The Baseline Matching Model: Job Destruction

[Sem0057]

Pietro Garibaldi

Academic Year 2024-2025

Contents

1 Introduction

2 Job Destruction With $\beta = 0$

- p is the productivity per worker on the job
  - When λ! λ strikes the productivity in the job falls and become a large negative number. Intuitively a job is destroyed because p becomes a a very negative and large number
- We now generalize and assume that the productivty per worker is px
  - p is the productivity component that is common across all jobs
  - x is the job specific component of productivity
  - When λ! the firm draws a new component of x from $F(x)$

    The value of the labor product is now px where x is drawn from a continuous distribution function $F(x)$ with support Ω, and where x^u is the upper support of Ω, and λ is the arrival rate of shocks.
  - Note that the stochastic process from which x is drawn is $i.i.d$
- Assume that firms have all the bargaining power (i.e. $\beta = 0$).
- Two assumptions are important:
  1. there is complete ex-post irreversibility, and
  2. free disposability is always an option.

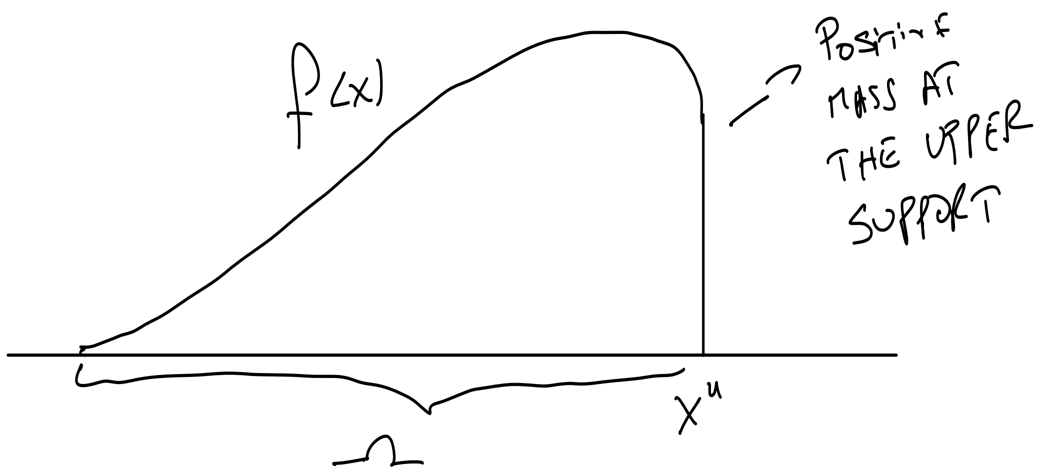


Figure 1: The Distribution of Productivity

- The outside option of the firm is V, and in equilibrium by free entry

$$V = 0$$

- The P.D.V. of a job is now index by x and is $J(x)$.
  - Firms observe x and then face an optimal stopping problem
  - Continue only if $J(x) > 0$
- The destruction rule is

$$\text{Destroy } \forall x : J(x) < 0$$

- If we are able to prove that $J(x)$ is monotonic and continuous, the destruction follows the reservation property and there exist a reservation productivity R such that the firm destroys is $x > R$, where R solves

$$\exists R : J(R) = 0.$$

- IT then follows

$$\begin{array}{llll} \text{if} & x > R & \Longrightarrow & J(x) > 0 \\ \text{if} & x < R & \Longrightarrow & J(x) \leq 0 \end{array}$$

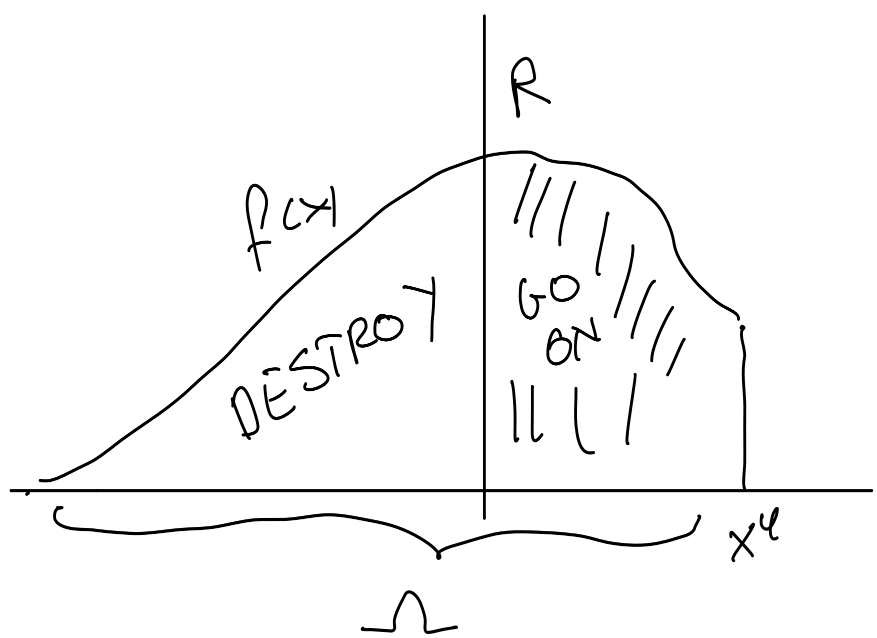


Figure 2: The Destruction Rule

- If $\beta = 0$ the P.D.V. of the firm $J(x)$ reads

$$rJ(x) = px - b + \lambda[\int_{z \ni \Omega} MAX[J(z), 0]dF(z) - J(x)]. \tag{1}$$

  where the integral can be interpreted as the expected value of a new job conditional on having a positive value.

- It is immediate to show that

$$J'(x) = \frac{p}{r+\lambda} > 0 \tag{2}$$

  or that the derivative is constant and independent of x, so that J is indeed monotonic and satisfies the *reservation rule*.

