Competitive Search Equilibrium and Directed Search

[Sem0057]

Pietro Garibaldi

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Contents

1 Introduction

- Direct search is a key feature of an economic environment.
- **Random search**
  - Meeting happen randomly with other partners
  - Prices are decided ex-post
- **Directed Search**
  - Agents post prices/terms of trade, and counterparts see the posted prices
  - Searching for a house is an obvious example
- **Competitive Search**

  Definition 1. – *In Competitive Search One side of the market posts prices (trems of trade) and the other side observes what is posted and search accordingly.*

- Let's classify economic environment on the basis of prices and meeting probabilities.
- At the two extreme there are Walrasian Markets and Random Search
- **Walrasian Markets**
  - Everyhing depends on prices (meeting probabilitis are irrelevant)
- **Random Search**
  - The most impostant dimension is meeting probabilities (prices are fixed ex post)
- **Directed Search**
  - Both dimensions are equally important
    1. prices
    2. trading probabilities

- **Who invented Competitive Search ?**
- Two smart graduate students invented competitive search in the labor market
  - Espen Moen at the London School of Economics published his thesis "Competitive Search Equiliibrium" in 1997 in the *Journal of Political Economy.* He was supervised by Chris Pissarides
  - Robert Shimer wrote his thesis at MIT with a very similar paper that was not never published (but still was very successfull in his career). He was supervised by Daron Acemoglu
  - Some early contributions on good prices
    - Peters in 1991
    - Mongomery in 1991
- Key "take aways" of competitive search
  - If you post more favourable terms of trade, you have more chances of trading but no certainty
  - competitive search is likely to be more efficient than directed search
  - Source of the lecture/survey *Kirchert et al. (2019); Journal of Economic Literature*

2 Directed Search in a Static Good Market

- Two type of agents
  - N_b is the stock of buyers
  - N_s is the stock of sellers
- $N = \frac{N_b}{N_s}$ is the buyer/seller ratio *Hint: it is the same as the vacancy unemployment ratio in labor*
- There is a good q that is indivisible (think of q as a tennis racket)
- Sellers produce good q
- One unit of good q costs $c > 0$
- Buyers obtain utility $u > c$ by consuming 1 unit of q
- p is the price of the 1 unit of good

- How is trading regulated
- We assume that there is another good that costs $c(x) = x$ to each party and yields utility x
- This is akin to assume that utility is transferable
- Sellers post price p
  - It is the amount of good x that buyer must pay to get x

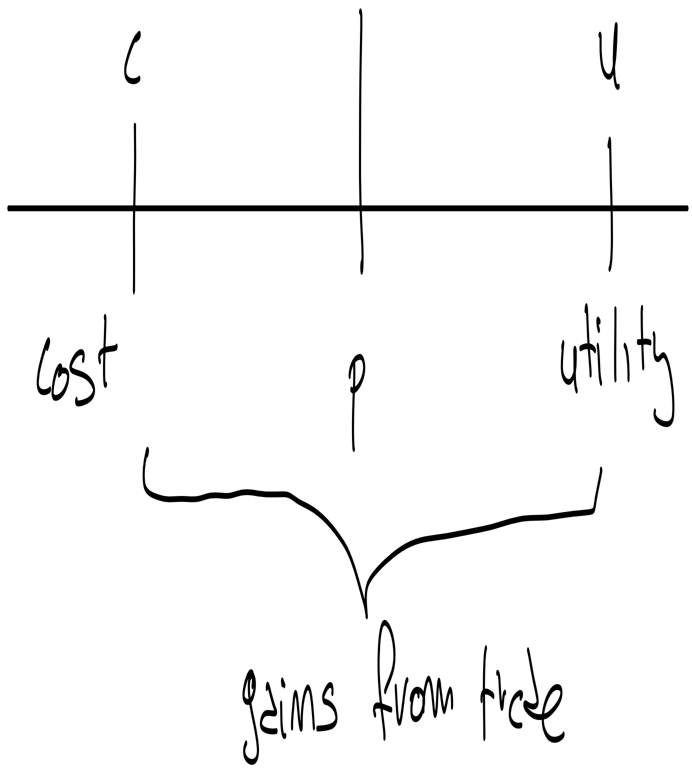

Figure 1: Unemployment Cyclical Dynamics

2.1 The Meeting Technology

- Traders meet pairwise
- n_b and n_s are the number of buyers and sellers that search for price p
- $$m = m(n_b, n_s)$$
- Standard assumptions
$$m_1 >; \qquad m_2 > 0$$
$$m_{11} < 0; \qquad m_{22} < 0$$
There are constant Returns to Scale
- Probability of sellers meeting buyers
  - $$\alpha_s = \frac{m(n_b, n_s)}{n_s} = m(\frac{n_b}{n_s}, 1) = \alpha(n)$$
  - Obviously
$$n = \frac{n_b}{n_s}; \quad \alpha'(n) > 0; \quad \alpha'' < 0$$

- Probability of Buyer meeting sellers
- By definition

$$\alpha_b = \frac{m(n_b, n_s)}{n_b} = \frac{m(n_b, n_s)}{n_s}\frac{n_s}{n_b}$$

$$\alpha_b = \alpha(n)\frac{n_s}{n_b}$$

that can be written as

$$\alpha_b = \frac{\alpha_n}{\frac{n_b}{n_s}} = \frac{\alpha(n)}{n}$$

- Can we proove that

$$\frac{\partial \alpha_b}{\partial n} \overbrace{<}^{??} 0$$

- In general

$$\frac{\partial \alpha_b}{\alpha_n} = \frac{\alpha'(n)n - \alpha(n)}{n^2} \overbrace{< 0}^{?}$$

- We can show that it is negative by assuming $\alpha(0) = 0$ and calling upon a property of concave functions.

