Rational expectations and the “new classical macroeconomics”

(1) Mechanisms of expectations formation (adaptive vs. rational)

(2) Comovements of output and inflation in a rational expectations model

(3) Policy implications: the "policy ineffectiveness proposition"

Adaptive expectations vs. rational expectations

- ***adaptive expectations*** (Friedman-Phelps):

$$p^e_{t,t+1} = p^e_{t-1,t} + \lambda \underbrace{(p_t - p^e_{t-1,t})}_{\text{forecast error in } t} \qquad 0 < \lambda < 1$$

$p^e_{t,t+1}$ is the expectation of p_{t+1} formed at t

- ***adaptive expectations*** (Friedman-Phelps):

$$p^e_{t,t+1} = p^e_{t-1,t} + \lambda \underbrace{(p_t - p^e_{t-1,t})}_{\text{forecast error in } t} \qquad 0 < \lambda < 1$$

$p^e_{t,t+1}$ is the expectation of p_{t+1} formed at t

$\Rightarrow$ possibility of *systematic* forecast errors

- ***rational expectations*** (Muth-Lucas-Sargent):

$$p^e_{t,t+1} = \underbrace{E(p_{t+1} \mid \Omega_t)}_{\substack{\text{mathematical expectation} \\ \text{of } p_{t+1} \text{ conditional on } \Omega_t}} \equiv E_t p_{t+1}$$

mathematical expectation

of p_{t+1} conditional on Ω_t

$\Rightarrow$ *only non-systematic forecast errors* ("surprises"):

$$E_t\,(p_{t+1} - E_t p_{t+1}) = 0$$

Hyperinflation model (Cagan 1956, Sargent-Wallace 1973)

Only one good and money.

Money market equilibrium (in logs, no stochastic elements):

$$m_t - p_t = -\alpha \underbrace{(p^e_{t,t+1} - p_t)}_{\substack{\text{expected inflation rate} \\ \text{between } t \text{ and } t+1}} \qquad \alpha > 0$$

Note: money market equilibrium in levels

$$\frac{M_t}{P_t} = \bar{Y} e^{-\alpha(\bar{r}+\pi^e_{t,t+1})}$$

with nominal interest rate $i \equiv \bar{r} + \pi^e_{t,t+1}$

Taking logs:

$$m_t - p_t = \underbrace{(\log \bar{Y} - \alpha \bar{r})}_{\text{constant}} - \alpha \pi^e_{t,t+1}$$

Then, with $\pi^e_{t,t+1} \simeq p^e_{t,t+1} - p_t$

$$m_t - p_t = -\alpha (p^e_{t,t+1} - p_t)$$

Money market equilibrium (in logs, no stochastic elements):

$$m_t - p_t = -\alpha \underbrace{(p^e_{t,t+1} - p_t)}_{\substack{\text{expected inflation rate} \\ \text{between } t \text{ and } t+1}} \qquad \alpha > 0$$

$\Rightarrow$ price level at t:

$$p_t = \frac{1}{1+\alpha} m_t + \frac{\alpha}{1+\alpha} p^e_{t,t+1}$$

Adaptive expectations

$$p^e_{t,t+1} = \lambda p_t + (1-\lambda) p^e_{t-1,t}$$

Substituting backward for $p^e_{t-1,t}$:

$$p^e_{t,t+1} = \lambda p_t + \lambda(1-\lambda) p_{t-1} + \quad (1-\lambda)^2 p^e_{t-2,t-1}$$

By repeated backward substitution

$$p^e_{t,t+1} = \lambda p_t + \lambda(1-\lambda)p_{t-1} + \lambda(1-\lambda)^2 p_{t-2} + \ldots +$$

$$+ \lambda(1-\lambda)^{T-1} p_{t-T+1} + \underbrace{(1-\lambda)^T p^e_{t-T,t-T+1}}_{\text{tends to 0 as } T\to\infty}$$

$$\Rightarrow \quad p^e_{t,t+1} = \lambda \sum_{i=0}^{\infty} (1-\lambda)^i p_{t-i}$$