- Basically, we are now confident that

$$\exists R : J(x) > 0 \qquad \forall x > R$$

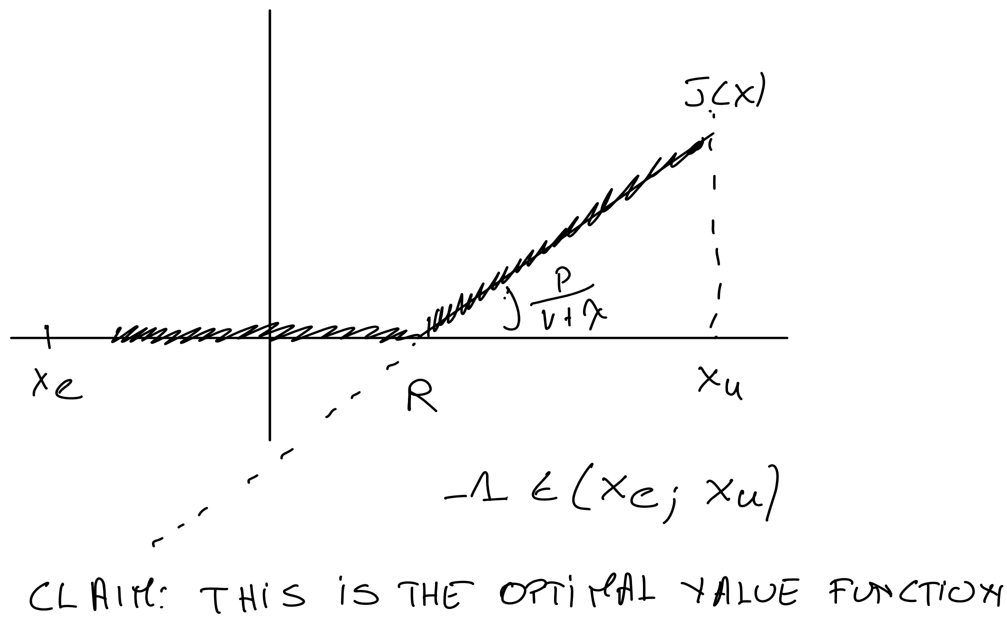


Figure 3: The Optimal Value Function with Endogenous Destruction

- This allows us to solve the maximization problem in the equation (1) so that the maximization inside the integral can be written as

$$\int_{z\ni\Omega} MAX[J(z),0]dF(z) = \int_{x_l}^{R} 0dF(z) + \int_{R}^{x^u} J(z)dF(z)$$

- and $J(x)$ becomes

$$(r+\lambda)J(x) = px - b + \lambda\int_{R}^{x^u} J(z)dF(z).$$

- The expected value of the job can be further simplified with an integration by parts
  - Recall
$$\int udv = uv - \int vdu$$
  . Here $v = F(z)$ and $u = J(z)$
- The integral is then

$$\int_{R}^{x^u} J(z)dF(z) = |J(z)F(z)|_{R}^{x^u} - \int_{R}^{x^u} J'(z)F(z)dz.$$

  which by virtue of equation (2) becomes

$$\int_{R}^{x^u} J(z)dF(z) = J(x^u) - \underbrace{\frac{p}{r+\lambda}}_{J'}\int_{R}^{x^u} F(z)dz.$$

  and substitute into $J(x)$ becomes

$$(r+\lambda)J(x) = px - b + \lambda\left\{J(x^u) - \frac{p}{r+\lambda}\int_{R}^{x^u} F(z)dz\right\}.$$

- while the reservation productivity solves

$$pR - b + \lambda\{J(x^u) - \frac{p}{r+\lambda}\int_R^{x^u} F(z)dz\} = 0 \tag{3}$$

- To obtain an expression for $J(x^u)$, we can write

  –
  $$(r+\lambda)J(x^u) = px^u - b + \lambda\left\{J(x^u) - \frac{p}{r+\lambda}\int_R^{x^u} F(z)dz\right\} \tag{4}$$
  $$(r+\lambda)\underbrace{J(R)}_{0} = pR - b + \lambda\left\{J(x^u) - \frac{p}{r+\lambda}\int_R^{x^u} F(z)dz\right\} \tag{5}$$

  –
  $$(r+\lambda)J(x^u) = px^u - pR$$

  so that
  $$J(x^u) = p\frac{x^u - R}{\lambda + r} = \frac{p}{r+\lambda}\int_R^{x^u} dz$$

  – or
  $$J(x) = \frac{p(x-R)}{r+\lambda} \qquad \forall x \geq R$$

- Substituting this expression into equation 3 we obtain the final expression for the reservation productivity

$$pR - b = -\frac{\lambda p}{r+\lambda}\int_R^{x^u} (1 - F(z))dz \tag{6}$$

- This expression uniquely solves for R.

- This expression is very deep

$$pR - b = \underbrace{-\frac{\lambda p}{r+\lambda} \int_{R}^{x^u} \underbrace{(1 - F(z))}_{positive} dz}_{negative} \tag{7}$$

- What is $pR - b$?
  - It is the net income to the firm at the marginal job. A sort of operational profits
  - It seems that
  - Yet, we know that $J(R) = 0$
  - Recall

$$\underbrace{rJ(R)}_{0} = \underbrace{\text{dividend}}_{negative} + \underbrace{\text{Capital Gain}}_{Positive}$$

- At the marginal job things can only improve (there is the optinm to destroy if things get worse) and the expected capital gain is thus positive. Thus the firm optimally takes a loss.