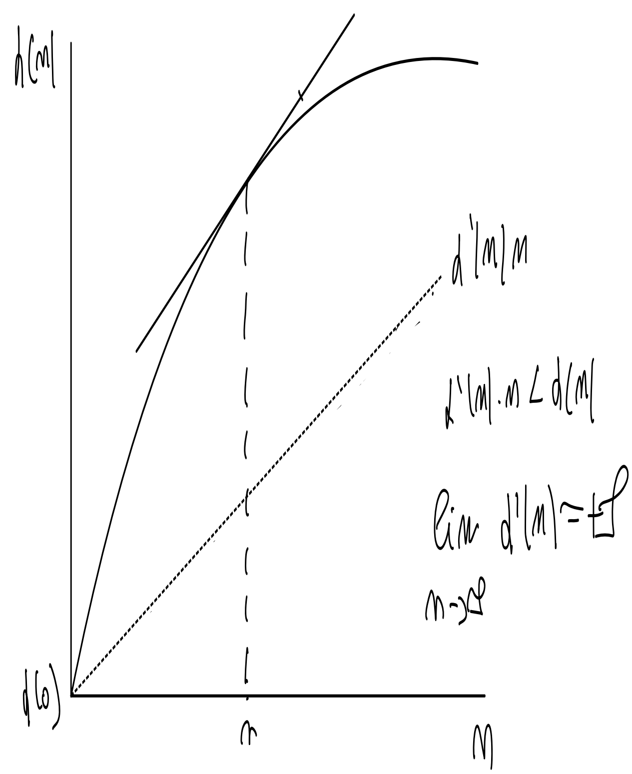

Figure 2: Unemployment Cyclical Dynamics

- It then follows

$$\alpha_b = \frac{\alpha(n)}{n}; \quad \alpha_b' < 0;$$

$$\alpha_s = \alpha(n); \quad \alpha'(n) > 0$$

2.2 Sub-Markets

- In general a buyer seeks for a seller with a particular price p but whether she actually finds one is random
- We introduce the concept of **Sub Market**

  Definition 2. *A Sub-Market is a set of sellers posting the same price p and a set of buyers searching for them. A submarket is a set* (p, n)

- Payofss are V_b and V_s
- What is the problem of the seller?
  - The seller wants to maximize V_s by posting in a sub market (p, n)
  - We will show that it is enough posting a price p and buyers working out n by themselves
- How do you solve this problem ?

2.3 The Market Equilibrium Approach

- For a **seller** to be in business the post (p, n) must deliver to the buyer V_b that is taking as given by the individual sellers

$$\begin{aligned} V_s = & Max_{p,n} \quad \alpha(n)(p-n) \\ \text{s.t.} \quad V_b = & \frac{\alpha(n)}{n}(u-p) \end{aligned} \tag{1}$$

- Note that the payoffs are

$$V_s = \underbrace{\text{trading probability}}_{\alpha(n)} \times \underbrace{\text{payoff}}_{p-c} \tag{2}$$

2.3.1 The Key Indifference Curve

- How do you obtain some utility V_s
  - Either with higher p that increase profits
  - Or with higher $n = \frac{n_b}{n_s}$ that increase meeting probability

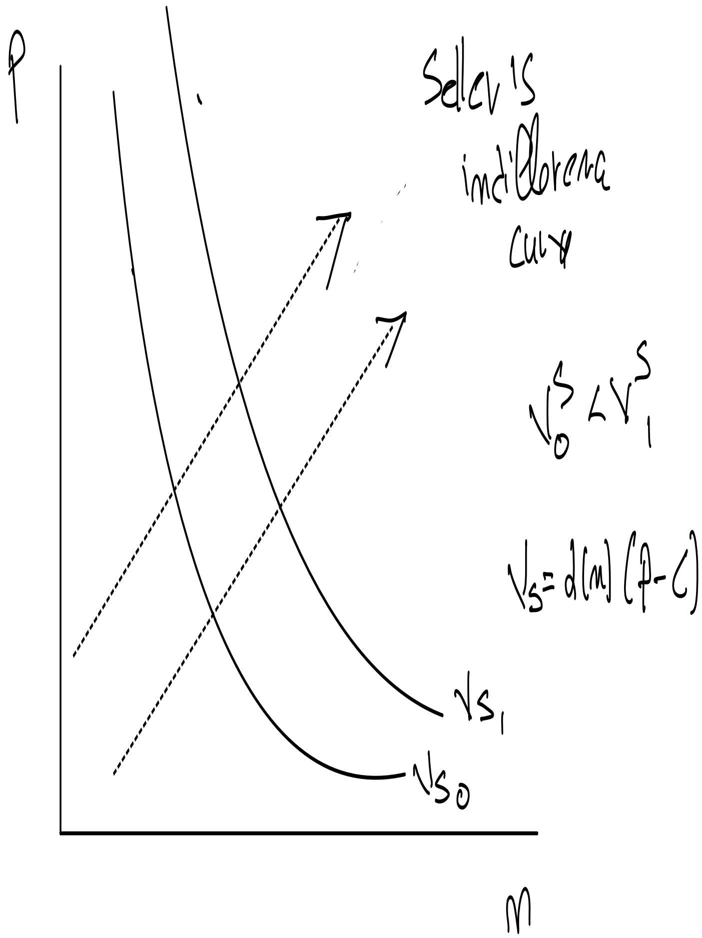

Figure 3: Unemployment Cyclical Dynamics

  - Insert chart

- Let's look at the indifference curves

$$V_s = \alpha(n)(p - c)$$

- Along the indifference curve the total differential is zero

$$dV_s = \alpha'(n)(p - c)dn - \alpha(n)dp = 0$$

so that we get

$$\frac{dp}{dn} = -\frac{\alpha'(n)(p - c)}{\alpha(n)} < 0$$

- It is useful to write it by multiplying and dividing by n as

$$\frac{dp}{dn} = -\frac{\alpha'(n)n(p - c)}{\alpha(n)n} = -\epsilon(n)\frac{p - c}{n} < 0$$

where

$$\epsilon(n) = 0 \leq \frac{\alpha'(n)n}{\alpha)n)} \leq 1$$

is the elasticity of α with respect to n

- Is it convex? We can study the second derivative

$$\frac{d^2p}{dn^2} = -\frac{\overbrace{\alpha''(n)}^{-nve}(p - c)\alpha(n) - (\alpha'(n)^2(p - c)}{\alpha(n)^2} > 0$$

