Chapter 5
Credit and Labor Market Frictions: the CL Model

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Outline

1. Random matching in financial markets
2. Integrating labor and financial market frictions
3. Efficiency and Hosios in the financial market

Random matching in financial markets

The Hosios Condition with Exogenous Separation

- Number of creditors ready to finance a project: $\mathcal{B}_c$
- Number of projects looking for liquidity: $\mathcal{N}_c$
- Subscript c stands for the search in the credit market by both the owner of the project and a creditor
- The rate at which the project meets the creditor: p
- the rate at which the creditor meets and implicitly both screens and accepts the project: $\check{p}$

The Hosios Condition with Exogenous Separation

- There is an identity in the financial market between the total number of matches, $\mathcal{M}_C$, the number of matched projects, and the number of matched creditors in a unit of time:

$$\mathcal{M}_C \quad = p\mathcal{N}_c = \quad \check{p}\mathcal{B}_c \tag{1}$$

- Ratio of projects to creditors searching in the credit market or *financial market tightness*:

$$\phi = \frac{\mathcal{N}_c}{\mathcal{B}_c} \tag{2}$$

- p and $\check{p}$ are necessarily linked through ϕ:

$$\check{p} = \phi p$$

The Hosios Condition with Exogenous Separation

- Assume a constant returns to scale matching function with the mass of investment projects and creditors searching in the financial market as arguments:

$$\mathcal{M}_C(\mathcal{N}_c, \mathcal{B}_c) \quad \text{with} \quad \partial \log \mathcal{M}_C / \partial \log \mathcal{B}_c = \eta_C(\phi) \qquad (3)$$
$$\text{and} \quad \partial \log \mathcal{M}_C / \partial \log \mathcal{N}_c = 1 - \eta_C(\phi)$$

- $\eta_C(\phi)$ is the elasticity of matching in the financial market with respect to searching creditors

The Hosios Condition with Exogenous Separation

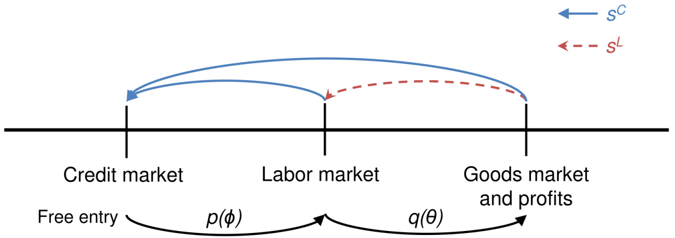
- The transition rates for investment projects and creditors are given by:

$$\frac{\mathcal{M}_C(\mathcal{N}_c,\mathcal{B}_c)}{\mathcal{N}_c} = p(\phi) \quad \text{with p'}(\phi)\text{¡0}$$

$$\frac{\mathcal{M}_C(\mathcal{N}_c,\mathcal{B}_c)}{\mathcal{B}_c} = \check{p}(\phi) = \phi p(\phi) \quad \text{with } \check{p}'(\phi) > 0$$

Integrating Labor and Financial Market Frictions

Markets and Transitions of the Firm



The Hosios Condition with Exogenous Separation

- In contrast with the benchmark model the posting of a vacancy needs to be externally financed
- To pay for the cash outflow γ, the investment project requires liquidity from a creditor
- The financial contract will specify that the creditor commits to paying this cost γ until the project has found the worker
- In return, the project will provide a flow return ψ to the creditor once the firm produces

Integrating Labor and Financial Market Frictions

- the sum of s^L and s^C is a *total* turnover rate
- $s^T = s^L + s^C$
- Assume $s^L = 0$

Integrating Labor and Financial Market Frictions

- $1 - \mathcal{U} = \mathcal{N}_\pi = \mathcal{B}_\pi$
- The rate of employment $1 - \mathcal{U}$, is equal to the stock of firms and creditors in the profit stage, $\mathcal{N}_\pi$ and $\mathcal{B}_\pi$, respectively
- Number of job vacancies the number of creditors in stage v
- $\mathcal{V} = \mathcal{N}_v = \mathcal{B}_v$