Solving for the equilibrium price level p_t:

$$p_t = \frac{1}{1+\alpha(1-\lambda)} m_t + \frac{\alpha\lambda}{1+\alpha(1-\lambda)} \sum_{i=1}^{\infty} (1-\lambda)^i p_{t-i}$$

Note that:

$$\frac{1}{1+\alpha(1-\lambda)} + \frac{\alpha\lambda}{1+\alpha(1-\lambda)} \sum_{i=1}^{\infty} (1-\lambda)^i = 1$$

Rational expectations

$$p_t = \frac{1}{1+\alpha} m_t + \frac{\alpha}{1+\alpha} E_t p_{t+1}$$

Leading forward by one period

$$p_{t+1} = \frac{1}{1+\alpha} m_{t+1} + \frac{\alpha}{1+\alpha} E_{t+1} p_{t+2}$$

taking the expected value as of *t*:

$$E_t p_{t+1} = \frac{1}{1+\alpha} E_t m_{t+1} + \frac{\alpha}{1+\alpha} E_t p_{t+2}$$

Note: by the *law of iterated expectations:* $E_t(E_{t+1} p_{t+2}) = E_t p_{t+2}$

More formally:

$$E_{t+1}p_{t+2} \equiv E(p_{t+2} \mid \Omega_{t+1})$$

then

$$E_t(E_{t+1}p_{t+2}) \equiv E[E(p_{t+2} \mid \Omega_{t+1}) \mid \Omega_t] = E(p_{t+2} \mid \Omega_t) \equiv E_t p_{t+2}$$

with

$$\Omega_t \subset \Omega_{t+1}$$

Example:

$$p_t = \rho p_{t-1} + u_t$$

so that

$$\begin{aligned} p_{t+2} &= \rho p_{t+1} + u_{t+2} \\ &= \rho^2 p_t + \rho u_{t+1} + u_{t+2} \end{aligned}$$

Now

$$E_{t+1}p_{t+2} = \rho^2 p_t + \rho u_{t+1} \quad \text{since } \Omega_{t+1} = \{p_t, u_{t+1}\}$$

and

$$\begin{aligned} E_t(E_{t+1}p_{t+2}) &\equiv E[E_{t+1}p_{t+2} \mid \Omega_t] \\ &= E[\rho^2 p_t + \rho u_{t+1} \mid \Omega_t] = \rho^2 p_t \quad \text{since } \Omega_t = \{p_t\} \end{aligned}$$

which is equal to

$$\begin{aligned} E_t p_{t+2} &\equiv E[p_{t+2} \mid \Omega_t] \\ &= E[\rho^2 p_t + \rho u_{t+1} + u_{t+2} \mid \Omega_t] = \rho^2 p_t \end{aligned}$$

Going back to:

$$p_t = \frac{1}{1+\alpha} m_t + \frac{\alpha}{1+\alpha} E_t p_{t+1}$$

and

$$E_t p_{t+1} = \frac{1}{1+\alpha} E_t m_{t+1} + \frac{\alpha}{1+\alpha} E_t p_{t+2}$$

we get by substitution:

$$p_t = \frac{1}{1+\alpha} m_t + \frac{\alpha}{1+\alpha} \left(\frac{1}{1+\alpha} E_t m_{t+1} + \frac{\alpha}{1+\alpha} E_t p_{t+2} \right)$$

Repeatedly substituting forward for $E_t p_{t+i}$ $(i = 2, 3, \ldots, T-1)$:

$$p_t = \frac{1}{1+\alpha} \sum_{i=0}^{T-1} \left(\frac{\alpha}{1+\alpha} \right)^i E_t m_{t+i} + \underbrace{\left(\frac{\alpha}{1+\alpha} \right)^T E_t p_{t+T}}_{\text{tends to 0 as } T \to \infty}$$

$$\Rightarrow \quad p_t = \frac{1}{1+\alpha} \sum_{i=0}^{\infty} \left(\frac{\alpha}{1+\alpha} \right)^i E_t m_{t+i}$$