- Is it convex?
  - Let's look at the indifference curve with a particular example

    $$\alpha(n) = n^\gamma; \quad \gamma < 1$$

  - The first derivative is $\alpha'(n) = \gamma n^{\gamma-1}$
  - The indifference curve is

    $$\frac{dp}{dn} = -\frac{\gamma n^{\gamma-1}(p - c)}{n^\gamma} = -\frac{\gamma(p - c)}{n} < 0$$

  - And the second derivative is

    $$\frac{d^2p}{dn^2} = \frac{\gamma(p - c)}{n^2} > 0$$

- The Indifference curve for buyer is

$$V_b = \frac{\alpha(n)}{n}(u-p)$$

- The differential is

$$dV_b = \frac{\left(\left(\alpha'(n)n - \alpha(n)\right)(u-p)\right)}{n^2}dn - \frac{\alpha(n)}{n}dp = 0$$

$$\frac{dp}{dn} = \frac{\alpha(n)\left(\frac{\alpha'(n)n}{\alpha(n)} - 1\right)(u-p)}{\alpha(n)n}$$

- This can be written as

$$\frac{dp}{dn} = \frac{(\epsilon(n)-1)(u-p)}{n} < 0$$

- Is it convex?

$$\frac{d^2p}{dn^2} = \frac{\overbrace{\epsilon'(n)}^{0 \text{ if Cobb Douglas}} \qquad -(\epsilon(n)-1)n}{n^2}(u-p) > 0$$

- with cobb Dougals

$$\left(\frac{\alpha'(n)n}{\alpha(n)}\right) = \gamma$$

$$\frac{dp}{dn} = -\frac{(\gamma-1)(u-p)}{n} > 0; \quad \text{since} \quad \gamma < 1$$

The second derivative is also negative

$$\frac{d^2p}{dn^2} = -\frac{(\gamma-1)(u-p)}{n^2} > 0$$

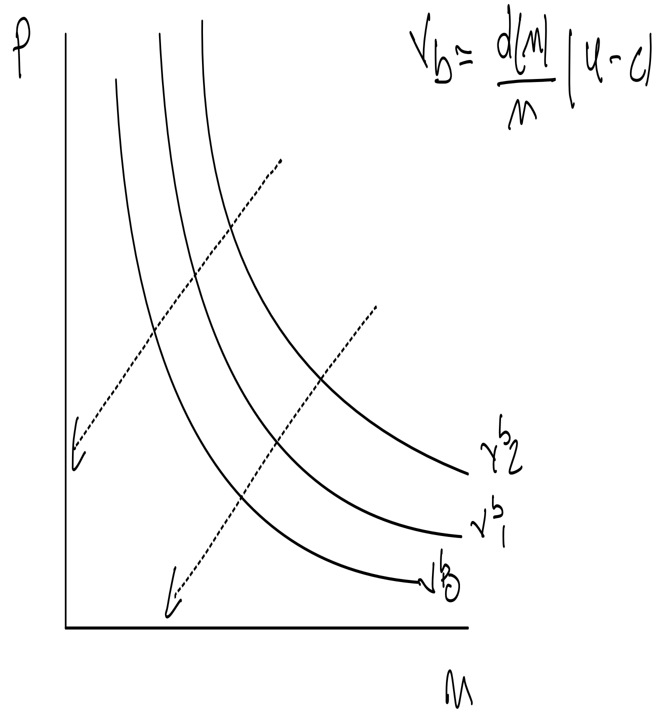

Figure 4: Unemployment Cyclical Dynamics

- The contract curve
- What is the property of the contract curve?
- Basically the slope of the two indifference curve is identical

$$\left|\frac{dp}{dn}\right|_{V_b} = \left|\frac{dp}{dn}\right|_{V_s}$$

- $$\frac{\epsilon(n)(p-c)}{n} = \frac{1-\epsilon(n)(u-p)}{n}$$

  or

  $$\epsilon(n)p - \epsilon(n)c = u - p - \epsilon(n)u + \epsilon(n)p$$

- which leads to

$$p = \epsilon(n)c + (u-p)\epsilon(n)$$

- With Cobb DOuglas

$$\underbrace{\frac{(\gamma-1)(u-p)}{n}}_{\text{slope of the buyer i.c.}} = \underbrace{\frac{\gamma}{n}(p-c)}_{\text{slope of the seller i.c.}}$$

