Business Cycle Properties : The Shimer Critique and the Fundamental Surplus

[Sem0057]

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Contents

1 Business Cycle Properties, and the Shimer Critique

- The MP or DMP model quickly became the key labor market model to study unemployment, Eursclerosis, and also labor market dynamics
- Yet the model was lackiing a quantitative assessmnet of its capability to deliver business cycle fluctuations in line with what we observe in the U.S.
- RObert Shimer in 2005 provide the quantative excercise that was still lacking ina very rigorous way.
- Shimer took the labor market empirical properties of key US business cycle statistics
  - u the unemployment rate
  - v the vacancy rate
  - $\theta = \frac{v}{u}$ the market tightness
  - y labor productivity

- How do you generate such time series and data?
  - That was part of the macro course in the first year
  - obtain time series
  - detrend them with the HO filter
  - generate business cycle statistics (mainly correlation and volatility)
- Shimer simply asked
  - How does the traditional SAM model behave in replicating those statistics? He used a simple stochastic version of the 1985 Pissarides model with NBW with exogenous job destruction
  - The answer was simply. Very poorly
  - The volatility of $\theta = \frac{v}{u}$ in real data is aprroximately an order of magnitude larger than what a basic model could predict
  - THe main problem was linked to the wage (NBW).
    * Wages are too volatile in the basic model, and θ consequently is not so volatile in the model.

- A huge amount of research was generated after the Shimer critique
  - Hall model with fixed wages
  - Credible bargaining model (Holmostrom and Hall)
  - Pissarides on entry wage versus average wage)
- Sargent Ljunkvist (2017) The Fundamental Surplus summarize this huge amount of research in a single paper that we will study in some details.
- THe Elasticity around steady state goes a long way for understanding and evaluating the Shimer critique and the rest of the literature.
- Before going into the literature and the model, we look at the data.

2 The Business Cycle Facts

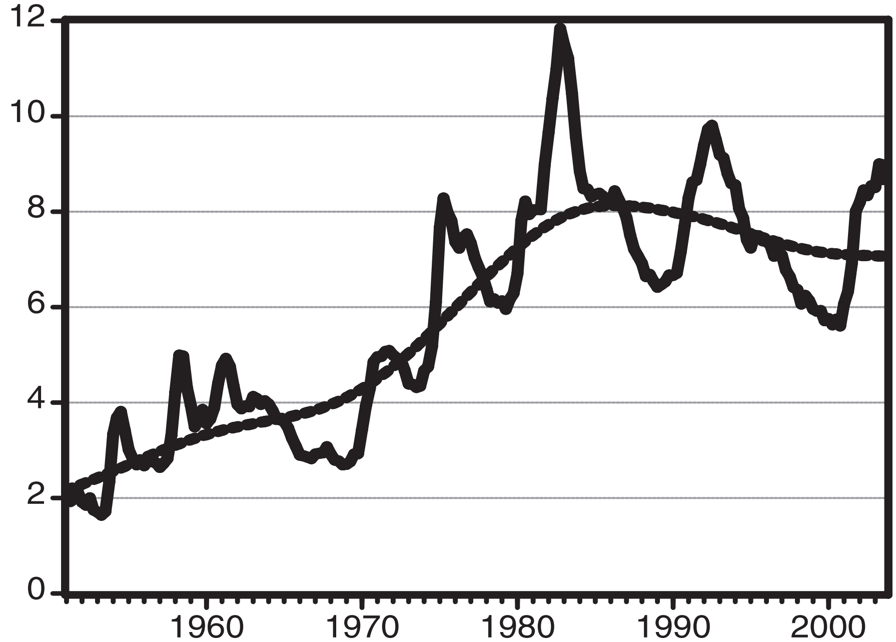


FIGURE 1. QUARTERLY U.S. UNEMPLOYMENT (IN MILLIONS) AND TREND, 1951–2003

Notes: Unemployment is a quarterly average of the seasonally adjusted monthly series constructed by the BLS from the CPS, survey home page http://www.bls.gov/cps/. The trend is an HP filter of the quarterly data with smoothing parameter 10^5.

