UNIVERSITÀ DI TORINO

The Baseline Matching Model: Job Creation

[Sem0057]

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Contents

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$F(K; L) = Y$

1 Introduction

- What is a Matching Function/Technology
  - It is a technology that describes how meeting take place in the market
  - It is somewhat similar in spirit to the production technology that describes how combined input generate output.
  - The key idea in the SAM literature is that matching technology can be described by a simple aggregate functions of few variables.

2 The Matching Function

- Production Function describes how input are combined to obtained a given output. It is a tool box of any introductory econ student
- Matching Function it describes how **meetings** take place among market participants. It is a key tool box for any SAM scholar and student.
  - It is a bit of black box as much as the production function is in production theory.
  - The key idea is that meetings among market particiapnts are a time consuming process.
  - The number of matches per unit of time is mL, (most of the time $L = 1$ is the size of the labor force)
  - m is thus the number of meeting per unit of time
  - time is continuous and we are in stationary environment

m L

- Key assumption: mL of m depends on very few variables
- There are two types of searching agents. We focus on the labor market
  - **Workers**. The unemployed is a worker searching for a job
  - **Firms**. A Vacant firms is a firm searching for a worker.

- The Basic Matching Function

$$\boxed{mL = m(uL, vL)}$$

with

  - u is the stock of unemployed
  - v is the stock of vacancies.
  - So that meetings depends on the absolute number of searchers.
  - u is a measure of job seekers' effort (read unemployment rate) while v is a measure of firm's recruiting effort (read vacancy rate, the number of vacancies normalized by the labor force).

- Basic Assumptions

$$\boxed{m < min(u, v)}$$

which implies that somebody goes "unmatched" ina unit of time.

- Typically there are more assumptions on the matching function

  - Positive first derivatives:
$$\boxed{m_u > 0;} \quad \text{and} \quad m_v > 0$$
it makes a lot of sense. The more searcher of one ype, the more meeting in the markets
  - Negative Second derivatives
$$\boxed{m_{uu} < 0;} \quad \text{and} \quad \boxed{m_{vv} \cancel{>} 0} \quad <$$
it is in line with diminishing returns in economics. At the margin the contribution of additional searchers falls.
  - There are CRS to matching
$$\Delta m = m(\Delta u, \Delta v)$$
so that the size of the market is irrelevant (a nice feature for cross country comparison). CRS is also "empirically plausible".
  - Sometime we also assume Inada conditions.
$$lim_{u \to 0} m_u = lim_{v \to 0} m_v = \infty$$
and
$$lim_{u \to \infty} m_u = lim_{v \to \infty} m_v = 0$$

Transition Rates

- Given the function m and its asumption we can derive transition rates
- Let0s take the firm standpoint. What is the probability that a vacant firm meets an unemployed.
- Let $q()$ be the rates at which vacancies are filled

$$q() = \frac{m(u,v)}{v} = m(\frac{u}{v}, 1)$$

$\frac{u}{v}$ Relative # OF [illegible]

- The key variable in the matching enviroment is **Market Tightness**

  Let

$$\boxed{\theta = \frac{v}{u}}$$

  so that Think of market tightness as to how tight is the market form the firm stand point. If θ is high, the search ~~environment is such that there~~ are many searcher of the firm type

- The probability that a firm meets a worker - GIVEN CRS- depends only on θ

$$q(\theta) = m(\frac{u}{v}; 1) \qquad q'(\theta) < 0$$

- We can make the Cobb douglas case right away. If Cobb-Douglas

$$m(u,v) = v^{1-\alpha}u^{\alpha}$$

  and

$$q(\theta) = \frac{v^{1-\alpha}u^{\alpha}}{v} = (\frac{v}{u})^{-\alpha} = \theta^{-\alpha}$$

$q(\theta) = \theta^{-\alpha}$ $\qquad$ $q'(\theta) = -\alpha\,\theta^{-\alpha-1} < 0$