- Which implies

  $$\gamma p - \gamma c = (1-\gamma)u - p + p\gamma$$

  or

  $$p = (1-\gamma)u + \gamma c$$

  which is **The set of prices that satisfy the contract curve**

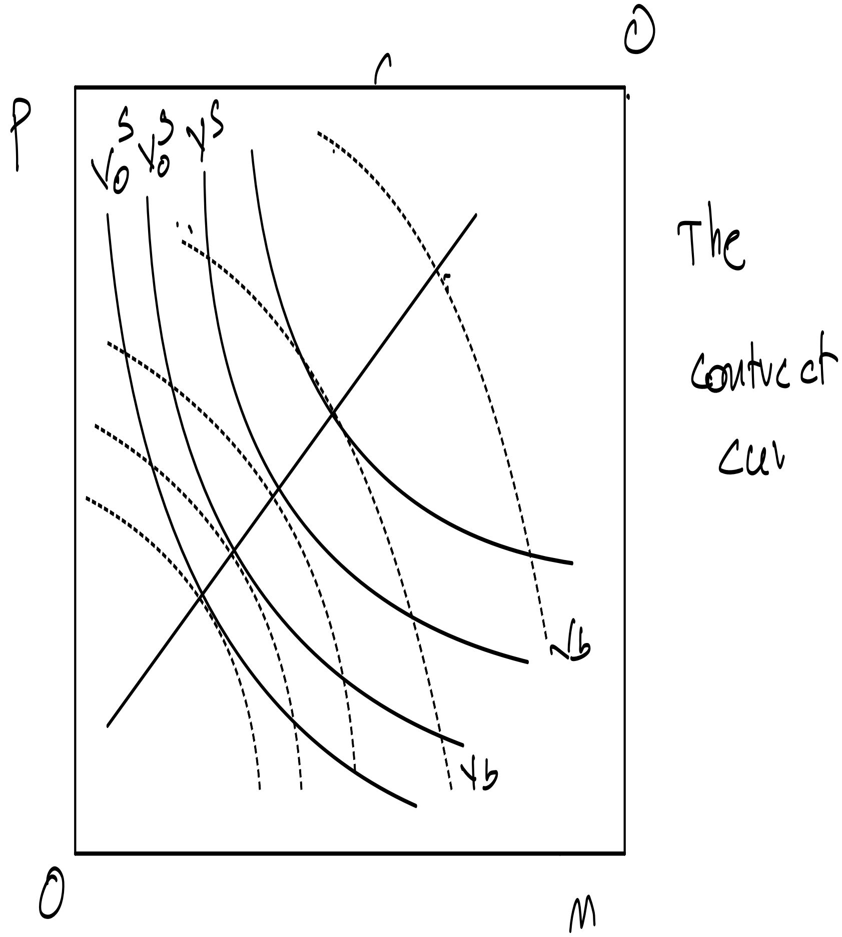

Figure 5: Unemployment Cyclical Dynamics

- Let's go back to the problem

$$
\begin{aligned}
V_s = & Max_{p,n} \quad \alpha(n)(p-n) \\
& \text{s.t.} \quad V_b = \frac{\alpha(n)}{n}(u-p)
\end{aligned}
\tag{3}
$$

- where $n = \frac{\text{buyer}}{\text{seller}}$ is the buyer seller ratio in sub markets (p, n)
- The simple way to solve it is to get rid of p into the objective function and maximize with respect to n
- Take p from teh constraint to obtain

$$p = u - \frac{n}{\alpha(n)} V_b$$

  and substitute it into the objective function
- The problem becomes

$$Max_n \left\{ \alpha(n) \left(\underbrace{u - \frac{n}{\alpha(n)} V_b}_{p} - c \right) \right\} \tag{4}$$

  or

$$Max_n \left\{ \alpha(n)(u-c) - nV_b \right\} \tag{5}$$

- The first order condition is

$$\alpha'(n)(u-c) = V_b; \qquad \text{FOC} \tag{6}$$

$$p = u - \frac{n}{\alpha(n)} V_b; \quad \text{budget constraint} \tag{7}$$

- To find the equilibrium there are two ways to go

2.3.2 The Solution with Fixed Buyers/Sellers

- The system 9 solves the first order condition but it does yet fully solves the system. We need some assumption of closing the supply.
- A first possible solution is assuming that

$$\overline{N}_b, \overline{N}_s; \qquad \text{exogenously given}$$

- Wit this assumption we have that

$$n = N = \frac{\overline{N}_b}{\overline{N}_s} \qquad \text{also given}$$

- Let's start from the system of first order condition with $n = N$

$$\alpha'(N)(u - c) = V_b; \qquad \text{FOC} \tag{8}$$

$$p = u - \frac{N}{\alpha(N)} V_b; \quad \text{budget constraint} \tag{9}$$

- Substitute out V_b to obtain

$$p = u - \underbrace{\frac{N\alpha'(N)}{\alpha(N)}}_{\epsilon(N)} (u - c) \tag{10}$$

  - Recall the elasticity of α with respect to the buyer/seller ratio

$$\epsilon = \frac{\frac{d\alpha}{\alpha}}{\frac{dn}{n}} = \frac{d\alpha}{dn}\frac{n}{\alpha} = \frac{\alpha'(n)n}{\alpha(n)}$$

  - In generally it depends endogenously on n given the underlying matching function. SO that $\epsilon(N)$
  - **but if N is fixed also $\epsilon(N)$ is fixed also if α is not Cobb DOuglas**

- THen the price can be written as

$$p = u - \epsilon(N)(u - c)$$

or

$$p = \epsilon(N)c + (1 - \epsilon(N)u; \qquad \text{With } N \text{ fixed this is detemrined}$$

- PRice becomes a weighted average of buyer and seller utility after trading.
with

$$S = u - c$$

and

$$S = \underbrace{(u - p)}_{S_b} + \underbrace{(p - c)}_{S_s}$$

- The Surplus of the buyer is

$$S = u - \underbrace{\epsilon c - (1 - \epsilon)}_{p} u$$

- The surplus of the seller is

$$S_s = p - c$$

or

$$S_s = \epsilon c + (1 - \epsilon)u - c = (1 - \epsilon)$$

- Which are the endogenous variables of the model ?