Integrating Labor and Financial Market Frictions

- Creditors funding project:

$$= \ \phi p(\phi)\mathcal{B}_c - q(\theta)\mathcal{V}(4)$$

- Vacant Jobs:

$$= \ \mathrm{p}(\phi)\mathcal{N}_c + s^L\mathcal{N}_\pi - \left[q(\theta) + s^C\right]\mathcal{V}(5)$$

- Profit Generating Jobs:

$$_\pi = q(\theta)\mathcal{V} - \left(s^C + s^L\right)\mathcal{N}_\pi(6)$$

- Unemployed workers:

$$= \ \left(s^C + s^L\right)\mathcal{N}_\pi - q(\theta)\mathcal{V}(7)$$

Projects Creditors and "the firm"

- Bellman equations for each stage:

$$
\begin{aligned}
rE_c &= -\kappa_I + p(\phi)\,(E_v - E_c) && (8)\\
rE_v &= -\gamma + \gamma + q(\theta)\,(E_\pi - E_v) + s^C(E_c - E_v) && (9)\\
rE_\pi &= x - w - \psi + s^C\,(E_c - E_\pi) + s^L(E_v - E_\pi) && (10)
\end{aligned}
$$

- Bellman equations for creditors:

$$
\begin{aligned}
rB_c &= -\kappa_B + \phi p(\phi)\,(B_v - B_c) && (11)\\
rB_v &= -\gamma + q(\theta)\,(B_\pi - B_v) + s^C(B_c - B_v) && (12)\\
rB_\pi &= \psi + s^C\,(B_c - B_\pi) + s^L(E_v - E_\pi) && (13)
\end{aligned}
$$

Entry in credit and labor markets

- Free entry in the market implies that $E_c = 0$ and $B_c = 0$
- Value of a project waiting to start production once a worker has been hired $= E_v$
- Expected costs of search in the credit market: $\kappa_I / p(\phi)$
- Projects will want to enter the market as long as $E_v > \kappa_I / p(\phi)$
- Value of being matched $= B_v$
- Expected cost of searching $= \kappa_B / \check{p}(\phi)$
- Creditors enter the market until $B_v = \kappa_B / \check{p}(\phi)$

Entry in credit and labor markets

- Value of being matched $= B_v$
- Expected cost of searching $= \kappa_B/\check{p}(\phi)$
- Creditors enter the market until $B_v = \kappa_B/\check{p}(\phi)$

Entry in credit and labor markets

- Applying the free entry condition to the values of searching the financial market leads to:

$$E_v = \frac{\kappa_I}{p(\phi)} \quad \text{and} \quad B_v = \frac{\kappa_B}{\check{p}(\phi)}$$

Entry in credit and labor markets

- $J_c = E_c + B_c$
- $J_v = E_v + B_v$
- $J_\pi = E_\pi + B_\pi$
-

$$\left(r + s^C\right) J_v = -\gamma + q(\theta)\left(J_\pi - J_v\right) \quad (14)$$

$$\left(r + s^C\right) J_\pi = x - w + s^L\left(J_v - J_\pi\right) \quad (15)$$

Entry in credit and labor markets

- free entry in the financial market leads to an equilibrium value of a vacancy to the firm:

$$J_v = \frac{\kappa_B}{\phi p(\phi)} + \frac{\kappa_I}{p(\phi)} \equiv K(\phi). \tag{16}$$

- $K(\phi)$ = the sum of all frictional costs in the financial market

Entry in credit and labor markets

- Value of the profit stage to the firm:

$$J_\pi = \frac{x - w + s^L K}{r + s^C + s^L}$$

(17)

- New job creation condition in the presence of financial market frictions:

$$K(\phi)\left(1 + \frac{r + s^C}{q(\theta)}\right) + \frac{\gamma}{q(\theta)} = \frac{x - w + s^L K}{r + s^C + s^L} \qquad (18)$$