- We can of course do comparative static.

$$R = R(b, p, \lambda, x^u, F)$$

- How about an increase in wages/unemployed income b?
  - How is
  $$\frac{\partial R}{\partial b} = ?$$
  - $$p\frac{\partial R}{\partial b} - 1 = -\frac{\lambda p}{r+\lambda}(-1)(1-F(R))\frac{\partial R}{\partial b}$$
  - $$\frac{\partial R}{\partial b}p\left(1 - \frac{\lambda}{r+\lambda}(1-F(R))\right) = 1$$
  - $$\frac{\partial R}{\partial b}p\left(\frac{r+\lambda F(R)}{r+\lambda}\right) = 1$$
  - $$\frac{\partial R}{\partial b} > 0$$
  - that makes economic sense. The firm destroy more jobs if wages increase

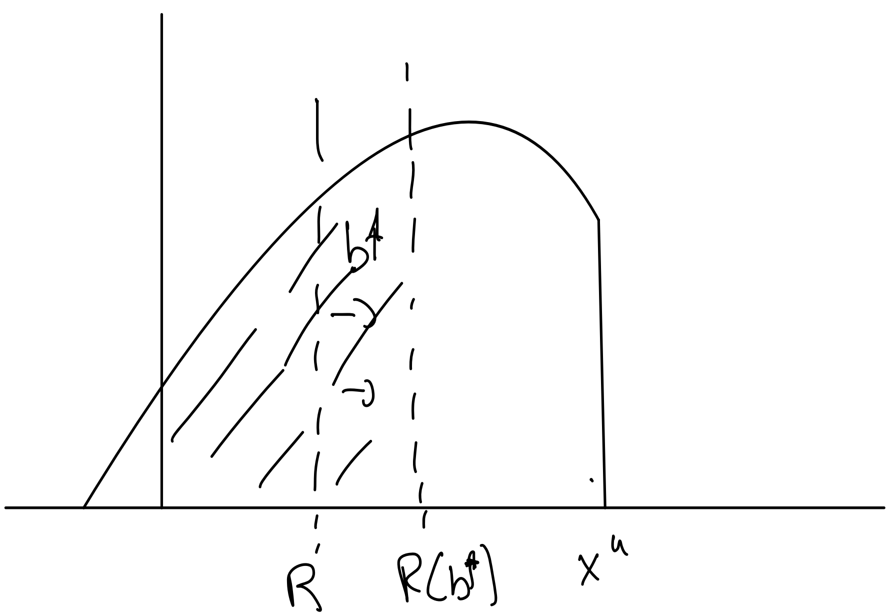


Figure 4: Increase in wages/unemployed income on reservation R

- How about job creation?
- To insert this into the model, assume that firms create jobs at the upper support of the distribution x^u, since the technology is ex-ante perfectly flexible.
- The value of vacancy reads

$$rV = -pc + q(\theta)[J(x^u) - V]$$

- with free entry this becomes

$$J(x^u) = \frac{pc}{q(\theta)}$$

- Using the fact that $J(x^u) = p\frac{x^u - R}{\lambda + r}$ we obtain

$$p\frac{x^u - R}{\lambda + r} = \frac{c}{q(\theta)} \tag{KEY2: JC}$$

  which uniquely solves for θ

- The Job creation curve is downward sloping. This is easy to show since

$$-\frac{p}{r+\lambda}\frac{\partial R}{\partial \theta} = -\frac{cq'(\theta)}{q(\theta)^2}$$

  Since $q'(\theta) < 0$ we have that

$$\frac{\partial \theta}{\partial R} > 0$$

- The model has now two key endogenous variables/Margin

$$\text{JD} \implies R$$

$$\text{JC} \implies \theta$$

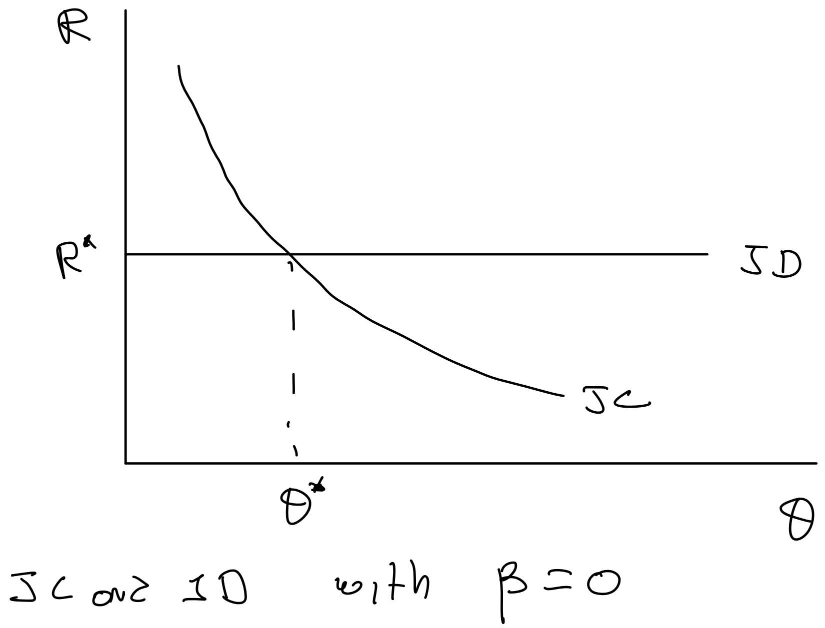


Figure 5: Equiliibrium θ and R with exogenous wages

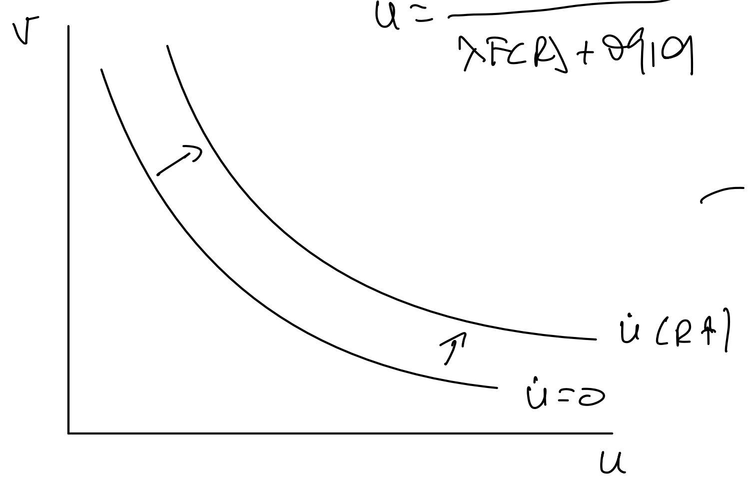
- To complete the model we need an expression for unemployment.
- Jobs are destroyed when hit by a reservation productivity that falls below the reservation rule R, while the are created when an unemployed worker is matched with a vacancy.
- This implies that

$$\lambda G(R)(1-u) = \theta q(\theta) u$$

so that

$$u = \frac{\lambda F(R)}{\lambda F(R) + \theta q(\theta)}$$

$$u = \frac{\lambda F(R)}{\lambda F(R) + \theta q(\theta)}$$



Beveridge curve with endogenous Job destruction.