$$\{V_b, V_s, n, p\}$$

- In the case of eogenous traders we have

$$n = N; \qquad N = \frac{\overline{N}^b}{\overline{N}^s} \text{ exogenously given} \tag{11}$$

$$p = \epsilon(N)c + (1 - \epsilon(N))U \qquad \text{Gives } p \text{ given } \epsilon(N) \tag{12}$$

$$V_b = \frac{\alpha(N)}{N}(u - p) = \frac{\alpha(N)}{N}\epsilon(u - c); \qquad \text{Gives } V_b \text{ given } N \text{ and } p \tag{13}$$

$$V_s = \alpha(N)(p - c) = \alpha(N)(1 - \epsilon)(u - c); \qquad \text{Gives } V_s \text{ given } N \tag{14}$$

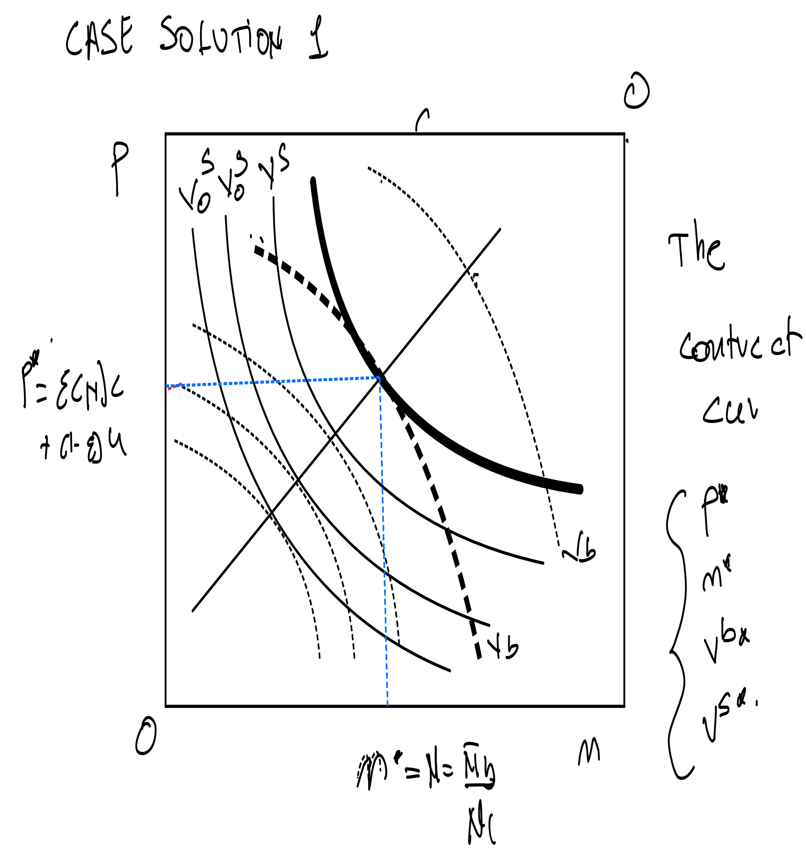

Figure 6: Unemployment Cyclical Dynamics

2.3.3 The SOluton with a Market Participant Cost

- The Second alternative is that on market participation cost
- The seller has a participation cost k
- As long as k is not too big or too small some but NOT all seller will enter.

$$V_s = k_s; \qquad \text{Entry condition}$$

Let's see how to solve it

$$V_b = \alpha'(n)(u - c) \tag{15}$$
$$nnp = \epsilon(n)c + (1 - \epsilon(n)u \tag{16}$$

- The four equilibrium quantities are the same

$$\{V_b, V_s, p, n\}$$

- The solution is

$$V_s = k_s \qquad \text{Entry condition} \tag{17}$$
$$k_s = \alpha(n)(p - c) \qquad \text{1 out of 3 equations for solving for } n, p, V_b \tag{18}$$
$$V_b = \alpha'(n)(u - p) \qquad \text{1 out of 3 equations for solving for } n, p, V_b \tag{19}$$
$$p = \epsilon(n)c + (1 - \epsilon(n))u \qquad \text{1 out of 3 equations for solving for } n, p, V_b \tag{20}$$