Figure 1: Unemployment Cyclical Dynamics

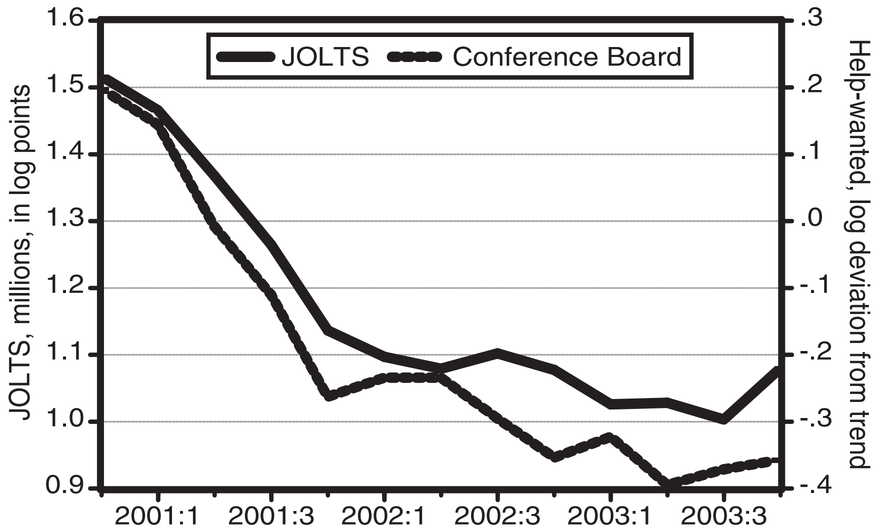


FIGURE 2. TWO MEASURES OF U.S. JOB VACANCIES, 2000Q4–2003Q4

Notes: The solid line shows the logarithm of the number of job openings in millions, measured by the BLS from the JOLTS, survey homepage http://www.bls.gov/jlt, quarterly averaged and seasonally adjusted. The dashed line shows the deviation from trend of the quarterly averaged, seasonally adjusted Conference Board help-wanted advertising index.

Figure 2: Vacancies Measures

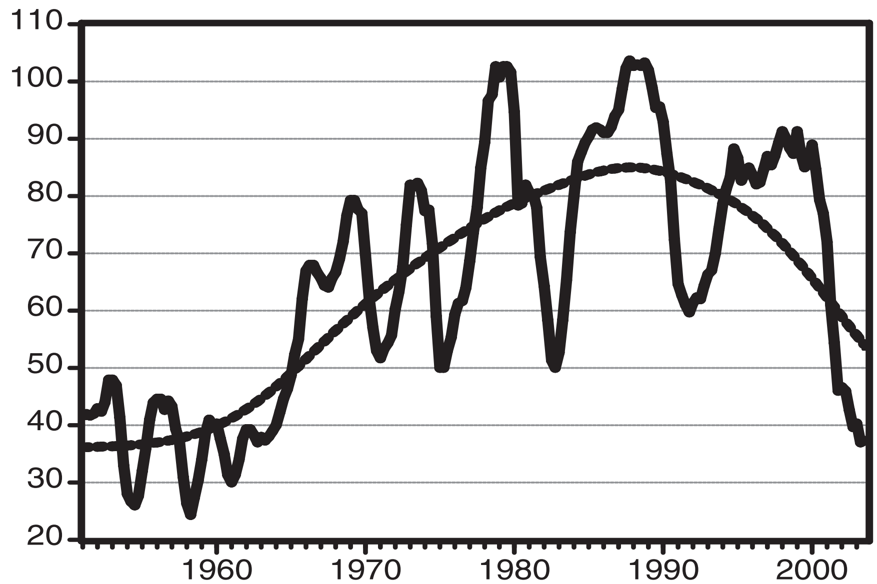


FIGURE 3. QUARTERLY U.S. HELP-WANTED ADVERTISING INDEX AND TREND, 1951–2003

Notes: The help-wanted advertising index is a quarterly average of the seasonally adjusted monthly series constructed by the Conference Board with normalization 1987 = 100. The data were downloaded from the Federal Reserve Bank of St. Louis database at http://research.stlouisfed.org/fred2/data/helpwant.txt. The trend is an HP filter of the quarterly data with smoothing parameter 10^5.

Figure 3: Vacancies Dynamics

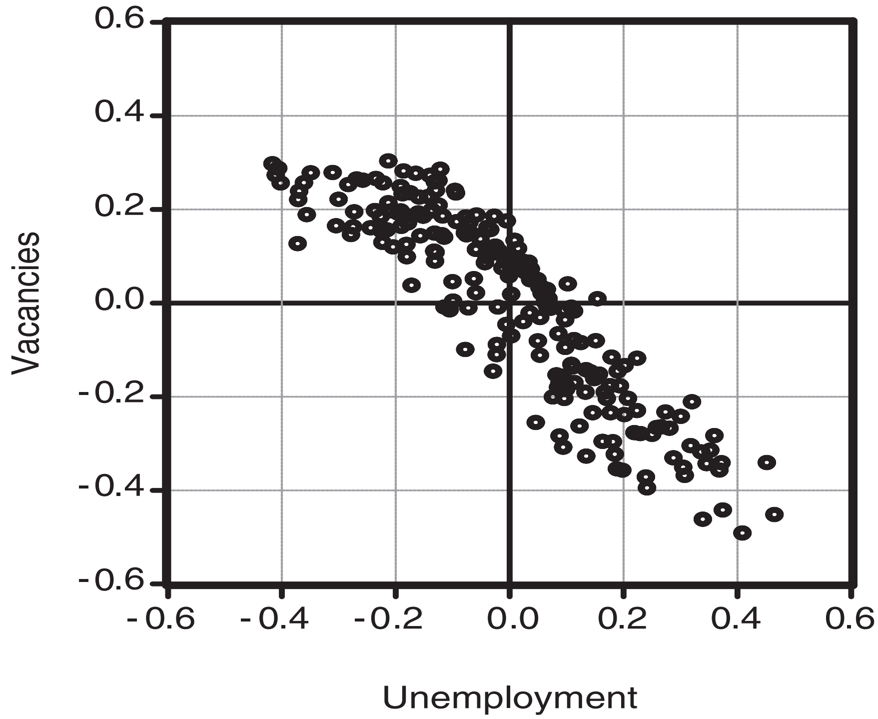


FIGURE 4. QUARTERLY U.S. BEVERIDGE CURVE, 1951–2003

Notes: Unemployment is constructed by the BLS from the CPS. The help-wanted advertising index is constructed by the Conference Board. Both are quarterly averages of seasonally adjusted monthly series and are expressed as deviations from an HP filter with smoothing parameter 10^5.