Figure 6: Equiliibrium θ and R with exogenous wages

2.1 EQUILIBRIUM

Definition 1. –

Definition 2. ***The Search equilibrium with endogenous job destruction is a triple*** (u, R, θ) *satisfying*

- *Free entry (*$V = 0$*)*
- *Optimal Job Destruction* $J(R) = 0$
- *Balance Flow* $(\lambda G(R)(1-u) = \theta q(\theta)$

- The reduced form is obtained by the three equations

$$\begin{aligned} pR - b &= -\frac{\lambda p}{r+\lambda}\int_R^{x^u}(1-F(z))dz; \quad \text{Uniquely solves for } R \\ \frac{x^u - R}{\lambda + r} &= \frac{c}{q(\theta)} \quad \text{given } R \text{ solves for } \theta \\ u &= \frac{\lambda F(R)}{\lambda F(R) + \theta q(\theta)} \quad \text{solves for } u \text{ given } \theta \text{ and } R \end{aligned}$$

The model is still fully recursive. The reservation productivity yields a unique R, free-entry determines θ given R while the Beveridge curve gives u given R and θ.

3 Job Destruction with Endogenous Wages

- The original Mortensen Pissarides (1994) paper obviusly has also endogenous wages
- With endogenous Wages also the value of employment to the worker is a function of the idiosyincratic productivity.
- THe value of employment is

$$(r+\lambda)W(x) = w(x) + \lambda \int_{z \in \Omega} Max\left[W(z), U\right] dF(z) \tag{8}$$

- while the value of unemployment is

$$rU = b + \theta q(\theta)\left[W(x^u) - U\right] \tag{9}$$

and the value of a job to the firm is as before

$$(r+\lambda)J(x) = px - w(x) + \lambda \int_{z \in \Omega} Max\left[J(z), 0\right] dF(z) \tag{10}$$

- The value of the vacancy is also standard

$$rV = -cp + q(\theta)\left(J(x^u) - V\right) \tag{11}$$

- Wages are the outcome of traditional Nash Bargaining and Nash splitting so that

$$W(x) - U = \beta\left(J(x) + W(x) - U - V\right)$$

- We proceed in three steps
  1. Obtain The two fundamental equations for the two margins JC and JD
  2. Obtain Wage
  3. Derive The entire Equilibrium

3.1 Obtaining Job Creation and Job Destruction

- To obtain the equation of the surplus it is useful to obtain the joint income from the job

$$M(x) = J(x) + W(x)$$

- Since

$$(r+\lambda)J(x) = px - w(x) + +\lambda \int_{z\in\Omega} Max\left[J(z), 0\right] dF(z) \tag{12}$$

$$(r+\lambda)W(x) = w(x) + \lambda \int_{z\in\Omega} Max\left[W(z), U\right] dF(z) \tag{13}$$

- Summing up the two and recalling that the summation goes inside the integral we obtain

$$(r+\lambda)M(x) = px + \lambda \int_{z\in\Omega} Max\left[M(z), U\right] dF(z) \tag{14}$$

- Recalling that

$$S(z) = J(x) + W(x) - U = M(x) - U$$

subtracting $(r+\lambda)U$ on both sides (and taking λU inside the integral)

$$(r+\lambda)\left(M(x) - U\right) = px + \lambda \int_{z\in\Omega} Max\left[M(z) - U, U - U\right] dF(z) - rU \tag{15}$$

- we obtain the key expression for the surplus

$$(r+\lambda)S(x) = px - rU + \lambda \int_{z\in\Omega} Max\left[S(z), 0\right] dF(z) \tag{16}$$

- Since the job destruction is such that

$$J(R) = 0$$

but since $J(x) = (1-\beta)S(x)$ it is clear that there is **full agreement within the match** on job destruction and that

$$\beta J(R) = (1-\beta)(W(R) - U); \qquad \text{further;} \qquad \frac{J(R)}{1-\beta} = 0 = S(R)$$

- The surplus clearly satisfies the reservation property and can be clearly expressed as

$$(r+\lambda)S(x) = x - R; \qquad S'(x) = \frac{p}{r+\lambda} \tag{17}$$

- Since the surplus satisfies the reservation productivity we can again obtain

$$\int_{z\in\Omega} Max\left[S(z), 0\right] dF(z) = \int_{R}^{x^u} S(z)dF(z) \tag{18}$$

  - Since the function is linear it can integrated by parts to obtain

$$\int_{R}^{x^u} S(z)dF(z) = \left[S(z)F(z)\right]_R^{x^u} - \frac{p}{r+\lambda}\int_{R}^{x^u} F(z)dz$$

    or

$$\int_{R}^{x^u} S(z)dF(z) = S(x^u) \times 1 - \frac{p}{r+\lambda}\int_{R}^{x^u} F(z)dz$$