Note:

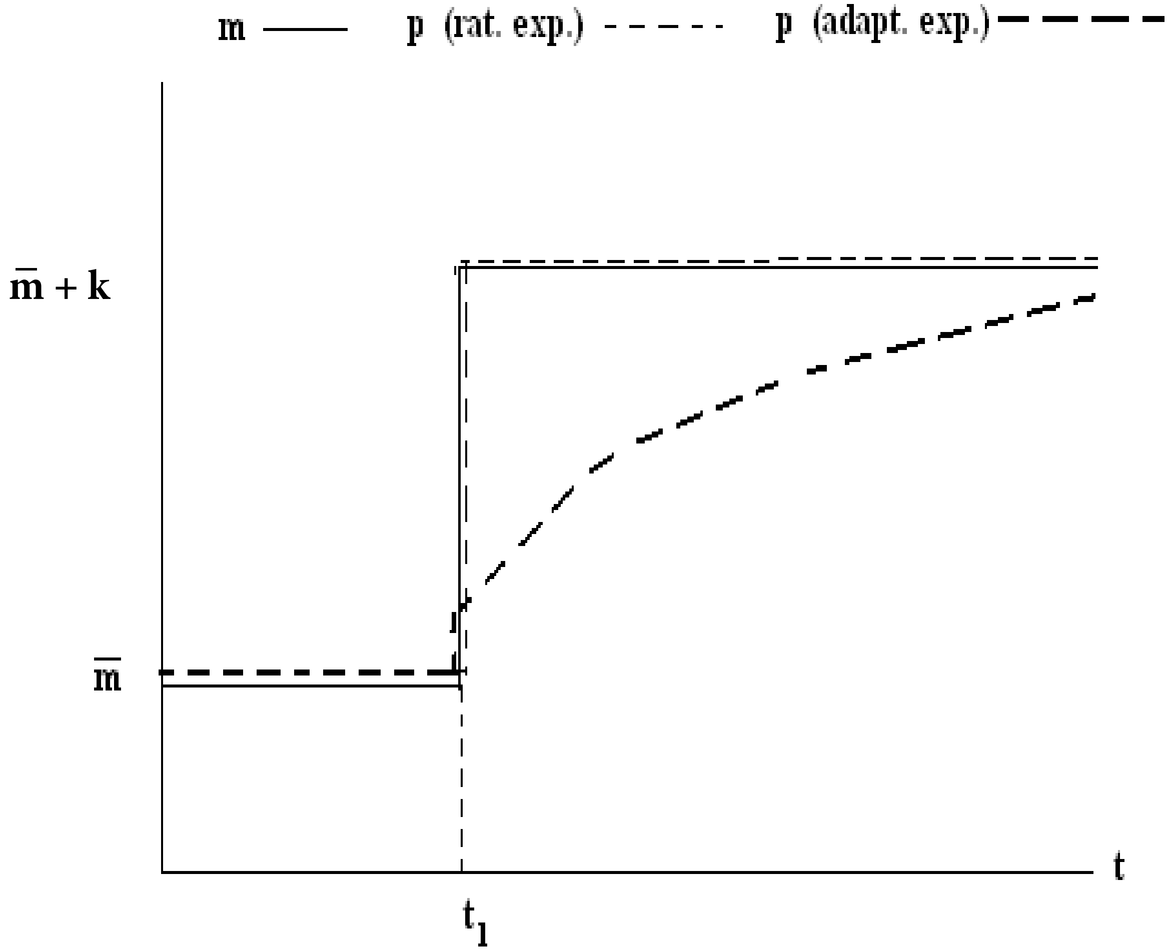
$$\frac{1}{1+\alpha} \sum_{i=0}^{\infty} \left(\frac{\alpha}{1+\alpha} \right)^i = 1$$