Entry in credit and labor markets

- Job creation condition in the presence of financial market frictions for a given wage:

$$\frac{\gamma_k}{q(\theta)} = \frac{x^{CL} - w}{r + s^C + s^L} \tag{19}$$

$$\text{with} \qquad \gamma_k = \gamma + k(\phi)$$

$$\text{and} \qquad x^{CL} = x - k(\phi)$$

Bargaining over credit

- Presence of frictions in financial markets implies the existence of a positive surplus to a match between a creditor and a project.
- The repayment is represented by ψ
- ψ is the solution to the following Nash bargaining game:

$$\psi = \text{argmax}(B_v - B_c)^{\alpha_C}(E_v - E_c)^{1-\alpha_C}$$

- $\alpha_C \in (0, 1)$

Bargaining over credit

- $\partial B_\pi / \partial \psi = -\partial E_\pi / \partial \psi = 1/(r + s^L + s^C)$
- $\partial B_v / \partial \psi = -\partial E_v / \partial \psi = Q_L \times 1/(r + s^L + s^C)$.
- The Nash sharing rule for ψ can be written as:

$$(1 - \alpha_C)(B_v - B_c) = \alpha_C (E_v - E_c) \tag{20}$$

- The sharing rule (20) is rearranged as:

$$B_v = \alpha_C J_v \text{ and } E_v = (1 - \alpha_C) J_v. \tag{21}$$

Bargaining over credit

- Equilibrium financial market tightness and repayment:

$$\phi^* = \frac{\kappa_B}{\kappa_I} \frac{1 - \alpha_C}{\alpha_C} \tag{22}$$

$$\psi = \alpha_C(x - w) + (1 - \alpha_C) \frac{\gamma(r + s^C + s^L)}{q(\theta)} \tag{23}$$

Block bargaining over wages

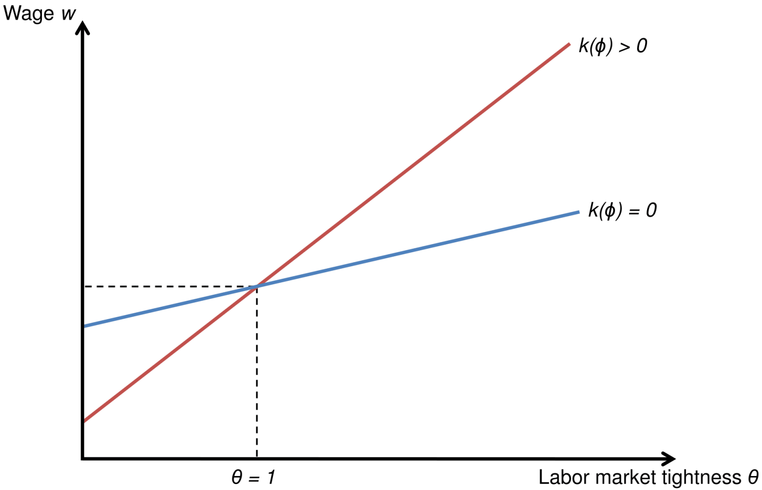
- Block bargained Nash wage in the presence of financial market frictions:

$$w = (1-\alpha_L)\,z + \alpha_L\left(x^{CL} + \gamma_k\theta\right) \quad (24)$$

or

$$w = (1-\alpha_L)\,z + \alpha_L\,(x + \gamma\theta) + (\theta - 1)\alpha_L k(\phi) \quad (25)$$