Figure 4: Beveridge CUrve Shifts

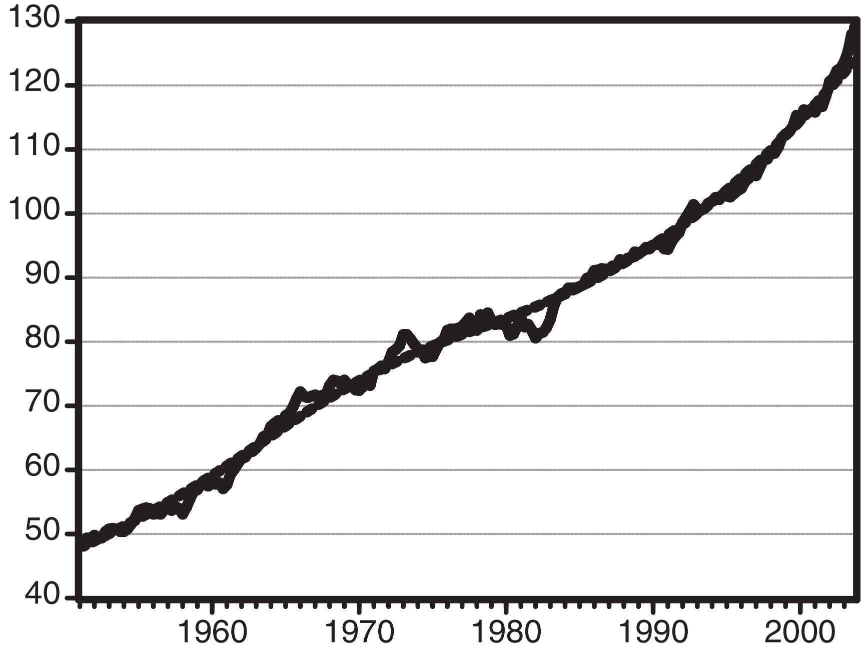


FIGURE 8. QUARTERLY U.S. AVERAGE LABOR PRODUCTIVITY AND TREND, 1951–2003

Notes: Real output per person in the non-farm business sector, constructed by the BLS Major Sector Productivity and Costs program, survey home page http://www.bls.gov/lpc/, 1992 = 100. The trend is an HP filter of the quarterly data with smoothing parameter 10^5.

Figure 5: Dynamics of Market TIghtness

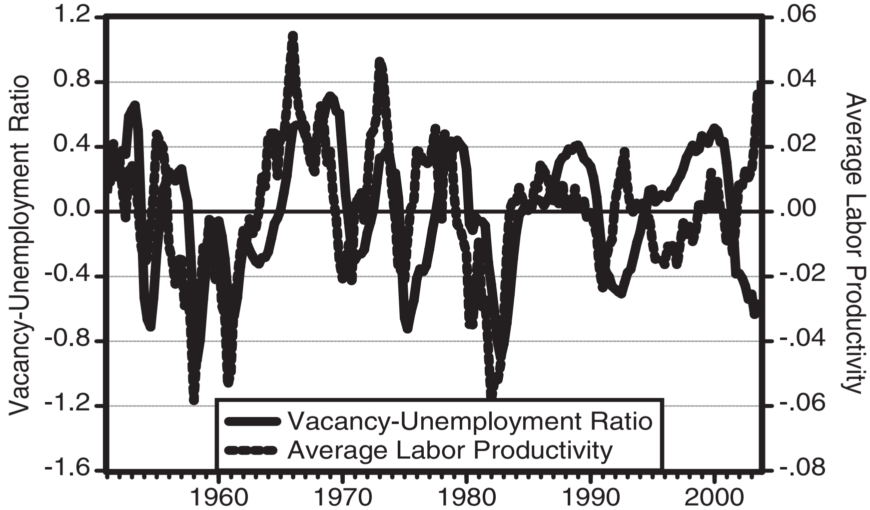


FIGURE 9. QUARTERLY U.S. VACANCY-UNEMPLOYMENT RATIO AND AVERAGE LABOR PRODUCTIVITY, 1951–2003

Notes: Unemployment is constructed by the BLS from the CPS. The help-wanted advertising index is constructed by the Conference Board. Both are quarterly averages of seasonally adjusted monthly series. Labor productivity is real average output per worker in the non-farm business sector, constructed by the BLS Major Sector Productivity and Costs program. The v-u ratio and labor productivity are expressed as deviations from an HP filter with smoothing parameter 10^5.