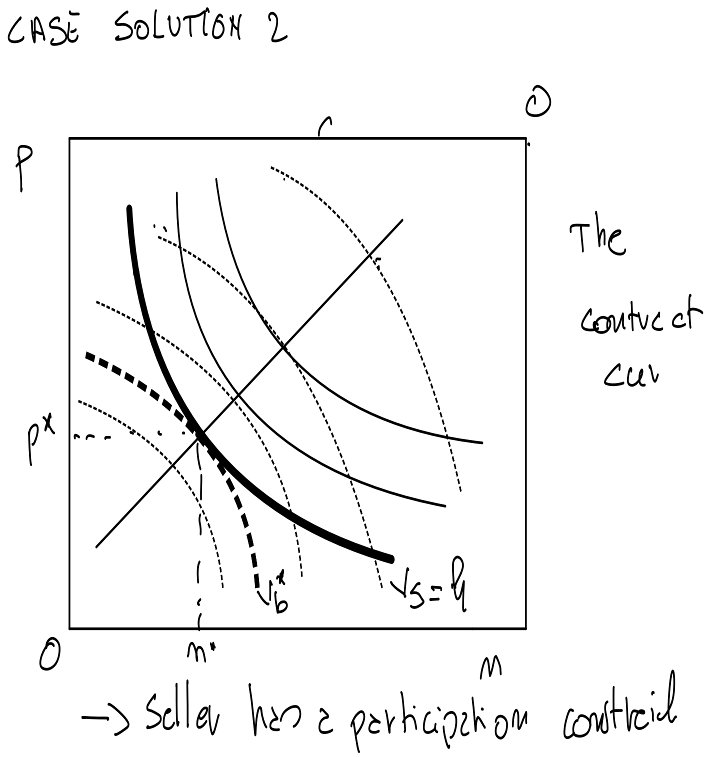

Figure 7: Unemployment Cyclical Dynamics

- Let's take a very simple example with Cobb DOugals Matching Function

$$\alpha(n) = n^{\gamma}; \qquad 0 < \gamma < 1; \qquad \epsilon(n) = \gamma$$

- The system is

$$k_s = n^{\gamma}(p - c); \qquad n^{\gamma} = \frac{k_s}{p - c}; \tag{21}$$

$$V_b = n^{\gamma - 1}(u - p) \tag{22}$$

$$p = \gamma c + (1 - \gamma)u \tag{23}$$

- So for obtaining n^* one has

$$n^* = \left(\frac{k_s}{p - c}\right)^{\frac{1}{\gamma}}$$

  or

$$n^* = \left(\frac{k_s}{(1 - \gamma)(u - c)}\right)^{\frac{1}{\gamma}}$$

and

$$V_b^* = \left(\frac{k_s}{(1 - \gamma)(u - c)}\right)^{\frac{\gamma - 1}{\gamma}} (u - c)$$

2.4 The Model when the buyer posts and the seller search.

- The problem is

$$V_b = max_{p;n} \quad \frac{\alpha(n)}{n}(u-p) \tag{24}$$

$$\text{s.t.} \qquad \alpha(n)(p-c) = V_s \tag{25}$$

- Let's work this out in the case of exogenous entry $V_s = k_s$ and the price is

$$p = c + \frac{k_s}{\alpha(n)}$$

and substituting out into the objection function we have

$$V_b = Max_n \left\{ \frac{\alpha(n)}{n} \left(u - c - \frac{k_s}{\alpha(n)} \right) \right\} \tag{26}$$

or

$$V_b = Max_n \left\{ \frac{\alpha(n)}{n}(u-c) - \frac{k_s}{n} \right\}$$

- The first order condition is

$$\frac{\alpha'(n)n - \alpha(n)}{n^2}(u-c) + \frac{k_s}{n^2} = 0$$

or

$$k_s = -\left(\alpha'(n)n - \alpha(n)\right)(u-c)$$

and collecting $\alpha(n)$

$$k_s = -\alpha(n) \left[\underbrace{\frac{\alpha' n}{\alpha(n)}}_{\epsilon(n)} - 1 \right] (u-c)$$

but recall that

$$p = c + \frac{k_s}{\alpha(n)}$$

$$p = c - \frac{\alpha(n)}{\alpha(n)}\left(\epsilon(n) - 1\right)(u-c)$$

or

$$p = c - (\epsilon(n) - 1)(u-c)$$

$$p = c\epsilon(n) + u(1 - \epsilon(n)) \qquad \text{QED}$$

- which proves that it is the same endogenous price

2.5 Efficiency and a third Alternative

- In principle (and Moen original contribution in particular) showed that equilibrium can be obtained by market makers that organize sub-markets and attract buyers and sellers
- We should focus on **Efficiency**
- What is the central planner problem with endogenous participation at cost k and a constrained central planner that face the same matching function.
- The social value is

$$W = \text{ Surplus per Buyer} - \text{ total entry costs of sellers per buyer}$$

or

$$Max_n W = \frac{\alpha(n)}{n}(u-c) - k\frac{N_s}{N_b}$$

since $n = \frac{N_b}{N_s}$

$$Max_n W = \frac{\alpha(n)}{n}(u-c) - \frac{k}{n} \qquad \text{Central Planner 1}$$

- Let's go back to the market problem and consider the problem of the buyer positing with $V_s = k$

$$Max_s \left\{ \frac{\alpha(n)}{n}(u-c) - \frac{\overbrace{V_s}^{k}}{n} \right\}$$

which is indeed idential to Central Planner 1

2.6 Random Matching with Ex-post bargaining

- We consider the identical model but we now assume that bargaining takes place after the parties meet.
- It is basically a **random matching** version of the model
- Asumme that θ is the bargaining share that goes to the buyer
- What is the Nash Maximand in this case?

$$\Omega = (u-p)^{\theta} p - c^{1-\theta}$$

which can be expressed through a monotonic transformation

$$Max_p \Omega = Max_p ln(\Omega)$$

$$Max_p = \theta ln(u-p) + (1-\theta) ln(p-c)$$

- And the first order condition is

$$-\frac{\theta}{u-p} + \frac{1-\theta}{(p-c)} = 0$$

or

$$(1-\theta)(u-p) = \theta(p-c)$$

by doing some algebra

$$(1-\theta)u - p + p\theta = p\theta - c\theta$$

- so that the price is

$$p = \theta c + (1-\theta)u$$

- And what is the implications?

$$\text{if} \qquad \theta = \epsilon(n^*); \quad \Longrightarrow \quad \text{Random Search} = \text{Competitive Search}$$

- But

$$\theta \qquad \text{is the ex poist bargaining share}$$

$$\epsilon(N) \qquad \text{is the elasticity of the matching funciton}$$

- But this is the Hosio conditions !
- IN other words only if the Hosios condition is satisfed ex post bargaining is efficient while Competitive search is always efficient!