Permanent increase of m from $\bar{m}$ to $\bar{m}+k$ at t_1

- with *adaptive* expectations:

$$p_{t_1} = \frac{1}{1+\alpha(1-\lambda)}k + \bar{m}$$

After t_1 the price level gradually increases up to $\bar{m}+k$

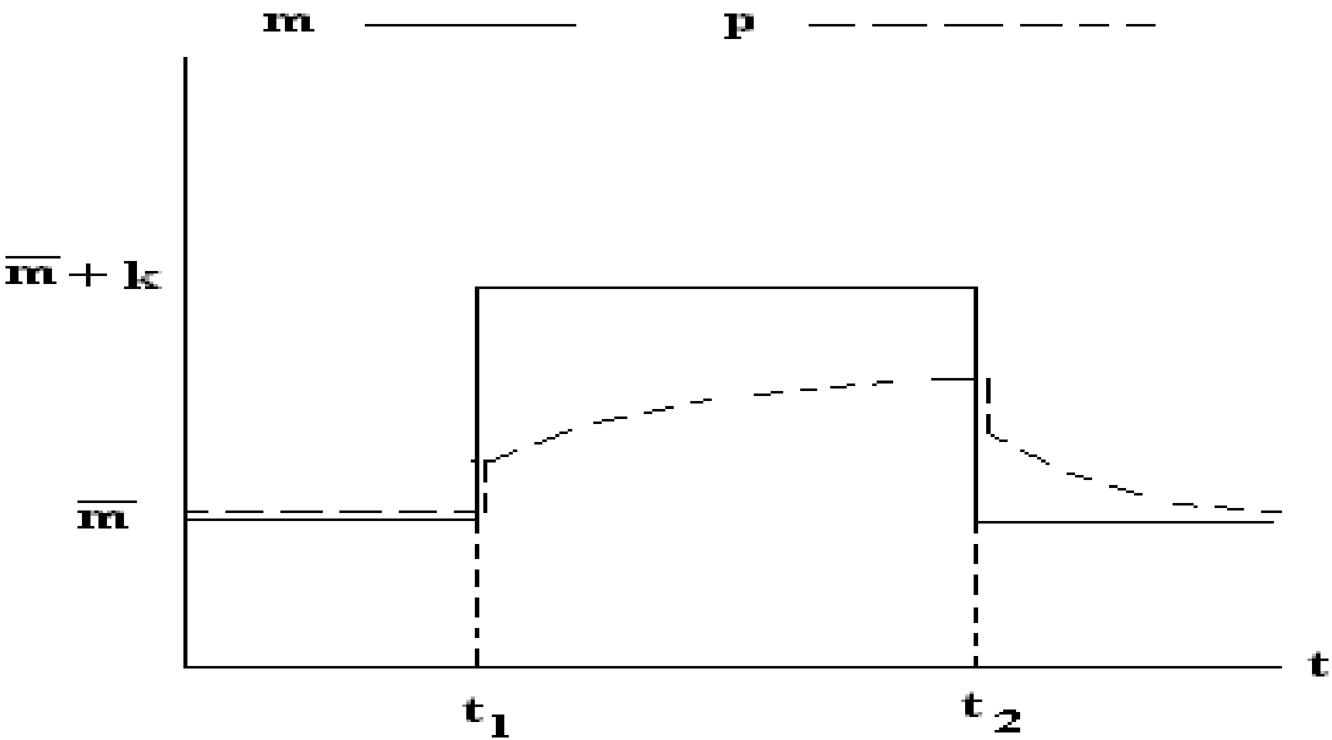


m
p (rat. exp.)
p (adapt. exp.)
$\bar{m} + k$
$\bar{m}$
t
t_1

- with *rational* expectations p jumps at t_1 to the new stationary equilibrium level

Temporary increase of m to $\bar{m} + k$ only between t_1 and $t_2 - 1$

- with *adaptive* expectations the behavior of p is the same as before until $t_2 - 1$; then the price level starts decreasing (with expectations gradually adjusting)