  - Using the fact $S(x^u) = \frac{x^u - R}{r+\lambda}$ we can write

$$\int_{R}^{x^u} S(z)dF(z) = \frac{p}{r+\lambda}\int_{R}^{x^u} (1 - F(z)dz$$

- And we obtain the expression for the reservation productivity as

$$S(R) = 0$$

  or

$$0 = pR - rU + \frac{p\lambda}{r+\lambda}\int_{R}^{x^u} (1 - F(z)dz$$

  where we then need an expression for rU

- To get an expression for rU recall that

$$rU = b + \theta q(\theta)\left[W(x^u) - U\right] \tag{19}$$

that can also be simplified as

$$\left[W(x^u) - U\right] = \beta S(x^u) = \frac{\beta}{1-\beta} J(x^u)$$

since $S(x) = \frac{J(x)}{1-\beta}$

- Recall job creation as

$$V = 0; \qquad J(x^u) = \frac{c}{q(\theta)}; \qquad S(x^u) = \frac{c}{(1-\beta)q(\theta)}$$

- The expression for rU is thus

$$rU = b + \theta q(\theta)\beta \frac{c}{(1-\beta)q(\theta)} \tag{20}$$

or

$$rU = b + \frac{c\theta\beta}{1-\beta}$$

- and the final expression for job destruction is

$$pR - b - \frac{c\theta\beta}{1-\beta} + \frac{p\lambda}{r+\lambda}\int_R^{x^u}(1-F(z))dz = 0$$

- Note that we can not determine job destruction without without knowledge of θ.
  - We can show that

$$\frac{\partial R}{\partial \theta} > 0$$

$$p\frac{\partial R}{\partial \theta} - \frac{p\lambda}{r+\lambda}(1-F(R))\frac{\partial R}{\partial \theta} = \frac{c\beta}{1-\beta}$$

    which clearly we have

$$p\frac{r+\lambda F(R)}{r+\lambda}\frac{\partial R}{\partial \theta} = \frac{c\beta}{1-\beta}; \qquad \rightarrow \frac{\partial R}{\partial \theta} > 0$$

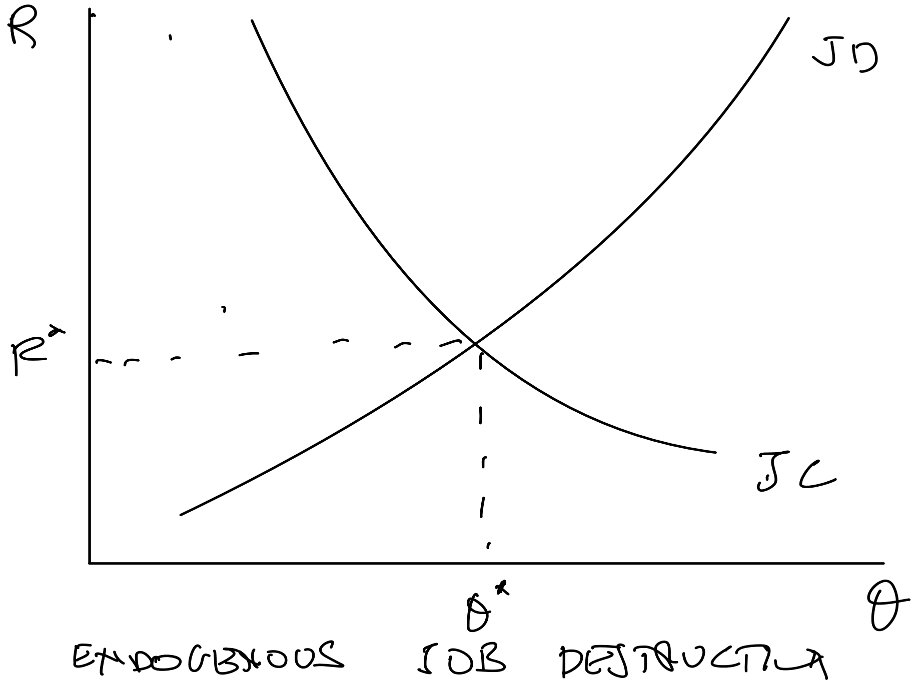


Figure 7: Equiliibrium θ and R with exogenous wages

- Job creation come from the posting of vacancies and the free entry conditions. As in the model without wages, jobs are created at the upper support. THis implis that the value of a vacancy

$$rV = -pc + q(\theta)\left[J(x^u) - V\right]$$

- And free entry implies

$$V = 0; \quad \Longrightarrow \quad \frac{pc}{q(\theta)} = J(x^u)$$

- Recalling the definition of surplus

$$S(x^u) = \frac{px^u - pR}{r+\lambda}; \quad J(x^u) = (1-\beta)S(x^u); \quad J(x^u) = (1-\beta)\frac{px^u - pR}{r+\lambda}$$

- And thus the job creation curve reads

$$\frac{pc}{q(\theta)} = (1-\beta)p\frac{x^u - R}{r+\lambda}; \quad \text{Job Creation Curve.}$$

- The Job creation curve is downward sloping. This is easy to show since

$$-\frac{p}{r+\lambda}\frac{\partial R}{\partial \theta} = -\frac{cq'(\theta)}{q(\theta)^2}$$

  Since $q'(\theta) < 0$ we have that

$$\frac{\partial \theta}{\partial R} > 0$$

3.2 Obtaining wages

- The model is basically complete with the job creation and job destruction curve, but it is useful to obtain the expression for the endogenous wages.
- We know that the expression of the surplus is

$$(r+\lambda)S(x) = px - rU + \lambda \int_{z\in\Omega} Max\left[S(z),0\right]dF(z)$$

- We also know the value of job is

$$(r+\lambda)J(x) = px - w(x) + \lambda \int_{z\in\Omega} Max\left[J(z),0\right]dF(z); \qquad \text{(J(x))}$$

- Using the wage rule

$$J(x) = (1-\beta)S(x)$$

  and multiplying $S(x)$ by $(1-\beta)$

-

$$(r+\lambda)\underbrace{(1-\beta)S(x)}_{J(x)} = (1-\beta)(px - rU) + \lambda \int_{z\in\Omega} Max\left[\underbrace{(1-\beta)S(z)}_{J(z)},0\right]dF(z)$$