2.7 A Labor Market Interpretation of the Static Directed Search

- What is going on in the Labor Market?
- FIrms
  - Buy time in exchange for w
  - It then follows
$$N_b = v; \qquad \text{Stock of Vacancies}$$
- Workers
  - Sell time in exchange for a salary
  -
$$N_s = u + e \qquad \text{Stock of Workers: unemployed plus employed}$$
- We solve the competitive search from workers' standpoing as seller.
- What is the payoff in the static model ?
$$U = \alpha(n)(w - b)$$

- Let write the problem

$$U = Max_{w;n} \quad \{\alpha(n)(w-b)\} \tag{27}$$

$$\text{s.t.} \quad V = \frac{\alpha(n)}{n}(y-w) \tag{28}$$

where

$$n = \frac{N_s}{N_s} = \frac{v}{u+e}$$

- Let's assume (as in the basic model) that free entry for the buyers implies

$$V = k$$

so that the constraint is

$$w = y + \frac{kn}{\alpha(n)}$$

- The problem becomes a simple maximization with respect to n

$$U = max_n \left\{ \alpha(n) \left(y + \frac{kn}{\alpha(n} - b \right) \right\}$$

or

$$U = max_n \left\{ \alpha(n) \left(y - b \right) - kn \right\}$$

- The first order condition is

$$\alpha'(n)(y-b) = k$$

- Going back to the wage

$$w = y + \frac{\overbrace{k}^{\alpha'(n)(y-b)} \quad n}{\alpha(n)}$$

- We obtain the wage

$$w = y - \frac{\alpha'(y-b)n}{\alpha(n)}$$

and recalling that

$$\epsilon(n) = \frac{\alpha'(n)n}{\alpha(n)}$$

- We obtain

$$w = y - (y-b)\epsilon(n)$$

or

- The final expression for the wage

$$w = \epsilon(n)b + (1 - \epsilon(n)y$$

- What is unemployment in the one period model?
  - Unemployed are the searching sellers that were unlucky and did not find a partner

$$u = (1 - \alpha(n)))N_s \qquad \text{The sellers who are not matached end up unemployed}$$

  - Employment is thus

$$e = \alpha(n)N_s$$

3 Dynamic Competitive Search- Moen 1997

- The Setting is identical to the basic SAM model with exogenous job destruction (the one also used by SHimer for BC)
- The matching function is standard

$$q(\theta) = \frac{x(u.v)}{v}; \qquad \eta(\theta) = -\frac{q'(\theta)\theta}{q(\theta)}$$

- The simplest way to solve the competitive search equilibrium is solve the **Rent Posting Game**. The model turns out to be much simpler than a pure wage positing gae,.
- We need to introduce the concept of rent.

$$R = \underbrace{W}_{\text{Value of Employment}} - \underbrace{U}_{\text{Value of Unemployment}} = \underbrace{S_w}_{WorkerSurplus}$$

- Basically we assume that firms post rents that workers observe in different submarkets.

- The value of unemployment is

$$rU = z + \theta q(\theta) \underbrace{[W - U]}_{R}$$

  or

$$rU = z + \theta q(\theta) R$$

- Note that if one fixes a value of U, this is the workers' indifference curve
  - How can you get a value of rU?
  - With a combination of the some θ and some R. IN principle both factors lead to increasing worker welfare. Thus to get the same rU there is a trade off between the two.
  - THe differential is

$$drU = q(\theta)(1 - \eta(\theta))Rd\theta + \theta q(\theta)dR \underbrace{=}_{\text{along an i.c.}} 0$$

  - which implies that the slope of the indifference curve is

$$\left.\frac{d\theta}{dR}\right|_{U=\overline{U}} = -\frac{\theta}{R(1 - \eta(\theta))} < 0 \qquad \text{Slope of Workers' indifference Curve} \tag{29}$$

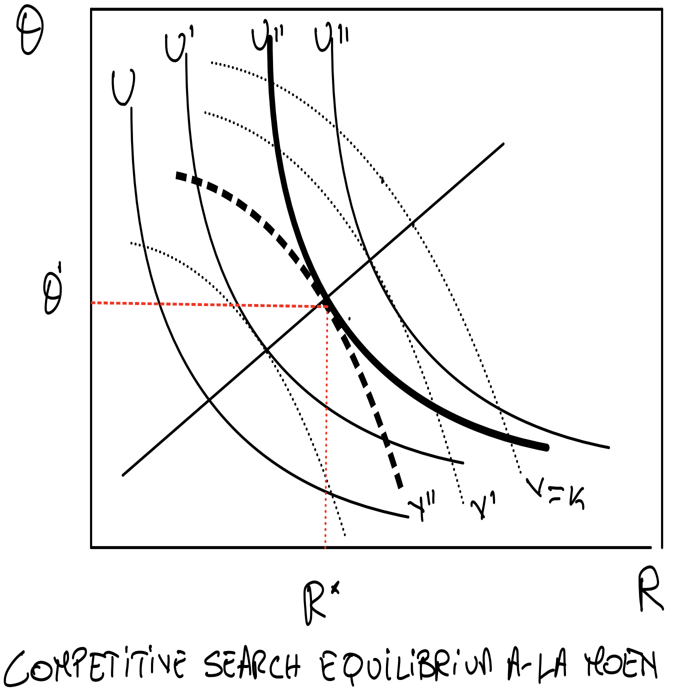

Figure 8: Competitive Search in the Rent Posting Game

- Do firms face a similar trade off?
- THe value of a vacancy reads

$$rV = -c + q(\theta)\underbrace{[J-V]}_{S-R}$$

- But we know that from the surplus definition

$$S = (J-V) + \underbrace{(W-U)}_{R}$$

or

$$J - V = S - W$$

- This implies that the vacancy can be written as

$$rV = -c + q(\theta)[S-R]$$

Let's look at the slope of the isovacancy o isoprofit

$$drV = q'(\theta)[S-R]d\theta - q(\theta)dR \qquad \underbrace{=}_{\text{Along a iso profit curve}} \qquad 0$$

- SO that we have

$$\left.\frac{d\theta}{dR}\right|_{rV=r\overline{V}} = \frac{q(\theta)}{q'(\theta)[S-R]} < 0 \tag{30}$$