Nash-bargained wage curve in the CL model



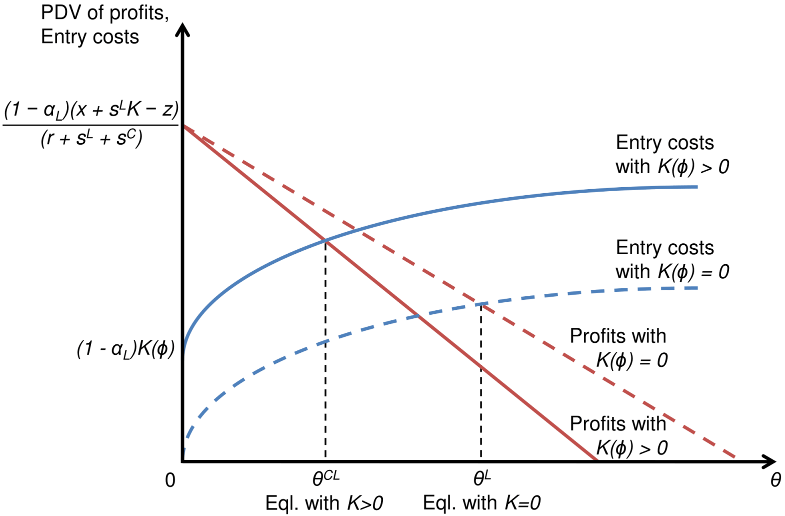
Nash-bargained wage curve in the CL model

- When the labor market is tight, workers' threat point is high and they can take advantage of financial frictions to raise wages
- The positive slope effect dominates if and only if $\theta > 1$

Equilibrium

- Summary of the equilibrium (ϕ, θ) in the model with both financial and labor market frictions:
  1. Credit market tightness:
  $$\phi^* = \frac{\kappa_B}{\kappa_I}\frac{1-\alpha_C}{\alpha_C}$$
  2. Labor market tightness:
  $$\frac{\gamma_k}{q(\theta^*)} = \frac{(1-\alpha_L)(x^{CL}-z)-\alpha_L\theta^*\gamma_k}{r+s^C+s^L}$$
  3. Wage:
  $$w = (1-\alpha_L)\,z + \alpha_L\left(x^{CL}+\gamma_k\theta\right)$$
  4. Adjusted Productivity:
  $$x^{CL} = x - k(\phi) = x - (r+s^C)K(\phi)\ ;\ K(\phi) = \frac{\kappa_B}{\phi p(\phi)} + \frac{\kappa_I}{p(\phi)}$$
  5. Adjusted labor entry costs:
  $$\gamma_k = \gamma + k(\phi)$$

Nash-bargained wage curve in the CL model



Equilibrium

- Labor market tightness in the CL model is decreasing in $K(\phi)$Â§
- Total costs must satisfy the following inequality:

$$\mathrm{K}(\phi^*) < \frac{x + s^L K(\phi^*) - z}{r + s^C + s^L} \quad \text{or simply} \quad K(\phi^*) < \frac{x - z}{r + s^C}$$

Equilibrium

- When $K \to 0$ the model converges to the conventional equation of the benchmark model
- Defines a level of labor market highness θ^L
- This level of labor market tightness is always higher than in the presence of frictions in the financial market
- The equilibrium rate of unemployment in the presence of friction in the credit market is always greater than in the benchmark model.

Efficiency and Hosios in the financial market

Social planner's problem with frictional labor and credit markets

- The social planner's problem:

$$\Omega^{SP} = \max_{\mathcal{B}_c, \mathcal{N}_c, \mathcal{U}, \mathcal{V}} \int_0^\infty e^{-rt}[x(1-\mathcal{U}) + z\mathcal{U} - \gamma\mathcal{V} - \kappa_B \mathcal{B}_c - \kappa_I \mathcal{N}_c]dt$$

$$\text{s.t.} \quad = \left(s^C + s^L\right)(1-\mathcal{U}) - \mathcal{M}_L(\mathcal{U}, \mathcal{V})$$

$$= \mathrm{M}_C(\mathcal{N}_c, \mathcal{B}_c) + s^L(1-\mathcal{U}) - \mathcal{M}_L(\mathcal{U}, \mathcal{V}) - s^C \mathcal{V}$$