Figure 6: Dynamics of Market TIghtness

TABLE 1—SUMMARY STATISTICS, QUARTERLY U.S. DATA, 1951–2003

		u	v	v/u	f	s	p
Standard deviation		0.190	0.202	0.382	0.118	0.075	0.020
Quarterly autocorrelation		0.936	0.940	0.941	0.908	0.733	0.878
	u	1	−0.894	−0.971	−0.949	0.709	−0.408
	v	—	1	0.975	0.897	−0.684	0.364
Correlation matrix	v/u	—	—	1	0.948	−0.715	0.396
	f	—	—	—	1	−0.574	0.396
	s	—	—	—	—	1	−0.524
	p	—	—	—	—	—	1

Notes: Seasonally adjusted unemployment u is constructed by the BLS from the Current Population Survey (CPS). The seasonally adjusted help-wanted advertising index v is constructed by the Conference Board. The job-finding rate f and separation rate s are constructed from seasonally adjusted employment, unemployment, and mean unemployment duration, all computed by the BLS from the CPS, as explained in equations (1) and (2). u, v, f, and s are quarterly averages of monthly series. Average labor productivity p is seasonally adjusted real average output per person in the non-farm business sector, constructed by the Bureau of Labor Statistics (BLS) from the National Income and Product Accounts and the Current Employment Statistics. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5.

Figure 7: Business Cycle Properties

- What is a Business Cycle SAM Model?
- It is basic SAM (take our exogenous job destruction) in which we need

  1. A Source of Shock
     - fluctuations of productivity
     - $y = \frac{Y}{L}$ and in the basic model is p
  2. Propagation and Amplification Mechanism.
     - The imperfection/features of a specific SAM model

- What is the key quantitative question?

  - How θ and u respond to a productivity shock

- The key approximation
  - The Elasticity of θ and u with respect to y around steady state
  - Two key elasticities

$$\eta_{\theta,y} = \frac{\frac{d\theta}{\theta}}{\frac{dy}{y}}$$

    and

  - Elasticity of u with respect to y

$$\epsilon_{u,y} = \frac{\frac{du}{u}}{\frac{dy}{y}}$$

- The Framework is Pissarides with exogenous Destruction
  - $\beta < 1$ is the discount rate
  - $\phi < 1$ is the bargaining share

2.1 A Bit of Algebra on the basic Model

- There is a Key claim

  Claim 1.

  The model in reduced form is simply

$$y - z = \frac{c(r + s + \phi\theta q((\theta))}{q(\theta)(1 - \phi)} \tag{1}$$

$$u = \frac{s}{s + \theta q(\theta)} \tag{2}$$

- We thus wnat to prove that equation 1 is correct

- THe value of a vacancy is

$$V = -c + \beta \left[q(\theta)J + (1 - (q(\theta))V \right]$$

- The value of unemployment is

$$U = z + \beta \left[\theta q(\theta)E + (1 - \theta q(\theta)U \right]$$

- The wage rule is

$$E - U = \phi S; \quad \text{S is the surplus}$$

$$J = (1 - \phi)S$$

- The value of job is

$$J = y - w + \beta \left[s \underbrace{V}_{0} + (1 - s)J \right]$$

- and the employment to a worker is

$$E = w + \beta \left[sU + (1 - s)E \right]$$

- In the process of proving Claim 1 we introduce a new claim (a sort of a problem set)

  Claim 2. *THe wage can be written as*

$$w = \frac{r}{1+r}U + \phi\left(y - \frac{r}{1+r}U\right)$$

- Introduce joint income

$$M = J + E$$

$$(1 - \beta(1-s))J = y - w$$

$$(1 - \beta(1-s))E = w + \beta sU$$

- So that the expression for M is

$$M = \frac{y + \beta sU}{1 - \beta(1-s)}$$

- Get the surplus from joint income

$$S = \underbrace{E + J}_{M} - U - \underbrace{V}_{0}$$

- So that

$$S = \frac{y + \beta sU - U + \beta U - \beta sU}{1 - \beta(1-s)} - U$$

  or

$$S = \frac{y - (1-\beta)U}{1 - \beta(1-s)}$$

  (recall that in continuous time

$$S = \frac{y - rU}{r + \lambda}$$

- Recall the value of a job as

$$J = (1 - \beta)S$$

where

$$J = \frac{y - w}{1 - \beta(1 - s)}; \quad \text{and} \quad S = \frac{y - (1 - \beta)U}{1 - \beta(1 - s)}$$

- SO that

$$\frac{y - w}{1 - \beta(1 - s)} = (1 - \phi)\left[\frac{y - (1 - \beta)U}{1 - \beta(1 - s)}\right]$$

and solve for w

$$y - w = (1 - \phi)y + (1 - \phi)(1 - \beta)U$$

- To obtain

$$w = (1 - \phi)(1 - \beta)U + \phi y \tag{3}$$

Now let

$$\beta = \frac{1}{1 + r}; \qquad (1 - \beta) = \frac{r}{1 + r}$$

and substitute into 3 to obtain

$$w = \frac{r}{1 + r}(1 - \phi)U + \phi U$$

- And finally

$$w = \frac{r}{1 + r}U + \phi\left[y - \frac{r}{1 + r}U\right]$$

which proves Claim 2.