$\alpha(\theta) = \theta q(\theta)$

- The rates at which workers find jobs is

$\theta q(\alpha)$

$$\alpha(\theta) = \frac{m(u,v)}{u} = \frac{v}{u}\frac{m(u,v)}{v} = \theta q(\theta) \qquad \alpha'(\theta) > 0$$

  - Since

$$\alpha'(\theta) = q(\theta) + q'(\theta)\theta =$$

  - and collecting $q(\theta)$

$$= q(\theta)[1 + \frac{q'(\theta)\theta}{q(\theta)}] = q(\theta)[1 - \eta(\theta)] > 0$$

    where

$$\eta(\theta) = \epsilon_{q,\theta} \left| \frac{q'(\theta)\theta}{q(\theta)} \right| < 1$$

  - with CRS in matching the elasticity of the matching function with respect to θ is less than one in absolute value.

- With a Cobb Douglas matching function

$$\boxed{m(u,v) = u^{\alpha}v^{1-\alpha}}$$

- Given market tightness $\theta = \frac{v}{u}$

$$q(\theta) = \left(\frac{u^{\alpha}v^{1-\alpha}}{v}\right) = \left(\frac{u}{v}\right)^{\alpha} = \theta^{-\alpha}$$

- Note that

$$q'(\theta) = -\alpha\theta^{-\alpha-1} < 0$$

$\theta q(\alpha) = \theta\theta^{-\alpha} = \theta^{1-\alpha}$

- With CObb DOuglas the elasticity $\eta(\theta)$ is also independent of θ and is equal to α.

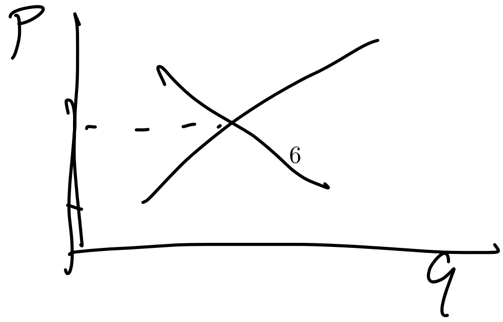
$$\alpha(\theta) = \theta^{(1-\alpha)}$$

- and

$$\eta(\theta) = \left| \underbrace{-\alpha\theta^{-\alpha-1}}_{q'(\theta)} \frac{\theta}{\theta^{-\alpha}} \right| = \alpha$$

$q(\theta)$

$$\frac{\partial \alpha(\theta)}{\partial \theta} = (1-\alpha)\theta^{-\alpha} > 0$$



2.1 Discussion and Implications

- How should $q(\theta)$ be interpreted?
  - $q(\theta)dt$ is the arrival rate of offer, a Poisson Process with arrival rate $q(\theta)$, so that $q(\theta)dt$ is the probability that a firm meets a workers
  - Note that $0 \leq q\theta) \leq +\infty$ since arrival rate can be infinitly large.
  - What has to bo bounded is the probabilty tha ta firm meets a worker

$$0 \leq q(\theta)dt \leq 1$$

- There is **stochastic rationing** (can not be eliminated by price adjustment):

$$(1 - q(\theta)dt) > 0$$

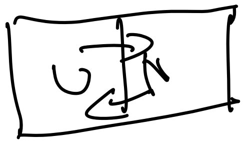
and (

$$1 - q(\theta)dt) > 0$$

.

- There are **trading externalities** (transition rates depends on the relative numbers of agents
  - If one type of the opposite side of the math enter the market, the probability of being mathced increases (we will call it the thick/thin market externality).
  - If one type of the same side of the math enter the market, the probability of being mathced increases. (we will call it the congestion externality)
  - THis is very important in the mating game in nature. Also relevant in marriage markets
  - This will be key as weill speak about efficeincy.