- $\mathcal{N}_c$ and $\mathcal{B}_c$ are control variables
- $\mathcal{U}$ and $\mathcal{V}$ are state variables

Social planner's problem with frictional labor and credit markets

- Constraints on the planner:
  1. Law of motion for unemployment
  2. Law of motion for job vacancies

Social planner's problem with frictional labor and credit markets

- Denote Ψ_i for $i = U, V$
- Hamiltonian to the problem:

$$
\begin{aligned}
\mathrm{H} \;=\; & e^{-rt}\left[x(1-\mathcal{U}) + z\mathcal{U} - \gamma\mathcal{V} - \kappa_B\mathcal{B}_c - \kappa_I\mathcal{N}_c\right] + \\
& \Psi_U\left[\left(s^C + s^L\right)(1-\mathcal{U}) - \mathcal{M}_L(\mathcal{U},\mathcal{V})\right] \\
& +\Psi_V\left[\mathcal{M}_C(\mathcal{N}_c,\mathcal{B}_c) + s^L(1-\mathcal{U}) - \mathcal{M}_L(\mathcal{U},\mathcal{V}) - s^C\mathcal{V}\right]
\end{aligned}
$$

Social planner's problem with frictional labor and credit markets

- First order conditions for the financial market are:

$$\begin{aligned}
\partial H/\partial \mathcal{B}_c = 0 \quad &\rightarrow \quad e^{-rt}(-\kappa_B) - \Psi_V \phi p(\phi)\eta_C = 0 \\
\partial H/\partial \mathcal{N}_c = 0 \quad &\rightarrow \quad e^{-rt}(-\kappa_I) - \Psi_V p(\phi)(1-\eta_C) = 0 \\
\dot{\Psi}_U = -\partial H/\partial \mathcal{U} = \quad &\rightarrow \quad \dot{\Psi}_U = e^{-rt}(x-z) + \Psi_U \left[s^C + s^L + \eta_L \theta q(\theta)\right] + \Psi_V[\eta_L \theta \\
\dot{\Psi}_V = -\partial H/\partial \mathcal{V} = \quad &\rightarrow \quad \dot{\Psi}_V = e^{-rt}\gamma + \Psi_U(1-\eta_L)q(\theta) + \Psi_V(1-\eta_L)q(\theta) + s^{C}
\end{aligned}$$

Socially optimal credit and labor market tightness

- Socially optimal level of credit market tightness:

$$\phi^{opt} = \frac{1-\eta_C}{\eta_C}\frac{\kappa_B}{\kappa_I} \tag{30}$$

Socially optimal credit and labor market tightness

- Socially optimal level of labor market tightness:

$$\frac{\gamma_k^{opt}}{q(\theta^{opt})} = \frac{(1-\eta_L)(x^{CL,opt} - z) - \eta_L \theta^{opt} \gamma_k^{opt}}{r + s^C + s^L} \tag{31}$$

$$\text{with} \quad \gamma_k^{opt} = \gamma + k(\phi^{opt})$$

$$\text{and} \quad x^{CL,opt} = x + k(\phi^{opt})$$

Hosios in credit and labor markets

- Social planner and decentralized job creation conditions with financial market frictions:

Social planner:

$$\frac{\gamma_k^{opt}}{q(\theta^{opt})} = \frac{(1-\eta_L)\left(x^{CL,opt} - z\right) - \eta_L \theta^{opt} \gamma_k^{opt}}{r + s^C + s^L}$$

$$\phi^{opt} = \frac{1-\eta_C}{\eta_C} \frac{\kappa_B}{\kappa_I}$$

Decentralized:

$$\frac{\gamma_k}{q(\theta^*)} = \frac{(1-\alpha_L)\left(x^{CL} - z\right) - \alpha_L \theta^* \gamma_k}{r + s^C + s^L}$$

