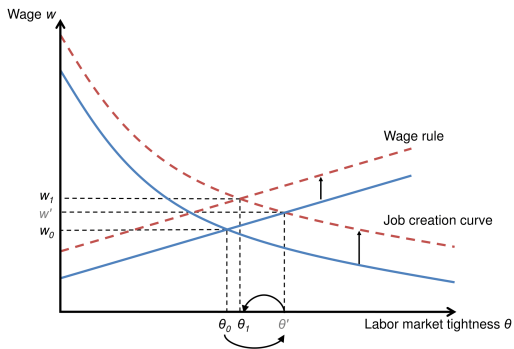
## Vacancy Costs $\gamma$

- ▶  $\gamma$  is set such that the mean rate of unemployment in the model corresponds to the empirical sample mean of 5.80 percent.
- ▶  $\gamma = 0.26$

## Bargaining Weight $\alpha_L$

- ▶ The Nash bargaining weight affects the importance of the time-varying components  $x_t$  and  $\theta_t$  in the wage.
- ▶ A low value of  $\alpha_L$  places most weight on the time-invariant flow value of unemployment  $z$  and the equilibrium wage fluctuates little over the business cycle.
- ▶ A high value of  $\alpha_L$  allows productivity  $x_t$  and labor market tightness  $\theta_t$  to make the wage pro-cyclical
- ▶  $\alpha_L = 0.5$  ; by using a symmetric sharing parametrization in the benchmark model

# The Business Cycle in the Calibrated Model



# The Business Cycle in the Calibrated Model

- ▶ Increase in productivity shifts job creation curve upwards
- ▶ Greater entry in labor market by firms
- ▶ Upward movement along the Nash wage curve from  $\theta_0$  to  $\theta'$
- ▶ Simultaneous increase in wage curve in proportion  $\alpha_I$  of the change in productivity
- ▶ Rise in wages limits amount of entry by firms
- ▶ New equilibrium tightness  $\theta_1$  is below  $\theta'$

# VOLATILITY IN ALTERNATIVE STRUCTURES

## Small Labor Surplus

- ▶ Increase the flow value of unemployment  $z$  up to 0.85. Keep all other parameters identical.
- ▶ New volatility of the labor market tightness is now 0.1 (up from 0.05).
- ▶ Narrowing the gap between the flow value of nonmarket activities and market productivity causes even small changes in productivity to induce proportionally large changes in the surplus from hiring labor.
- ▶ This amplifies the effect of productivity shocks on  $\theta_t$  over the business cycle by a factor of two



# Credible Bargaining

- ▶ Leads to less volatile wages
- ▶ Wage becomes less responsive to current labor market tightness
- ▶ Additional parameters:
  1. the negotiation breakdown probability  $\varphi = 0.1$
  2. the cost of delaying to the firm  $\zeta = 0.25$
  3. reduce the flow value of unemployment to 0.60 such that  $\zeta + z = 0.85$

# Entry Costs and Amplification

- ▶ Assume a job creation cost structure with a fixed post-match creation cost  $C > 0$  to be paid upon hiring a worker
- ▶ The resulting average cost of recruiting,  $\gamma/q(\theta_t) + C$ , is now augmented with  $C$
- ▶ Job creation condition with fixed entry costs:

$$\frac{\gamma}{q(\theta_t)} + C = \frac{1}{1+r} \mathbb{E}_t \left[ x_{t+1} - w_{t+1} + (1-s) \left( \frac{\gamma}{q(\theta_{t+1})} + C \right) \right] \quad (4)$$

## Entry Costs and Amplification

- ▶ Under assumption of a fixed wage the log-linearization of the job creation condition with extra costs is:

$$\hat{\theta}_t = \frac{1}{\eta_L} \times \frac{x}{x - \bar{w} - (r + s)C} \times \frac{r + s}{1 + r} \mathbb{E}_t \left[ \sum_{i=0}^{\infty} \left( \frac{1 - s}{1 + r} \right)^i \hat{x}_{t+1+i} \right] \quad (5)$$

- ▶ Amplification due to labor frictions with fixed wage and entry cost C:

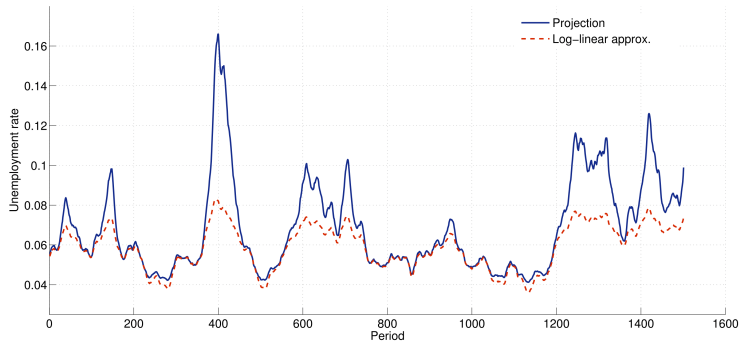
$$\frac{1}{\eta_L} \frac{x}{x - \bar{w} - (r + s)C} \quad (6)$$

# NONLINEAR DYNAMICS IN THE LABOR MARKET

# Solving Models with Search Frictions

- ▶ In order to determine the inaccuracy from log-linearization around a deterministic steady state for a search and matching model, solve the same model in two different ways:
  1. global (projection) method
  2. local (log-linearization)
- ▶ Then subject both solutions to a same path of labor productivity

# Example of Path of Unemployment



# Example of Path of Unemployment

- ▶ The solid line represents the accurate global model
- ▶ Takes into account the curvature of the matching function for the evolution of unemployment as labor market tightness changes over time
- ▶ Log-linearization model fails during recessions