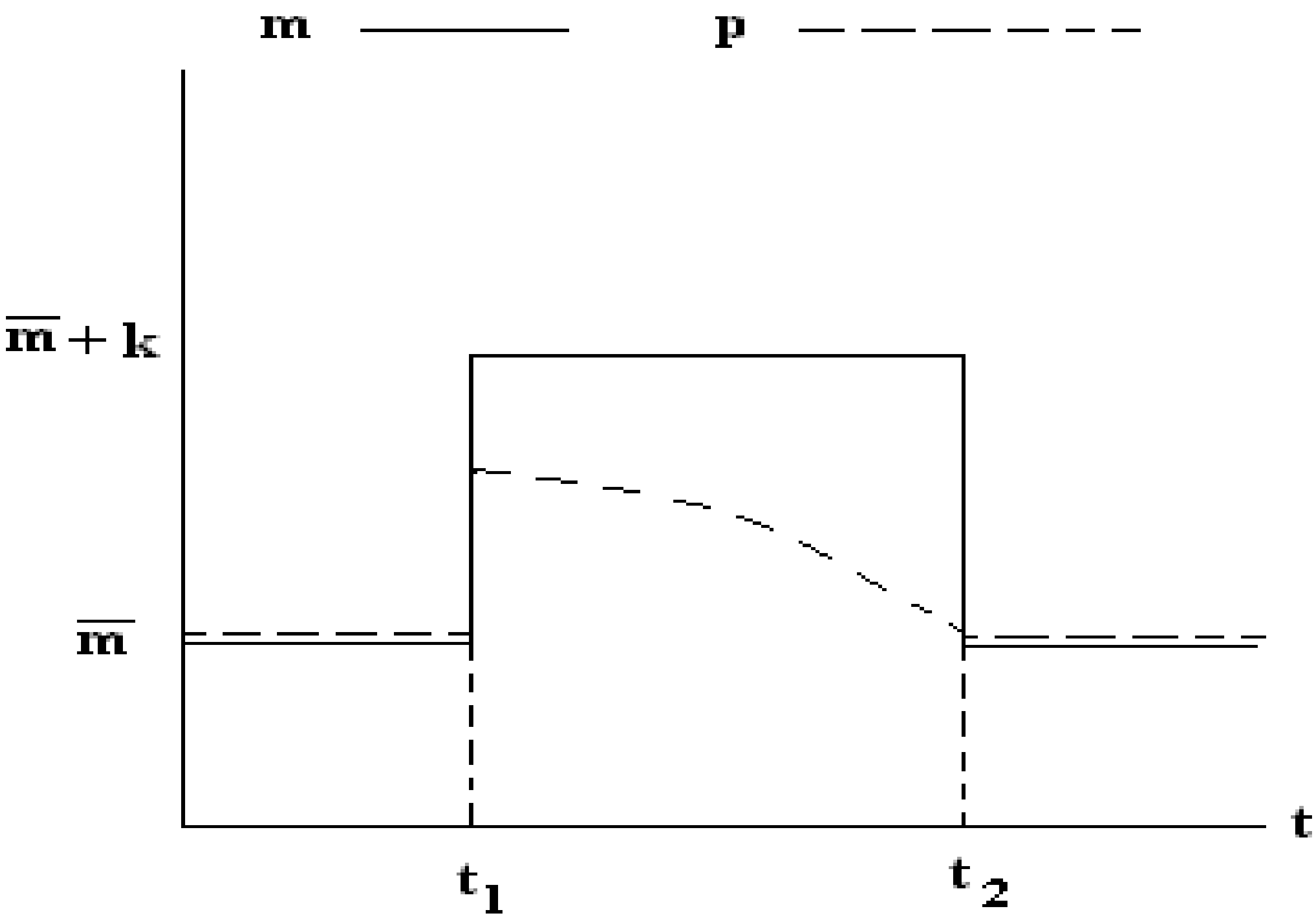


(a)

- with *rational* expectations the response of p in t_1 is:

$$p_{t_1} = \left[1 - \left(\frac{\alpha}{1+\alpha}\right)^{t_2 - t_1}\right] k + \bar{m}$$