- This implies that the previous equation is identical to equation $J(x)$ so that subtracing both the right hand side and the left hand side we get

$$0 = px - w(x) - (1-\beta)(px - rU)$$

- SO that a final expression for the wage is

$$w(x) = (1-\beta)rU + \beta px; \qquad \text{Key Wage 1}$$

- To get rid of rU we can use the expression that we laready have as

$$rU = b + \frac{\beta cp\theta}{1-\beta}$$

- to get the final expression for the wage

$$w(x) = (1-\beta)b + \beta(px + c\theta)$$

3.3 Equilibrium With JC, JD and Endogenous Wages

- **Definition 3.** ***The Search equilibrium with endogenous job destruction and NBwages is a triple*** (u, R, θ) *and a wage function satisfying*

  - *Free entry* ($V = 0$)
  - *Optimal Job Destruction* $S(R) = 0$
  - *Nash Bargaining* $(1-\beta)J(x) = \beta(W(x) - U)$
  - *Balance Flow* $(\lambda G(R)(1-u) = \theta q(\theta)$

- The reduced form is obtained by the three equations

$$\begin{aligned}
pR - b - \frac{c\theta\beta}{1-\beta} &+ \frac{p\lambda}{r+\lambda}\int_R^{x^u}(1-F(z))dz = 0; \quad \text{JD Curve, upward sloping in space } R, \theta \\
\frac{pc}{q(\theta)} &= (1-\beta)p\frac{x^u - R}{r+\lambda}; \quad \text{JC Curve, upward sloping in space } R, \theta \\
w(x) &= (1-\beta)b + \beta(px + c\theta); \qquad \text{Wage Equation} \\
u &= \frac{\lambda F(R)}{\lambda F(R) + \theta q(\theta)} \quad \text{solves for } u \text{ given } \theta \text{ and } R
\end{aligned}$$

- The model is still fully recursive. The reservation productivity yields a unique R, free-entry determines θ given R while the Beveridge curve gives u given R and θ.

4 Efficiency With Endogenous Job Destruction

- We work through efficiency in the case of exogenous job destruction both with $r = 0$ and the more general result with $r > 0$. The latter use the math of dynamic programming and the use of Hamiltonian.
- It is possible to obtain the Hosios Conditions also for the case of endogenous destruction model.
- The exercise requires a bit more of algebra, and most important the concept of average output per person in the labor force, that is not totally straightforward.
- We label such average output with y, and we show below how to obtain it
- Once we have y, the expression of social welfare is simply

$$\Omega = \int_0^\infty e^{-rt} \left(y + uz - pc\theta u \right) dt \tag{21}$$

- The evolution of unemployment is standard and reads

$$\dot{u} = \lambda F(R)(1-u) - \theta q(\theta)$$

  where job destruciton is endogenous and only jobs that get a productivity shock x below R are destroyed.
- The new concept is $y(t)$ and it expression reads

$$\dot{y} = \underbrace{px^u \theta q(\theta) u}_{1} + \underbrace{\lambda(1-u) \int_R^{x^u} pz dF(z)}_{2} - \underbrace{\lambda y}_{3}$$

  - where
    1. 1 Refers to new jobs created whose output is x^u
    2. 2. is the fraction of existing jobs that are hit by a productivity shock. If the new shocks is between R and x^u, the jobs continues to produce pz with the associated mass of jobs. If the shock is below R the job is destroyed and produces zero
    3. The final term shows that the existing output from a typical worker is lost every time a shock arrives

- The Problem is thus

$$Max_{\theta,\mathbb{R}}\Omega = \int_0^{\infty} e^{-rt}\left(y + uz - pc\theta u\right)dt \tag{22}$$

$$\text{s.t} \quad \dot{u} = \lambda F(R)(1-u) - \theta q(\theta)u \tag{23}$$

$$\dot{y} = px^u\theta q(\theta)u + \lambda(1-u)\int_R^{x^u} pzdF(z) - \lambda y \tag{24}$$

- The Hamiltonian reads

$$H(\theta, R, u, y\mu_1, \mu_2) = e^{-rt}\left(y + uz - pc\theta u\right) + \mu_1(\lambda F(R)(1-u) - \theta q(\theta)u) + \mu_2\left(px^u\theta q(\theta)u + \lambda(1-u)\int_R^{x^u} pzdF(z) - \lambda y\right)$$

- The first order conditions are

$$\frac{\partial H}{\partial \theta} = 0; \quad -e^{-rt}pcu - \mu_1(1-\eta(\theta))q(\theta)u + \mu_2 px^u(1-\eta(\theta))q(\theta)u = 0; \tag{25}$$

$$\frac{\partial H}{\partial R} = 0; \quad \lambda F'(R)(1-u)\mu_1 - (1-u)\lambda pRF'(R)\mu_2 = 0 \tag{26}$$

$$\frac{\partial H}{\partial y} = -\dot{\mu}_2; \quad e^{-rt} - \mu_2\lambda = -\dot{\mu}_2 \tag{27}$$

$$\frac{\partial H}{\partial u} = -\dot{\mu}_1; \quad e^{-rt}(z - pc\theta) - \left[\lambda F(R) + \theta q(\theta)\right]\mu_1 + \mu_2\left[x^u p\theta q(\theta) - \lambda\int_R^{x^u} pzdF(z)\right] = -\dot{\mu}_1 \tag{28}$$

- The steps to get the results are the following
- From 26 obtain

  $$\mu_1 = pR\mu_2$$

  and substitute into 25 after getting rid of u to obtain

  $$-e^{-rt}pc - pR\mu_2(1-\eta(\theta))q(\theta) + \mu_2 px^u(1-\eta(\theta))q(\theta) = 0$$