- We still need to prove Claim 1. We need to get rid of w and to obtain a single equation in θ
- Start from the value of unemployment as

$$U = z + \beta\left[\theta q(\theta)E + (1-\theta q)\theta))U\right] \tag{4}$$

or

$$U(1-\beta) = z + \beta\theta q)\theta)\left[E - U\right]$$

- Recall that

$$E - U = \phi S; \quad J = (1-\phi)S; \qquad S = \frac{1}{1-\phi}J$$

- Free entry implies

$$J = \frac{c}{\beta q(\theta)}$$

- So that

$$S = \frac{1}{1-\phi}\frac{c}{q(\theta)\beta}$$

and then $E - U$ is

$$E - U = \phi S = \frac{\phi}{1-\phi}\frac{c}{q(\theta)\beta}$$

- And the value of unemployment reads

$$U(1-\beta) = z + \beta\theta q(\theta)\frac{\phi}{1-\phi}\frac{c}{q(\theta)\beta}$$

or

$$U(1-\beta) = z + \frac{c\theta\phi}{1-\phi}$$

and using the result for $1-\beta)$

$$\frac{r}{1+r}U = z + \frac{c\theta\phi}{1-\phi}$$

- The expression we just obtained for $(1-\beta)U$ can be used into the wage. Recall

$$U(1-\beta) = z + \frac{c\theta\phi}{1-\phi}$$

and

$$w = (1-\phi)(1-\beta)U + \phi y$$

so that

- The wage is then the standard wage of the basic model

$$w = (1-\phi)z + \phi(y + c\theta)$$

- Take the value of a job

$$\underbrace{J}_{\frac{c}{\beta q(\theta)}}(1-\beta(1-s)) = y - w$$

and substituting the wage found above one has

- We are then close to Claim 1.

$$\frac{c}{\beta q(\theta)}(1-\beta(1-s)) = y - (1-\phi)z - \phi(y + c\theta)$$

- We can simplify the coefficients in the LHS as

$$\frac{1-\beta(1-s)}{\beta} = \frac{1}{\beta} - 1 + s = \frac{1}{\frac{1}{1+r}} - 1 + s = (r+s)$$

and the equation looks like

$$\frac{c(r+s)}{q(\theta)} = (y-z)(1-\phi) - c\theta\phi$$

- Which **proves Claim 1** since

$$y - z = \frac{c\left((r+s) + \phi\theta q(\theta)\right)}{q(\theta)(1-\phi)} \tag{5}$$

$$u = \frac{s}{s + \theta q(\theta)} \tag{6}$$

2.2 Toward the Calibration of Key Elasticities

- Is it true that this Model fail to match the business cycle statistics
  - The key elasticities are
  - The Elasticity of θ and u with respect to y around steady state
  - Two key elasticities

$$\eta_{\theta,y} = \frac{\frac{d\theta}{\theta}}{\frac{dy}{y}}$$

    and

  - Elasticity of u with respect to y

$$\epsilon_{u,y} = \frac{\frac{du}{u}}{\frac{dy}{y}}$$

- THe road map is into three steps
  1. Obtain analytic expression
  2. Calibrate the elasticity with empirical parameters
  3. Compare them with reality

- We use the Cobb DOuglas matching function

$$M(u,v) = Au^{\alpha}v^{-\alpha}$$

from which $q(\theta) = A(\frac{u}{v})^{\alpha} = A\theta^{-\alpha}$ where

$$\alpha = -\frac{q'(\theta)}{q(\theta)}\theta$$

- The Elasticity we need to play with is

$$\eta_{u,y} = \frac{\frac{du}{u}}{\frac{dy}{y}}$$

- As a tool we also need the elasticity of unemployment with respect to θ)

$$\eta_{u,\theta} = \frac{\frac{du}{u}}{\frac{d\theta}{\theta}} = \frac{du}{d\theta}\frac{\theta}{u}$$

where clearly

$$u = \frac{s}{s+\theta q(\theta)}$$

- To get the elasticity start from the derivative

$$\frac{du}{d\theta} = -\frac{s[q(\theta)+\theta q'(\theta)]}{[s+\theta q(\theta)]^2}$$

or

$$\frac{du}{d\theta} = -\underbrace{\frac{s}{s+\theta q(\theta)}}_{u}\frac{q(\theta)+\theta q'(\theta)}{s+\theta q(\theta)} = -\frac{s}{s+\theta q(\theta)}\frac{q(\theta)\left[1+\overbrace{\frac{\theta q'(\theta)}{q(\theta)}}^{-\alpha}\right]}{s+\theta q(\theta)}$$

- So that we have

$$\frac{du}{d\theta} = -\frac{uq(\theta)(1-\alpha)}{s+\theta q(\theta)}$$

- We can thus obtain the elasticity that we are looking for

$$\eta_{u,\theta} = \frac{du}{d\theta} - \frac{\theta}{u} = -\frac{uq(\theta)(1-\alpha)}{s+\theta q(\theta)}\frac{\theta}{u} = -(1-\alpha)\frac{\theta q(\theta)}{s+\theta q(\theta)}$$