λdt



$u + n = 1$

2.2 Unemployment Flows

- Labor force is constant and normalized to 1

$$\boxed{u + n = 1}$$

15-64

where u is the unemployment rate and n is employment.

- The labor force is thus fixed. Remember that the Working Age Population is defined as the population between the age of 16 and 64 that cna be in one of three states

$$WAP = u + n + \cancel{o} \tag{1}$$

where o is the stock of people out of the labor force. We are thus assuming $\cancel{o = 0}$. Garibaldi and Wassmer (2005) conside a mathcing model with also flow in and out of the labor force.

- In the matching model the stock of unemployment is derived by the flows. It is the famous "bath-tab" approach to stock. When is the bath water constant? When inflows are equal to outflows.
  - Steady state unemployment: inflows=outflows.
- We need some assumptions on unemployment inflows.
- In the simple model unemployment inflows are equal to total job destruction.
  - Jobs are destroy at an exogenous rate equal to λ, where λ is the arrival rate (Poisson) of idiosyncratic shocks.
  - λ ~~sdt~~ is the probability of destruction.
  - Total Jobs Destruction

$$JD = \cancel{s}\,\lambda(1 - u)$$

λn

$\lambda(1-u)$

- How many meeting take place in a given time interval?
  - How about unemployment outflows?
  - The simple idea is that **any meeting** in the labor market leads to a new jobs.
  - $$JC = \theta q(\theta)u$$
    which corresponds to the number of meeting formed by the matching function.
  - $$JC = \frac{m(u, v)}{u}u = \theta q(\theta)u$$
- There is no on the job search

$$JC = m(u;v) = \frac{m(u;v)}{u}u$$

$$JC = \theta q(\theta) u$$

$u\,q(\theta)u = \lambda - \lambda u$

- What is the dynamics of unemployment

$$\dot{u} = \frac{du}{dt} = \text{Unemployment Inflows} - \text{Unemployment outflows}$$

$$\dot{u} = \frac{du}{dt} = \text{Job Destruction} - \text{Job Creation}$$

- Unemployment is constant if

$$uq(\theta)u = \lambda(1-u)$$

which implies

$$u = \frac{\lambda}{\lambda + \theta q(\theta)}, \tag{UV-KEY1}$$

where θ is one of the key endogenous variables and λ is an exogenous parameter.

- This is a negative relationship between u and θ. Alternatively, in the $u-v$ space the unemployment relationship is

$$\lambda(1-u) = x(u,v)$$

.

$\lambda(1-u) = x(u,v)$

This is called **Beveridge curve**.

- The **Beverdige Curve** is the combination of $v-u$ for which unemployment is constant.

Totally differentiating yields

$$-\lambda du = x_u du + x_v dv$$

which implies

$$\frac{dv}{du} = -\frac{\lambda + x_u}{x_v} < 0$$

- Is it convex?

$$\frac{d^2v}{du^2} = -\frac{\overbrace{m_{uu}m_v}^{-\times+=-} - \overbrace{m_{vu}m_u}^{+\times+=+}}{m_{vv}^2} = -\frac{-}{+} = + \tag{2}$$

it is convex if $m_{uv} > 0$ so that the cross derivative of the vacancy and unemployment rate is postive.