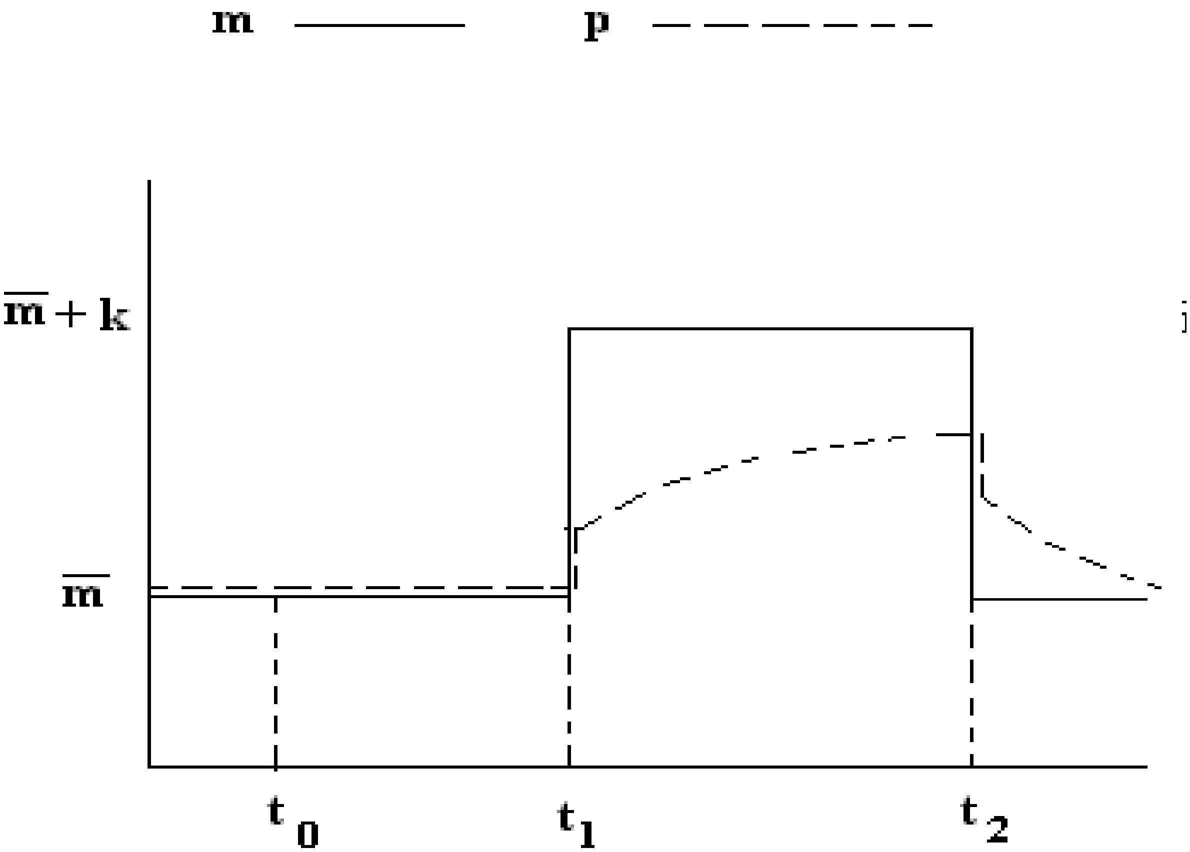
Between t_1 and t_2, p decreases and gets back to $\bar{m}$ at t_2



(b)

Expected temporary increase of m (announced in t_0)