- which leads to the final expression

$$\eta_{u,\theta} = -(1-\alpha)(1-u) \tag{7}$$

  and we will soon need this elasticity

2.3 The Key Elasticity and the Fundamental Surplus

- THe Elasticity of θ with respect to y is

$$\eta_{\theta,y} = \frac{\frac{d\theta}{\theta}}{\frac{dy}{y}}$$

- Start from Claim 1

$$y - z = \frac{c\left((r+s) + \phi\theta q(\theta)\right)}{q(\theta)(1-\phi)} \tag{8}$$

$$u = \frac{s}{s + \theta q(\theta)} \tag{9}$$

- Write the first equation as

$$\frac{(1-\phi)}{c}(y-z) = \frac{(r+s)}{q(\theta)} + \phi\theta \tag{10}$$

This equation can be differentiated with respect to y since it defines implicitly $\theta(y)$ as

$$\frac{1-\phi}{c} = -\frac{(r+s)}{q(\theta)^2} q' \frac{\partial\theta}{\partial y} + \phi\frac{\partial\theta}{\partial y}$$

and collecting

$$\frac{\partial\theta}{\partial y}\left[\phi - \frac{(r+s)q'(\theta)}{q(\theta)^2}\right] = \frac{1-\phi}{c}$$

or

$$\frac{\partial\theta}{\partial y} = \frac{-\frac{1-\phi}{c}}{-\left(\phi - \frac{(r+s)q'(\theta)}{q(\theta)^2}\right)}$$

- Use 10 in the numerator to obtain

$$\frac{\partial\theta}{\partial y} = \frac{-\left[\frac{r+s}{q(\theta)} + \phi\theta\right]\frac{1}{y-z}}{-\left(\phi - \frac{(r+s)q'(\theta)}{q(\theta)^2}\right)}$$

- MUltiply and divide by θ in the denominator

$$\frac{\partial\theta}{\partial y} = \frac{-\left[\frac{r+s}{q(\theta)} + \phi\theta\right]\frac{1}{y-z}}{-\left(\phi - \frac{(r+s)}{q(\theta)\theta}\frac{q'(\theta)\theta}{q(\theta)}\right)}$$

or

$$\frac{\partial\theta}{\partial y} = \frac{-\left[\frac{r+s}{q(\theta)} + \phi\theta\right]\frac{1}{y-z}}{-\left(\frac{\alpha(r+s)+\phi\theta q(\theta)}{\theta q(\theta)}\right)}$$

$$\frac{\partial\theta}{\partial y} = \frac{-\left[\frac{r+s+\phi\theta q(\theta)}{q(\theta)}\right]\frac{1}{y-z}}{-\left(\frac{\alpha(r+s)+\phi\theta q(\theta)}{\theta q(\theta)}\right)}$$

- Simplifying $q(\theta)$ we get the final expression for the derivative

$$\frac{\partial\theta}{\partial y} = \frac{r+s+\phi\theta q(\theta)}{\alpha(r+s)+\phi\theta q(\theta)}\frac{\theta}{y-z}$$

- Recall that the elasticity is

$$\eta_{\theta,y} = \frac{\partial \theta}{\partial y} \frac{y}{\theta}$$

so that we can write

$$\eta_{\theta,y} = \underbrace{\frac{\partial \theta}{\partial y} = \frac{r + s + \phi\theta q(\theta)}{\alpha(r+s) + \phi\theta q(\theta)}}_{\Gamma^{Nash}} \qquad \underbrace{\frac{y}{y-z}}_{\text{Fundamental Surplus}}$$

- We are thus arrived at

$$\eta_{\theta,y} = \Gamma^{Nash} \frac{y}{y-z}$$

where

$$\Gamma^{Nash} = \frac{\partial \theta}{\partial y} = \frac{r + s + \phi\theta q(\theta)}{\alpha(r+s) + \phi\theta q(\theta)}$$

- and $eta_{u,\theta} = -(1-\alpha)(1-u)$

2.4 Calibrating the Elasticity and the Fundamental Surplus

- We use quarterly data

  1. $\phi = .5$ is the baseline value of the bargaining share
  2. $\theta q(\theta)$ is the average probability that an unemployed finds a job in a given quarter

  $$\theta q(\theta) = .5$$

  3. r is the pure interest rate $r = .01$
  4. $s = 0.035$ since approximately 3.5% of jobs are lost in a given quarter. It is the EU flow
  5. $\alpha = .5$ is the baseline elasticity of the matching function (The paper by Petrongolo and PIssarides that surveyd the empirical work on the matching function

- We can then calibrate the Γ^{Nash}

$$\Gamma^{Nash} = \frac{\partial \theta}{\partial y} = \frac{r + s + \phi \theta q(\theta)}{\alpha(r+s) + \phi \theta q(\theta)}$$

$$\Gamma^{Nash} = \frac{\overbrace{0.01}^{r} + \overbrace{0.035}^{s} + \overbrace{0.5}^{\phi} \times \overbrace{0.5}^{\theta q(\theta)}}{\underbrace{0.5}_{\alpha}(\underbrace{0.01}_{r} + \underbrace{0.035}_{s}) + \underbrace{0.5}_{\phi} \times \underbrace{0.5}_{\theta q(\theta)}}$$