- and get an expression for μ_2 as

  $$\mu_2 = \frac{e^{-rt}c}{(x^u - R)(1-\eta(\theta))q(\theta)}$$

  so that the time derivative is

  $$\dot{\mu}_2 = \frac{-re^{-rt}c}{(x^u - R)(1-\eta(\theta)q(\theta)}$$

- And substituting into 26

  $$e^{-rt} - \lambda\frac{e^{-rt}c}{(x^u - R)(1-\eta(\theta))q(\theta)} = \frac{+re^{-rt}c}{(x^u - R)(1-\eta(\theta)q(\theta)}$$

  so that it leads to

  $$(x^u - R)(1-\eta(\theta))q(\theta) = (\lambda + r)c$$

- which leads to the fundamental job creation in efficency term

  $$\frac{x^u - R}{r+\lambda}(1-\eta(\theta)) = \frac{c}{q(\theta)} \tag{29}$$

- To get job destruction start from equation 28.

$$e^{-rt}(z-pc\theta)-[\lambda F(R)+\theta q(\theta)]\,\mu_1+\dot{\mu}_1+\mu_2\left[x^u p\theta q(\theta)-\lambda\int_R^{x^u} pzdF(z)\right]=0$$

- and substitute μ_1, μ_2 and $\dot{\mu}_2$

$$\begin{aligned} e^{-rt}(z-pc\theta)-[\lambda F(R)+\theta q(\theta)]\,\underbrace{pR\frac{e^{-rt}c}{(x^u-R)(1-\eta(\theta)q(\theta)}}_{\mu_1}+ \\ \underbrace{-rpR\frac{e^{-rt}c}{(x^u-R)(1-\eta(\theta)q(\theta)}}_{\dot{\mu}_1}+\underbrace{\frac{e^{-rt}c}{(x^u-R)(1-\eta(\theta)q(\theta)}}_{\mu_2}\left[x^u p\theta q(\theta)-\lambda\int_R^{x^u} pzdF(z)\right]=0 \end{aligned} \tag{30}$$

- that simplifies to

$$\begin{aligned} (z-pc\theta)-[\lambda F(R)+\theta q(\theta)]\,pR\frac{c}{(x^u-R)(1-\eta(\theta))q(\theta)}+ \\ -rpR\frac{c}{(x^u-R)(1-\eta(\theta)q(\theta)}+\frac{c}{(x^u-R)(1-\eta(\theta)q(\theta)}\left[x^u p\theta q(\theta)-\lambda\int_R^{x^u} pzdF(z)\right]=0 \end{aligned} \tag{31}$$

- since

$$\frac{(x^u-R)(1-\eta(\theta))}{1}=\frac{c(r+\lambda)}{q(\theta)}$$

$$(z-pc\theta)(x^u-R)(1-\eta(\theta)q(\theta)-[\lambda F(R)+\theta q(\theta)]\,pRc-rpRc+c\left[x^u p\theta q(\theta)-\lambda\int_R^{x^u} pzdF(z)\right]=0$$

or getting rid of c

$$(z-pc\theta)(r+\lambda)-[\lambda F(R)+\theta q(\theta)]\,pR-rpR+x^u p\theta q(\theta)-\lambda\int_R^{x^u} pzdF(z)=0$$

or

$$(z-pc\theta)(r+\lambda))-\lambda F(R)pR+\theta q(\theta)p(x^u-R)-rpR-\lambda\int_R^{x^u} pzdF(z)=0$$

- DIviding by $(r+\lambda)$

$$(z-pc\theta)-\frac{p\lambda F(R)R}{r+\lambda}-\frac{rpR}{r+\lambda}+\theta q(\theta)\frac{p(x^u-R)}{r+\lambda}-\frac{\lambda}{r+\lambda}\int_R^{x^u} pzdF(z)=0$$

- adding and subtracting $\frac{pR\lambda}{r+\lambda}$

$$(z-pc\theta)-\frac{p\lambda F(R)R}{r+\lambda}-\frac{rpR}{r+\lambda}+\frac{pR\lambda}{r+\lambda}-\frac{pR\lambda}{r+\lambda}+\theta q(\theta)\frac{p(x^u-R)}{r+\lambda}-\frac{\lambda}{r+\lambda}\int_R^{x^u} pzdF(z)=0$$

$$(z-pc\theta)+\underbrace{\frac{\lambda pR}{r+\lambda}(1-F(R))}_{\int_R^{x^u} RdF(z)}+\theta q(\theta)\frac{p(x^u-R)}{r+\lambda}-\frac{\lambda}{r+\lambda}\int_R^{x^u} pzdF(z)=R$$

- Using again the job creation condition since

$$p\frac{x^u-R}{r+\lambda}=\frac{c}{q(\theta)(1-\eta(\theta)}$$

 we arrive at

$$(z-pc\theta)+\frac{\theta c}{(1-\eta(\theta)}-\frac{p\lambda}{r+\lambda}\int_R^{x^u}(z-R)dF(z)=pR$$

 or

$$pR-z-\frac{pc\theta\eta(\theta)}{1-\eta(\theta)}+\frac{p\lambda}{r+\lambda}\int_R^{x^u}(z-R)dF(z)=0 \tag{32}$$

5 Firing Taxes in a Model with Two Margins

- We study the effect of of firing taxes on equilibrium unemployment.
- The effect of firing costs in the temporary contract will be analyzed in week 5. is ambiguous
- Here we study the traditional effect of firing tax in a model with two margins, (.i.e. with a model with endogenous job destruction).
- Yet, to keep the notation as simple as possible we do not work explicitly with wages, and we set $\beta = 0$. Firms are hit by productivity shocks at rate λ and they draw from a distribution function $F(x)$
- The value of a vacancies read
$$rV = -pc + q(\theta)[J(x_u) - V]$$
where $J(\epsilon_u)$ is the value of a job at the upper support of the distribution.
- The value of an existing job at idiosyncratic productivity x is
$$rJ(x) = px - b + \lambda[\int_{x_l}^{R} -TdF(z) + \int_{R}^{x_u} J(z)dF(z) - J(x)] \tag{33}$$
where the reservation rule for the firm is
$$J(R) = -T$$