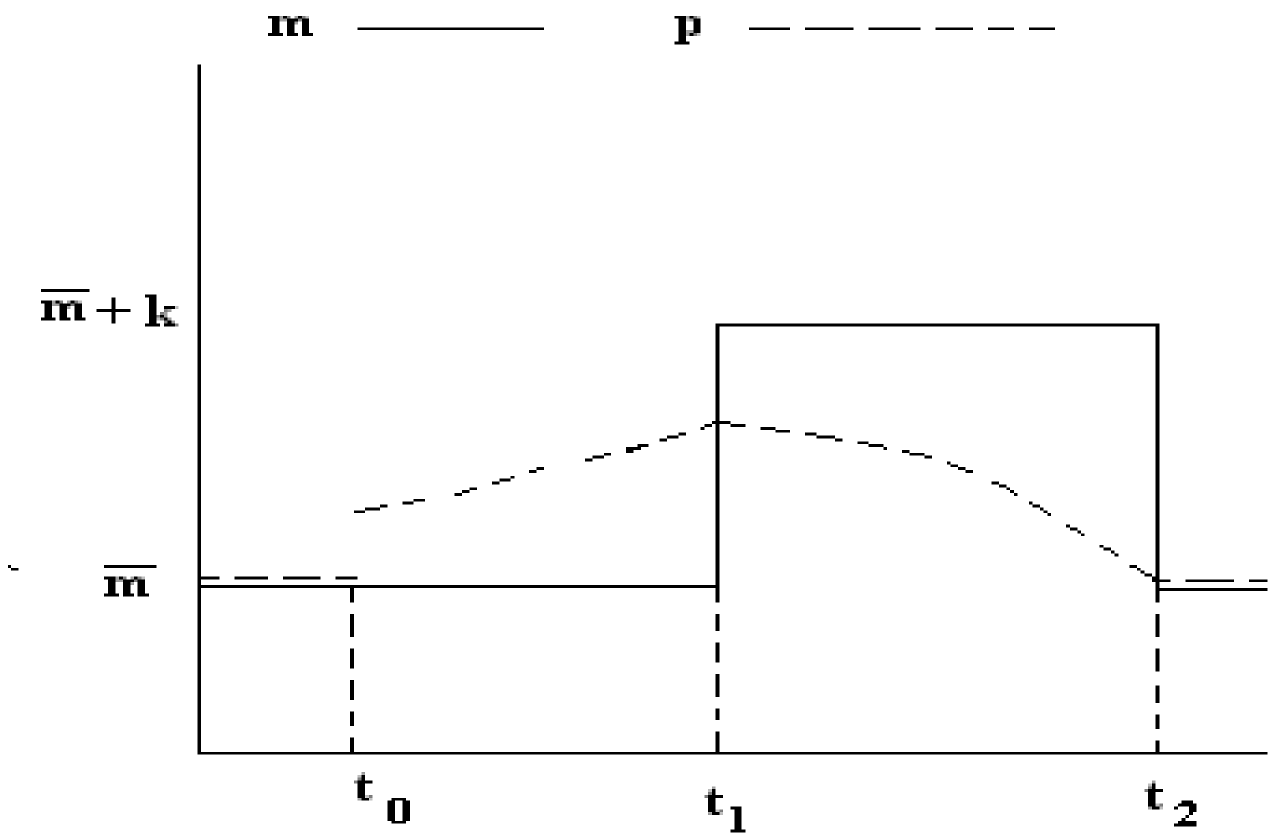
- with *adaptive* expectations, the behavior of p is the same as in the case of an unanticipated money increase



(a)

- with *rational* expectations p at the announcement time t_0 "jumps" to the level:

$$p_{t_0} = \left(\frac{\alpha}{1+\alpha}\right)^{t_1-t_0}\left[1-\left(\frac{\alpha}{1+\alpha}\right)^{t_2-t_1}\right]k+\bar{m}$$



(b)

Extension to stochastic shocks with rational expectations

$$m_t - p_t = -\alpha\,(p^e_{t,t+1} - p_t) + u_t \qquad \text{with} \quad E_{t-1}u_t = 0$$

$$\Rightarrow \qquad p_{t+1} = \frac{1}{1+\alpha}\,m_{t+1} + \frac{\alpha}{1+\alpha}E_{t+1}\,p_{t+2} - \frac{1}{1+\alpha}\,u_{t+1}$$

$$E_t\,p_{t+1} = \frac{1}{1+\alpha}E_t m_{t+1} + \frac{\alpha}{1+\alpha}E_t p_{t+2}$$