# Labor market moments - the inaccuracy of local approximations

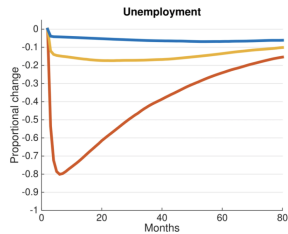
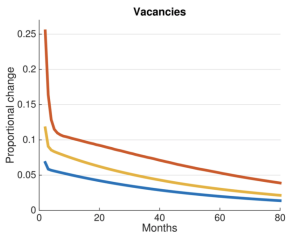
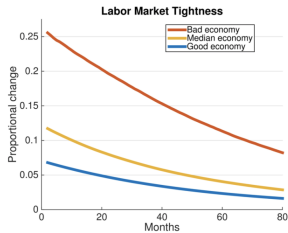
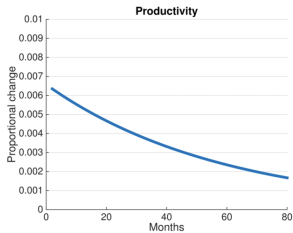
	Log-linear approximation				Global solution method			
	$U$	$\mathcal{V}$	$\theta$	$x$	$U$	$\mathcal{V}$	$\theta$	$x$
Standard deviation	0.108	0.115	0.291	0.013	0.291	0.158	0.216	0.013
Correlation matrix		-0.861	-0.818	-0.934	$U$	-0.496	-0.542	-0.593
			0.817	0.986	$\mathcal{V}$		0.841	0.873
				0.842	$\theta$			0.992



## Labor market moments - the inaccuracy of local approximations

- ▶ The first set of columns follow a calibration in the assumption of increasing the value of nonemployment  $z$  and it is a log-linearization approximation that is solved.
- ▶ The second set of columns uses a global method
- ▶ The volatility of unemployment is 3 times greater in panel B than in panel A
- ▶ the correlation between unemployment and vacancies is nearly halved

# Impulse Response Functions



- ▶ Consider a shock during a midpoint of a business cycle when unemployment is at its median
- ▶ The strongest increase in labor market tightness  $\theta_t$  is contemporaneous to the shock, returning to trend at the same rate as labor productivity
- ▶ Job vacancies exhibit a pronounced decline following the initial jump, then a progressive return to trend
- ▶ The rise in market tightness pushes job creation above job destruction, and the unemployment rate declines
- ▶ Job vacancies at a given level of labor productivity are increasing in unemployment
- ▶ During the first several months following the productivity shock, job vacancies are also responding to the sharp decline in unemployment
- ▶ In the subsequent months unemployment and job vacancies follow a similar paths back towards their trends

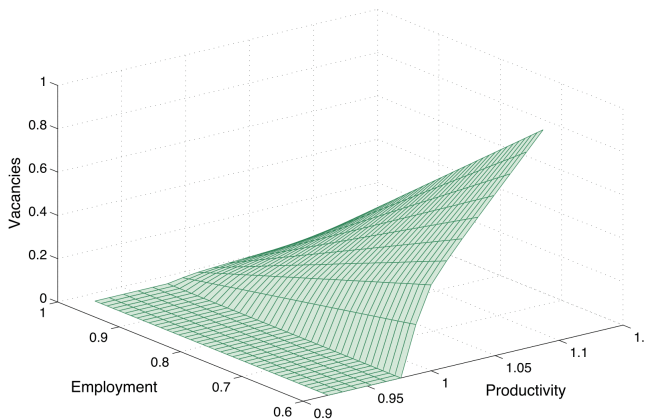
# Impulse Response Functions

- ▶ Job vacancies and unemployment are complementary in producing new meetings per unit of time
- ▶ The dynamics of the labor market are more sensitive to shocks when the labor market is slack than when it is tight

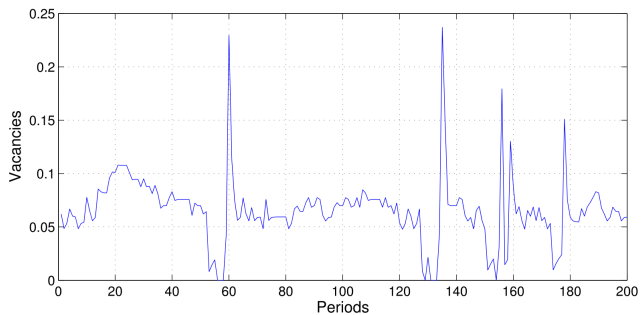
## Other nonlinearities: the small surplus assumption and the zero bound for vacancies

- ▶ Firms enter the labor market posting job vacancies as long as the expected benefit from filling the job exceeds the cost of posting a vacancy.
- ▶ If the cost  $\gamma$  is greater than  $\mathbb{E}_t[J_{\pi t+1}]$ , then there is no surplus to entering and the result is a shutting down of hiring with  $\mathcal{V}_t = 0$ .

# Job Vacancies in $(x, \mathcal{N})$ space



# Model Job Vacancies Time Series



# Model Job Vacancies Time Series

- ▶ First 50 periods are standard response of model to stochastic change in productivity
- ▶ Series of negative shocks pushes productivity under its boundaries
- ▶ Corner solution  $\nu_t = 0$  for several periods
- ▶ Existing jobs continue to be destroyed at rate  $s$  and there is a sharp rise in unemployment
- ▶ When productivity finally returns to a value such that firms enter the labor market once again, employment is very low and there is massive entry by firms into the market.