- The firm gets $-T$ if the new shock is below the reservation rule while it gets the expected value of a the job if the new shock is above R. From equation (), it is immediate to see that (by taking the difference between $J(x)$ and $J(R) = -T$ that

$$(r+\lambda)J(x) + (r+\lambda)T = p(x-R)$$

or that

$$J(x) = p\frac{x-R}{r+\lambda} - T$$

- The integral in equation (), after an integration by parts become

$$\int_R^{x_u} J(z)dF(z) = J(x_u) + TF(R) - p\frac{1}{r+\lambda}\int_R^{x_u} F(z)dz \tag{34}$$

and using the fact that $J(x^u) = p\frac{x^u-R}{r+\lambda} - T$ we get

$$\int_R^{x_u} J(z)dF(z) = \frac{p}{r+\lambda}\int_R^{x_u}(1-F(z))dz - (1-F(R))T$$

- Substituting back into equation (33) and substituting for $J(R) = -T$ we get an expression for the reservation productivity which reads

$$-(\lambda+r)T = pR - b - \lambda F(R)T + \frac{\lambda p}{r+\lambda}\int_R^{x_u}(1-F(z))dz - \lambda(1-F(R))T$$

which simplifies to

$$pR - b = -\frac{\lambda p}{r+\lambda}\int_R^{x_u}(1-F(z))dz - rT \tag{35}$$

which is the final expression for the reservation productivity R the job destruction margin.

- To get an expression for job creation, simply use $V = 0$ to obtain

$$\frac{cp}{q(\theta)} = p\frac{x_u - R}{r + \lambda} - T \tag{36}$$

Equilibrium unemployment is then

$$u = \frac{\lambda F(R)}{\lambda F(R) + \theta q(\theta)}$$

Definition 4. *The equilibrium with firing taxes and endogenous reservation productivity is a triple* (R, θ, u) *that satisfies:*

i) optimal job destruction ($J(x) = -R$)
ii) optimal job creation ($V = 0$)
iii) steady state unemployment

- What happens when T goes up?
- First, one can see that R reduces the reservation productivity. To see this, take the derivative of T with respect to R
$$p\frac{\partial R}{\partial T} = \frac{\lambda p}{r+\lambda}(1-F(R))\frac{\partial R}{\partial T} - r$$
which simplifies to
$$p\frac{\partial R}{\partial T}[\frac{r+\lambda F(R)}{r+\lambda}] = -r$$
which implies that
$$\frac{\partial R}{\partial T} < 0$$
so that the firm holds on to less productive jobs when firing taxes increase.
- The effect of T on θ is
$$-\frac{cpq'(\theta)}{q(\theta)^2}\frac{\partial \theta}{\partial T} = -\frac{p}{r+\lambda}\frac{\partial R}{\partial T} - 1$$
which looks ambiguous. But substituting the value of $\frac{\partial R}{\partial T}$ one obtains
$$\begin{aligned} -\frac{cpq'(\theta)}{q(\theta)^2}\frac{\partial \theta}{\partial T} &= \frac{1}{r+\lambda}\frac{r(r+\lambda)}{r+\lambda F(R)} - 1 \\ -\frac{cpq'(\theta)}{q(\theta)^2}\frac{\partial \theta}{\partial T} &= \frac{-\lambda F(R)}{r+\lambda F(R)} \end{aligned}$$
so that
$$\frac{\partial \theta}{\partial T} < 0$$
which shows that fewer jobs come to the market.
- Basically, firing taxes reduce job destruction and job creation at give unemployment. The effect on unemployment is indeed ambiguous since
$$\begin{aligned} \frac{\partial u}{\partial T} &= \frac{\lambda f(R)\frac{\partial R}{\partial T}[\lambda F(R) + \theta q(\theta)] - [\lambda f(R)\frac{\partial R}{\partial T} + q(\theta)(1-\eta(\theta))\frac{\partial \theta}{\partial T}]\lambda F(R)}{(\lambda F(R) + \theta q(\theta))^2} \\ \frac{\partial u}{\partial T} &= \frac{\lambda f(R)\frac{\partial R}{\partial T}\theta q(\theta) - q(\theta)(1-\eta(\theta))\frac{\partial \theta}{\partial T}\lambda F(R)}{(\lambda F(R) + \theta q(\theta))^2} =? \end{aligned}$$

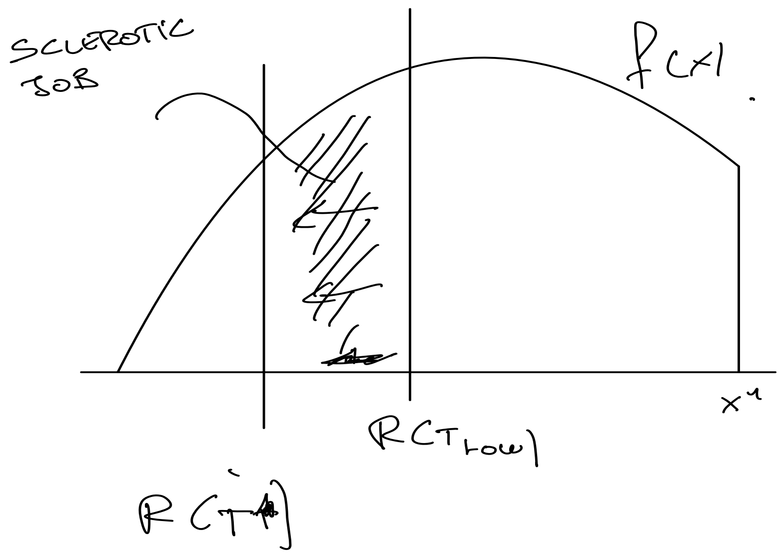


Figure 8: The Effect of higher Firing taxes