- This implies that

$$\Gamma^{Nash} = \frac{0.295}{0.2727} \approx 1.07$$

- We now need to calibrate the fundamental surplus
- $y = 1$ is set as a numeraire in the calibration
- $z = .7$ is a reasonable (albeit high!) value for the flow value of leisure and unemployment benefit
- This implies that

$$\frac{y}{y-z} = \frac{1}{1-0.7} = \frac{1}{\frac{3}{10}} \approx 3.34$$

Definition 1. *The* **SHimer Critique**. *The baseline matching model of unemployment can not replicate the stylized facts since*

$$\hat{\eta}_{\theta,y} = 1.07 \times 3.33 \approx 3.4$$
$$\eta_{\theta,y}^{data} \approx 20$$

2.5 The Fundamental Surplus with FIxed Wages

- Note that the method of playing with business cycle elasticities is very general
- We now apply it to the simplest SAM model with fixed wage
- LEt's take a model with fixed wage and continuous time
  - The value of a job is
    $$rJ = y - \overline{\omega} - \lambda J$$
    and Obviously
    $$V = 0; \qquad \Longrightarrow \quad J = \frac{c}{q(\theta)}$$
  - THe model is then
    $$\frac{(r+\lambda)c}{q(\theta)} = y - \overline{\omega} \tag{11}$$
    $$u = \frac{\lambda}{\lambda + \theta q(\theta)} \tag{12}$$
    And obtain the differentiation implicit $\theta(y)$
  - The differential is
    $$1 = -\frac{c(r+\lambda)}{q(\theta)} \frac{q'(\theta)}{q(\theta)} \frac{\partial \theta}{\partial y}$$
    that can be written as (multiplying and dividing by θ
    $$\underbrace{\frac{c(r+\lambda)}{q(\theta)}}_{y-\overline{\omega}} \underbrace{\left[-\frac{q'(\theta)\theta}{q(\theta)}\right]}_{\eta(\theta)} \frac{\partial \theta}{\partial y} \frac{1}{\theta} = 1$$

- SO that

$$1 = (y - \overline{\omega})\eta(\theta)\frac{1}{\theta}\frac{\partial\theta}{\partial y}$$

- Recall that the elasticity is

$$\eta_{\theta,y} = \frac{\partial\theta}{\partial y}\frac{y}{\theta}$$

or

$$\eta_{\theta,y} = \frac{\theta}{\eta(\theta)}\frac{1}{y - \overline{\omega}}\frac{y}{\theta}$$