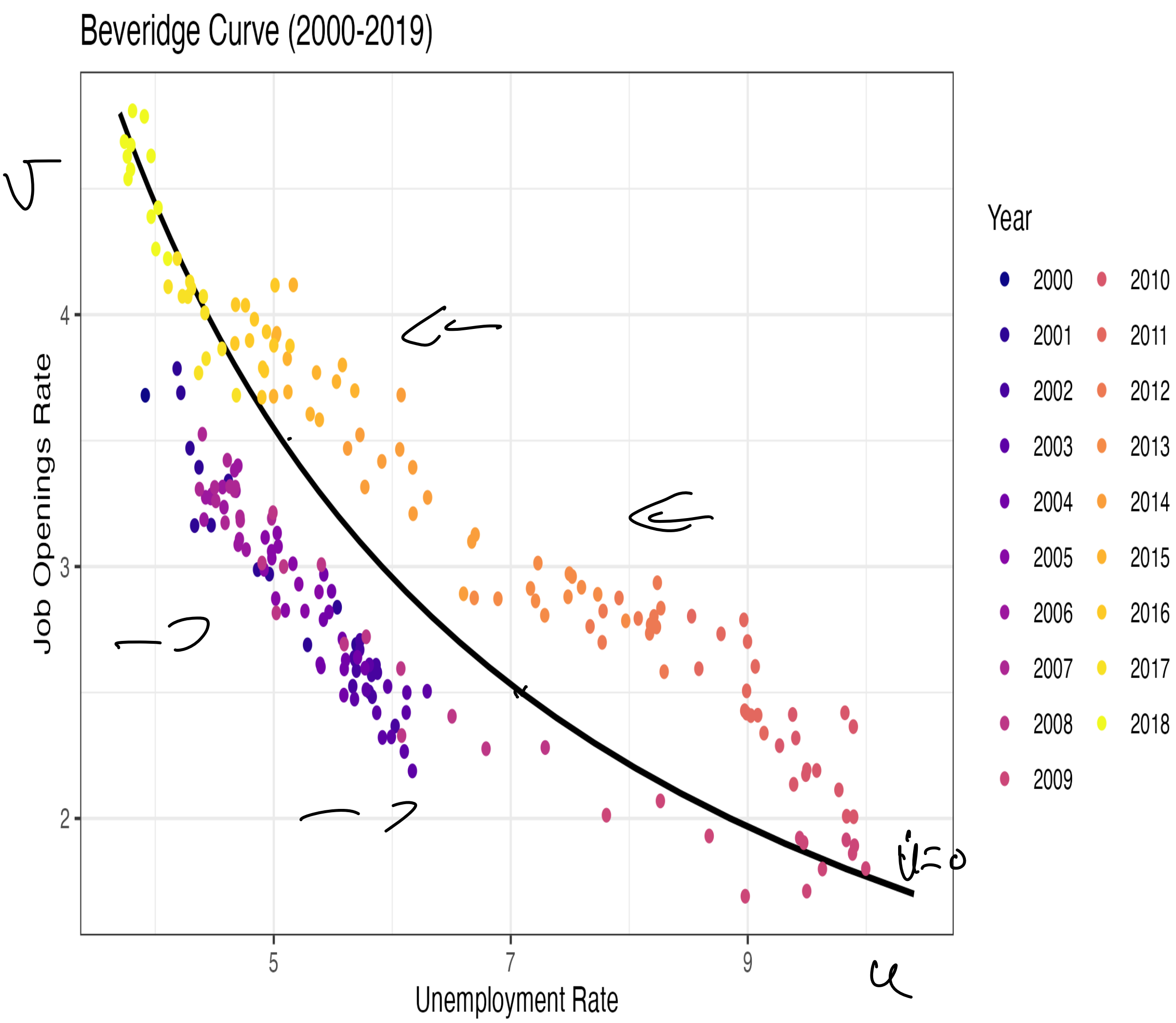


Figure 1: The Actual Beveridge CUrve from the US

- We now have the first key equation of equilibrium unemployment theory.

$$u = \frac{\lambda}{\lambda + \theta q(\theta)}, \tag{UV-KEY1}$$

  where θ is one of the key endogenous variables and λ is an exogenous parameter.

- From the previous equation we need three elements

  1. λ is an exogenous parameter the govern job destruction
  2. The function $q()$ is the matching function
  3. Market tightness $\theta = \frac{v}{u}$ is the key endogenous variable

- Once we have θ we can get unemployment from the beveridge curve as $u = u(\theta)$

- The basic model derives θ from value functions for workers and firms