$$\phi^* = \frac{1-\alpha_C}{\alpha_C} \frac{\kappa_B}{\kappa_I}$$

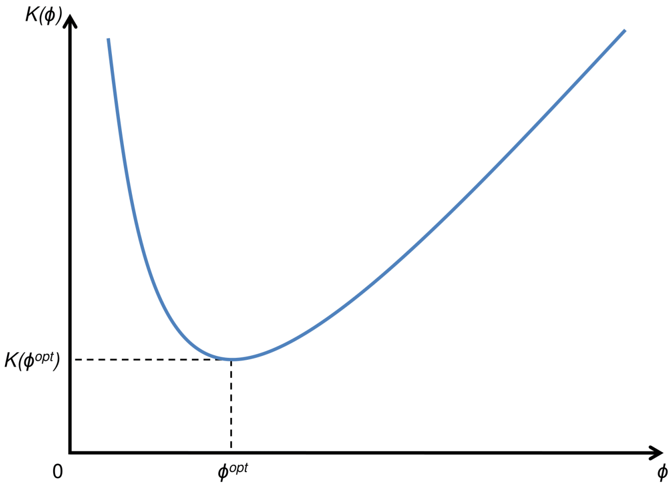
Hosios in credit and labor markets

- Hosios constrained efficiency conditions in the CL model:

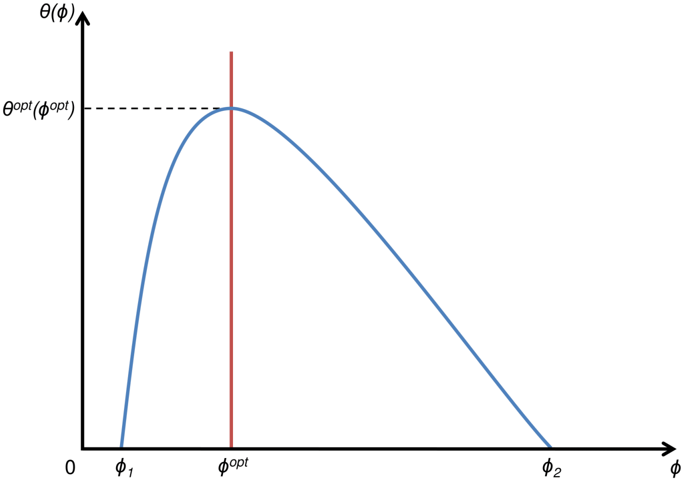
$$\phi^* = \phi^{opt} \quad \text{if and only if} \quad \alpha_C = \eta_C(\phi^{opt})$$

$$\theta^* = \theta^{opt} \quad \text{if and only if} \quad \alpha_L = \eta_L(\theta^{opt}) \text{ and } \phi^* = \phi^{opt}$$

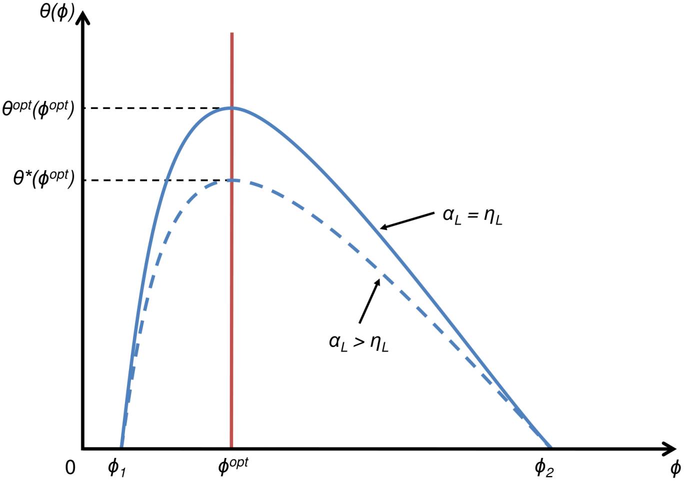
Total credit market transactions costs as a function of credit market tightness



Labor market tightness as a function of credit market tightness



Deviations from Hosios in the labor market



Deviations from Hosios in the credit market