- Which implies

$$\eta_{\theta,y} = \Gamma^{Fixed}\frac{y}{y - \overline{\omega}}$$

- Now we can calibrate it

1. $\eta(\theta) = .5$
2. $y = 1$
3. $\overline{\omega} = .8$

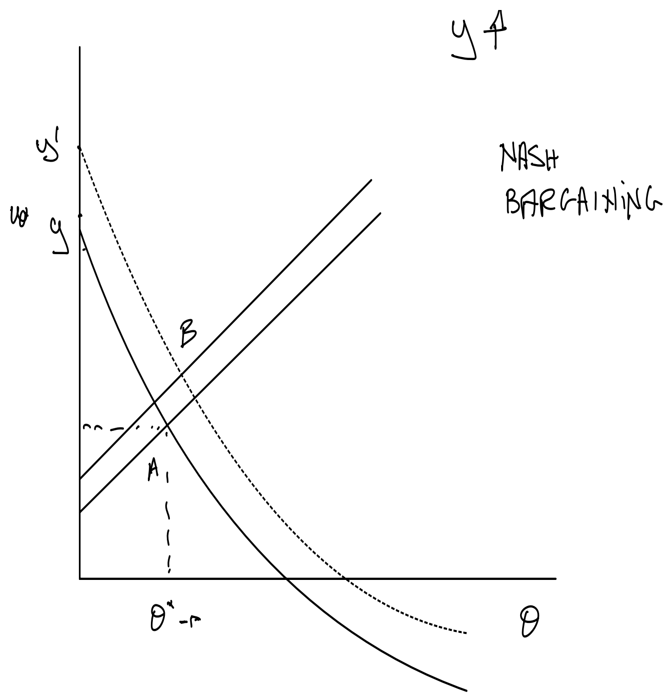
- This implies

$$\Gamma^{Fixed} = \frac{1}{\eta(\theta)} = 2.$$

- And Furhter

$$\eta^{Fixed}_{\theta,y} = 2\frac{1}{0.2} = 10!!$$

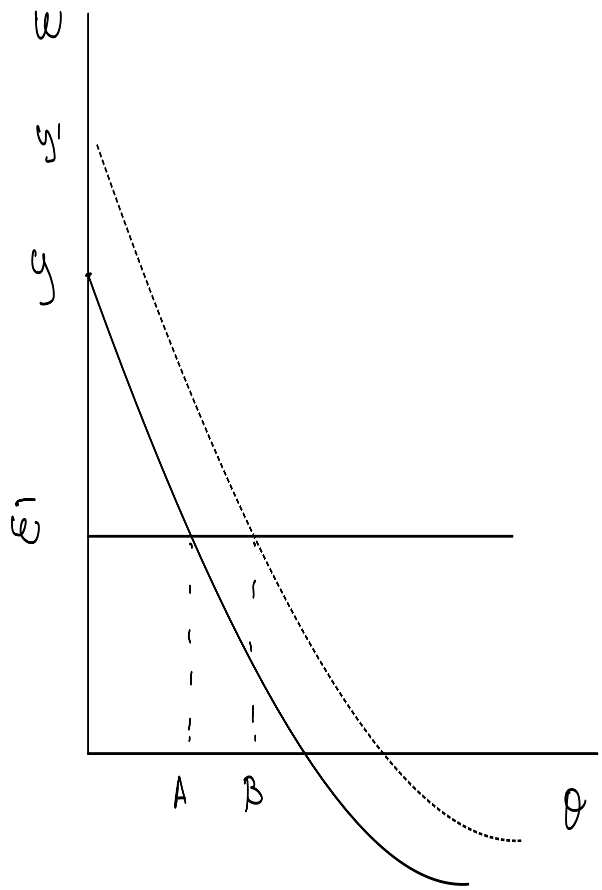
so that **with fixed wages clearly the elasticity goes up**



$$w = y - \frac{c}{\beta q(\theta)}\left(1-\beta(1-s)\right)$$

$$w = (1-\phi)z + \phi\left(y + c\theta\right)$$

Figure 8: The small response to y shock with NBW



Impact with Fixed Wgl. θ Moves more

Figure 9: The larger response to y shock with NBW

- The intuition is that with NBW thh wage responds too much to changes in y

$$w^{NBW} = (1-\phi)z + \phi(y + c\theta)$$

and the intuition is that if prices move too much you do not move with quantitites

2.6 FUrhter Amplification Mechanisms

- In general the literature looked for amplification mechanisms in the real world
- **Adjustment costs** such as hiring and firing costs, can push up the fundamental surplus.
- How can we show it
- Take the model with fixed wages **plus** hiring costs
- Assume that the firm when meets a worker has to pay a fixed costs H

$$rV = -c + q(\theta)\left[J - H - V\right] \tag{13}$$

  - With $V = 0$ we obtain

    $$\frac{c}{q(\theta)} = J - H$$

    and J is just

    $$rJ = y - \overline{\omega} - \lambda J$$

    so that the equation for θ

    $$\frac{c}{q(\theta)} = \frac{y - \overline{\omega}}{r + \lambda} - H$$

  - And the fundamental equation is

    $$\frac{c(r+\lambda)}{q(\theta)} = y - \overline{\omega} - H(r+\lambda)$$

    and obtain a function $\theta(y)$

- How Do you proceed? Implictly differentiate

$$\frac{c(r+\lambda)}{q(\theta)} = y - \overline{\omega} - H(r+\lambda)$$

- $$-\frac{c(r+\lambda)}{q(\theta)^2} q'(\theta) \frac{\partial\theta}{\partial y} = 1$$

  or

  $$-\frac{q'(\theta)}{q(\theta)} \left(\frac{c(r+\lambda)}{q(\theta)} \right) = 1$$

- Using the original equation

$$-\frac{q'(\theta)}{q(\theta)} \left(y - \overline{\omega} - (r+\lambda)H\right) \frac{\partial\theta}{\partial y} \frac{1}{\theta} = 1$$

- If the function is Cobb Douglas we get

$$\alpha(y - \overline{\omega} - (r+\lambda)H) \frac{\partial\theta}{\partial y} = \theta$$

- The Elasticity is

$$\eta_{\theta,y} = \frac{\frac{d\theta}{\theta}}{\frac{dy}{y}} = \frac{\partial\theta}{\partial y}\frac{y}{\theta}$$

- Or

$$\eta_{\theta,y} = \frac{1}{\alpha}\frac{\theta}{(y - \overline{\omega} - (r+\lambda)H)}\frac{y}{\theta}$$

and

$$\eta_{\theta,y} = \frac{1}{\alpha}\frac{y}{(y - \overline{\omega} \quad \underbrace{-(r+\lambda)H)}_{\text{Amplification}}}$$

2.7 A Caveat on Calibrting Matching Function

- When the model is calibrated we typically work with Cobb DOuglas

$$x(u,v) = Au^{\alpha}v^{1-\alpha}$$

- With $q(\theta) = A\theta^{-\alpha}; \qquad \theta q(\theta) = A\theta^{1-\alpha}$
- Remember that these are instantaneous rates and furhter

$$lim_{\theta \to \infty} q(\theta) = +\infty$$

  and the probability is $q(\theta)dt$

- In Discrete Model $q(\theta)$ is a real probability and it should be bounded

$$V = -c + \beta \left[q(\theta)J + (1 - q(\theta))V \right]$$

  with

$$0 \leq q(\theta) \leq 1$$

- THere is amatching function that ensures that $q(\theta)$ is bounded

$$M = \frac{v_t u_t}{(v_t^{\nu_L} + u_t^{\nu_L})^{\frac{1}{\nu_L}}}$$

- And

$$q(\theta_t) = \frac{M}{V_t} = (1 + \theta^{\nu_L})^{-\frac{1}{\nu_L}}$$

  with $0 \leq q(\theta) \leq 1$