UNIVERSITÀ DI TORINO

The Baseline Matching Model: Efficiency

[Sem0057]

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1 Introduction

- Suppose that in the equilibrium $\theta \uparrow$
- What happens to welfare of the workers?
  - Workers' welfare goes up, since it's easier to find a job.
  - THis is an exmple of a **thick market externality**
- What happens to firms' welfare?
  - welfare (profits) of the firms fall, as it is harder to find a worker
  - This is an example of **congestion externality**
- In the search equilibrium there are trading externalities, since agents utility depends on the relative number of traders in the market. An additional participant on the same side decreases an individual effort (*congestion effect*) while an additional participant of the other side increase welfare (*strategic complements*)
- Does the decentralized market wage internalize these externalities ?
- The natural question to ask in whether the unemployment resulting from the decentralized equilibrium is constrained-efficient, in the sense that it maximizes net output of the aggregate economy.
- We will learn that in general the answer to this question is no. Yet, search scholars discover tha under particular conditions (eventually known in the literature as the Hosios conditions), the decentralized equilibrium can be efficient.

- To answer those questions we need a measure of social welfare
- The key concept is the **Constrained Central Planner**
  - It is a central planner that can move around people with "brute force" (without the makret) but it is constrained in using the matching function.
- We do this in several steps
  1. Social welfare (SW) in a model with exogenous job destrcution and $r = 0$
  2. Social Welfare with exogenous job destruction but a fully forward looking model
  3. Social welfare with endogenous job destruciton (we will get back once we are done with endogenous job destruction)

2 A First Take on the Hosios COnditions: A Micky Mouse Basline Model

- We begin with a measure of social welfare when $r = 0$
- The concept is net output in steady state with $r = 0$

$$SS = \underbrace{(1-u)p}_{\text{Employed Output}} + \underbrace{ub}_{\text{unemployment output}} - \underbrace{cv}_{\text{Cost of vacancies}} \tag{1}$$

- We can always say that

$$cv \equiv c\theta u; \qquad \text{since} \quad \theta = \frac{v}{u}$$

- Further we have a constraint that

$$u = \frac{s}{s + \theta q(\theta)} \tag{2}$$

- Basically we can set up the problem a constrained optimization problem.

- The problem is

$$Max_{\theta,u} SS = (1-u)p + ub - c\theta u \tag{3}$$

$$\text{s.t.} U = \frac{s}{s+\theta q(\theta)} \tag{4}$$

- The Lagrangean is then

$$Max_{\theta,u,\mu} \mathscr{L} = (1-u)p + ub - c\theta u + \mu\left[u - \frac{s}{s+\theta q(\theta)}\right]$$

- The first order conditions are

$$\frac{\partial \mathscr{L}}{\partial u} = 0; \quad -p + b - c\theta + \mu = 0 \tag{5}$$

$$\frac{\partial \mathscr{L}}{\partial \theta} = 0; \quad -cu + \mu \frac{q(\theta) + \theta q'(\theta)}{(s+\theta q(\theta))^2} s = 0 \tag{6}$$

$$\frac{\partial \mathscr{L}}{\partial \mu} = 0; \quad u = \frac{s}{s+\theta q(\theta)} \tag{7}$$

- Note that in the second equation unemployment can be simplified from both terms so that one has

$$\mu = \frac{q(\theta) + \theta q'(\theta)}{(s+\theta q(\theta))^2} s$$

- From the first equation

$$\mu = (p-b) + c\theta$$

- Substituting out μ one has

$$c(s+\theta q(\theta)) = (p-b)\left(q(\theta)+\theta q'(\theta)\right) + \theta c\left[q(\theta)+\theta q'(\theta)\right]$$

- Let's recall some definition we had on $q(\theta)$

$$\underbrace{\eta(\theta)}_{\text{Absolute Value fo Elasticity of the matching function}} = \left|-\frac{\frac{dq}{q}}{\frac{d\theta}{\theta}}\right| = -\frac{q'(\theta)}{q(\theta)}\theta$$

  - general with CRS matching function
$$0 \leq \eta(\theta) \leq 1$$
  - Further if $q = \theta^{-\alpha}$
  -
$$\eta(\theta) = \alpha; \quad \text{independent of } \theta$$

- Recall also that
$$\frac{\partial \theta q(\theta)}{\partial \theta} = q(\theta) + q'(\theta)\theta = q(\theta)\left[1+\frac{\theta q'(\theta)}{q(\theta)}\right]$$
  or
$$\frac{\partial \theta q(\theta)}{\partial \theta} = q(\theta)(1-\eta(\theta))$$

- it follows

$$c(s+\theta q(\theta)) = (p-b)\underbrace{(q(\theta)+\theta q'(\theta))}_{q(\theta(1-\eta(\theta))} + \theta c\underbrace{[q(\theta)+\theta q'(\theta)]}_{q(\theta(1-\eta(\theta))}$$

- And dividing by $q(\theta)$

$$c\frac{(s+\theta q(\theta))}{q(\theta)} = (p-b)(1-\eta(\theta) + \theta c(1-\eta(\theta)$$

- We thus have a single equation in (θ)

$$(p-b)(1-\eta(\theta) + \theta c - \eta(\theta)\theta c = \frac{cs}{q(\theta)} + c\theta$$

or

$$(p-b)(1-\eta(\theta) - \eta(\theta)\theta c = \frac{cs}{q(\theta)}; \quad \text{Central Planner Solution} \tag{8}$$

- We call θ^*_{CP} the efficient solution for θ

- Let's go back to the decentralized SAM model with NBW in continuous time
- At some point with showed that the general equilibrium i sobtained by a single equation in θ

$$(p-b)(1-\beta)-\beta c\theta=\frac{(r+s)c}{q(\theta)}$$

but we need to have with $r=0$ for a full comparison

- THe market solution is thus

$$(p-b)(1-\beta)-\beta c\theta=\frac{(r+s)c}{q(\theta}\quad \text{Decentralzied Solution} \tag{9}$$

- We call θ^*_{MK} the decentralized solution for θ
- Let's compare the two

$$(p-b)(1-\beta)-\beta c\theta=\frac{(r+s)c}{q(\theta)}\quad \text{Decentralzied Solution} \tag{10}$$

$$(p-b)(1-\eta(\theta)-\eta(\theta)\theta c=\frac{cs}{q(\theta)};\quad \text{Central Planner Solution} \tag{11}$$

- Under what conditions do we have that Market SOlution= Central Planner SOlution?

$$\underbrace{\beta}_{\text{Bargaining Share of the Worker}} = \underbrace{\eta(\theta)}_{\text{Elasticity of the Matchinf Function}} \quad \text{Hosios Conditions} \tag{12}$$

- What are the two elements of the Hosios Condition?

  1. $0 \leq \beta \leq 1$; Strucutral Parameter of Negotiation
  2. $0 \leq \eta(\theta) \leq 1$; Strucutral Parameter of matching Function

- If the matching function is Cobb DOuglas

$$\beta = \alpha \quad \text{Hosios Conditions with Cobb Dougals Matching Function}$$

- There is no ecomomics mechanism that imply that the these 2 parameters should be identical
- The idea is that at the table of **negotiation, the parties just look at their own surplus and do not internalize the effect of their negoatiation on the wlefare of the agents who are outside (unemployed and vacancies)**
- Only under the (non obvious) Hosios conditions such condition is satisfied
- NOte also that here we are using the Hosios NBW. We said that there are many wages that satisfy the search equilibrium. Among allo those wages, the net output is clearly inefficient.