- What is the Contract Curve in this case

$$\underbrace{\left|\frac{d\theta}{dR}\right|_{U=\overline{U}}}_{\text{Slope of the isoprofit}} = \underbrace{\left|\frac{d\theta}{dR}\right|_{rV=r\overline{V}}}_{\text{Slope of the i.c.}} \tag{31}$$

- Equating the two expressions in absolute value

$$-\frac{q(\theta)}{q'(\theta)\,[S-R]} = \frac{\theta}{R(1-\eta(\theta)}$$

and multiplying by sides by $\frac{1}{\theta}$

$$-\frac{1}{\underbrace{\frac{\theta q'(\theta)}{q(\theta)}}_{\eta(\theta)}\,[S-R]} = \frac{\theta}{R(1-\eta(\theta))\theta}$$

- We obtain

$$\frac{1}{\eta(\theta)\,[S-R]} = \frac{1}{R\,[1-\eta(\theta)]}$$

or

$$R\,(1-\eta(\theta)) = \eta(\theta)\,(S-R)$$

- So that along the contract curve we have

$$R = \eta(\theta)$$

or

$$W - U = \eta(\theta)S$$

which implies that **along the contract curve the Hosios condition is satisfied**

3.1 The Reduced Form

- Let's work out the reduced from of the dynamic competitive search (this helps also for Problem Set 3)
- We already know that on the rent positing game we have

$$R = W - U = \eta(\theta)S$$

- Moen use and entry cost to close the model
  - The Equilibrium value of the vacancy must be be such that

$$V^* = K$$

  - But the value of the vacancy in general solves the following problem

$$rV = Max_R \left[-c + q(\theta)\left(S - R\right)\right] \tag{32}$$

$$\text{s.t.} \qquad rU = z + \theta q(\theta) R \tag{33}$$

  - And θ^* is such that

$$R = \eta(\theta^*)S$$

- Let's work out the reduced form

$$V^* = k \tag{34}$$
$$rV^* = -c + q(\theta^*)(J - V) \tag{35}$$
$$W - U = \eta(\theta^*)S \tag{36}$$

- This implies that in equilibrium

$$rk + q(\theta^*)k = -c + q(\theta^*)J \tag{37}$$

or

$$\frac{rk}{q(\theta^*)} + k + \frac{c}{q(\theta^*)} = J$$

- But recall that

$$J - V = (1 - \eta(\theta))S$$

or

$$(J - k) = (1 - \eta(\theta))S$$

so that

$$\frac{rk}{q(\theta^*)} + \frac{c}{q(\theta^*)} = \underbrace{J - k}_{(1-\eta(\theta))S}$$

- The value of the surplus is

$$S = \frac{y - rU - rk}{r + \lambda}$$

- and

$$rU = z + \theta q(\theta)R; \qquad R = \eta(\theta)S$$

- The reduced form is thus

$$rU = z + \theta q(\theta)\eta(\theta)S \tag{38}$$

$$\frac{rK}{q(\theta)} + \frac{c}{q(\theta)} = (1-\eta(\theta))S \tag{39}$$

$$S = \frac{y - rU - rk}{r+\lambda} \tag{40}$$

$$u = \frac{\lambda}{\lambda + \theta q(\theta))} \tag{41}$$

  where equations 38 39 and 40 determine a system with S, θ, U and then unemployment is solved.

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3.2 Random Search with Entry cost of Vacancies

- We only want to check whether it is true that the competitive search a la Moen is identical to random search when Hosios condition is satisfied.
- It is also a problem set to see the model with $V = k > 0$ as entry condition
- Recall the value of a vacancy
$$rV = -c + q(\theta)\left[J - V\right]$$
with $V = k$ we get
$$rk + q(\theta)k + c = q(\theta)J$$
- Or
$$\frac{rk}{q(\theta)} + k + \frac{c}{q(\theta)} = J$$
and also
$$\frac{rk}{q(\theta)} + \frac{c}{q(\theta)} = J - k$$
- The value of a job is
$$rJ = y - w + \lambda\left[\overbrace{V}^{k} - J\right]$$

- Start from join income

$$M = J + W; \qquad S = M - U - k$$

- The wage ruls is thus

$$\underbrace{W - U}_{R} = \beta \underbrace{\left[\underbrace{J + W}_{M} - k - U \right]}_{S}$$

- Further

$$J - k = (1-\beta)S; \qquad S = \frac{J-k}{1-\beta}$$

- Indeed if $\beta = \eta(\theta)$ the two models are identical
- Recall

$$S = \frac{y - rU - rk}{r + \lambda}$$

- The reduced form is thus

$$rU = z + \theta q(\theta)\beta S \tag{42}$$

$$\frac{rK}{q(\theta)} + \frac{c}{q(\theta)} = (1-\beta)S \tag{43}$$

$$S = \frac{y - rU - rk}{r + \lambda} \tag{44}$$

$$u = \frac{\lambda}{\lambda + \theta q(\theta))} \tag{45}$$

where equations 38 39 and 40 determine a system with S, θ, U and then unemployment is solved.

3.2.1 A CAveat on Entry Cost versus Hiring Costs

- It is easy to mix up entry cost k with hiring cost H
- With hiring costs H the firm incurs the costs when form the job (meets the worker)

$$rV = -c + q(\theta)\left[J - H - V\right]$$

$V = 0$ implies

$$\frac{c}{q(\theta)} = J - H$$

- and the surplus is

$$S = J - 0 + W - U$$

- H is more similar to F firing tax

$$rV = -c + q(\theta)\left[J - V\right]$$

or

$$rJ = y - w + s\lambda\left[V - F - J\right]$$

with $V = 0$

$$(r + \lambda)J = y - w - sF$$

and the surplus is

$$S = J - (V - F) + W - U$$

or

$$S = J + F + W - U$$