Repeating this step for p_{t+2}, p_{t+3}, ... and letting $T \to \infty$:

$$p_t = \frac{1}{1+\alpha} \sum_{i=0}^{\infty} \left(\frac{\alpha}{1+\alpha} \right)^i E_t m_{t+i} - \frac{1}{1+\alpha} u_t$$

The forecast error at t is:

$$p_t - E_{t-1} p_t = \frac{1}{1+\alpha} \sum_{i=0}^{\infty} \left(\frac{\alpha}{1+\alpha} \right)^i \underbrace{(E_t m_{t+i} - E_{t-1} m_{t+i})}_{\text{revisions in expectations of } m_{t+i}} - \frac{1}{1+\alpha} u_t$$

Imperfect information and business cycles: Lucas (1973)

- large number (N) of geographically separated markets (index z)
- *local* output supply determined by the local price of the good *relative* to the economy-wide general price level
- two *demand shocks*:
  - *aggregate* $\rightarrow p$
  - *local* $\rightarrow p(z)$
- *imperfect information*: producers observe $p(z)$, *not* p

  $\Rightarrow$ the *expected* relative price determines local output

Local markets:

homogeneous good, perfect competition, market-clearing price

- *supply function:*

$$y(z) = y^* + \gamma\,(p(z) - E(p \mid I(z))) \qquad \gamma > 0$$

with

y^* : “natural” level of output (same for all markets)

γ : price elasticity (“slope”) of supply (“structural” parameter)

$p(z) \in I(z)$

- assumptions on *probability distributions* of p and $p(z)$:

$$\begin{aligned} p &= \bar{p} + v \qquad \text{with} \quad v \sim N(0, \sigma^2) \\ p(z) &= p + z \qquad \text{with} \quad z \sim N(0, \tau^2) \\ &= \bar{p} + v + z \end{aligned}$$

Moreover:

$$\sum_1^N z = 0$$

$$\text{cov}(v, z) = 0$$

- Producers face a *signal extraction* problem:

  from observation of $p(z)$ get $E(p \mid p(z))$

  Solution: from joint distribution

$$\begin{pmatrix} p \\ p(z) \end{pmatrix} \sim N\left[\begin{pmatrix} \bar{p} \\ \bar{p} \end{pmatrix}, \begin{pmatrix} \sigma^2 & \sigma^2 \\ \sigma^2 & \sigma^2 + \tau^2 \end{pmatrix}\right]$$

we get the conditional expected value:

$$E(p \mid p(z)) = \bar{p} + \frac{\sigma^2}{\sigma^2 + \tau^2}\left(p(z) - \bar{p}\right)$$

$$= \underbrace{\frac{\tau^2}{\sigma^2 + \tau^2}}_{\theta}\ \bar{p} + \underbrace{\frac{\sigma^2}{\sigma^2 + \tau^2}}_{1-\theta}\ p(z)$$

$$\equiv \theta \bar{p} + (1 - \theta)\, p(z)$$

- *local supply function*:

$$y(z) = y^* + \gamma\,\theta\,(p(z) - \bar{p})$$

non structural

slope

In the aggregate:

measure of (average) *aggregate supply*:

$$\frac{\sum_1^N y(z)}{N} = y^* + \gamma\,\theta\,\frac{\sum_1^N (p(z) - \bar{p})}{N}$$

$$= y^* + \gamma\,\theta\,\frac{\sum_1^N v + \sum_1^N z}{N}$$

$$= y^* + \gamma\,\theta\,v$$

$$\Rightarrow \qquad y = y^* + \gamma\,\theta\,(p - \bar{p})$$

Lucas supply curve

Aggregate demand:

$$y = m - p$$

m is nominal quantity of *money* or an indicator of *nominal* aggregate demand (from $y + p = m$)

Equilibrium:

price level (as a function of expected price):

$$p = \frac{1}{1+\gamma\theta} m - \frac{1}{1+\gamma\theta} y^* + \frac{\gamma\theta}{1+\gamma\theta} \bar{p}$$

expected value of the general price level $\bar{p} \equiv E(p)$:

$$\bar{p} = E(m) - y^*$$

equilibrium price level:

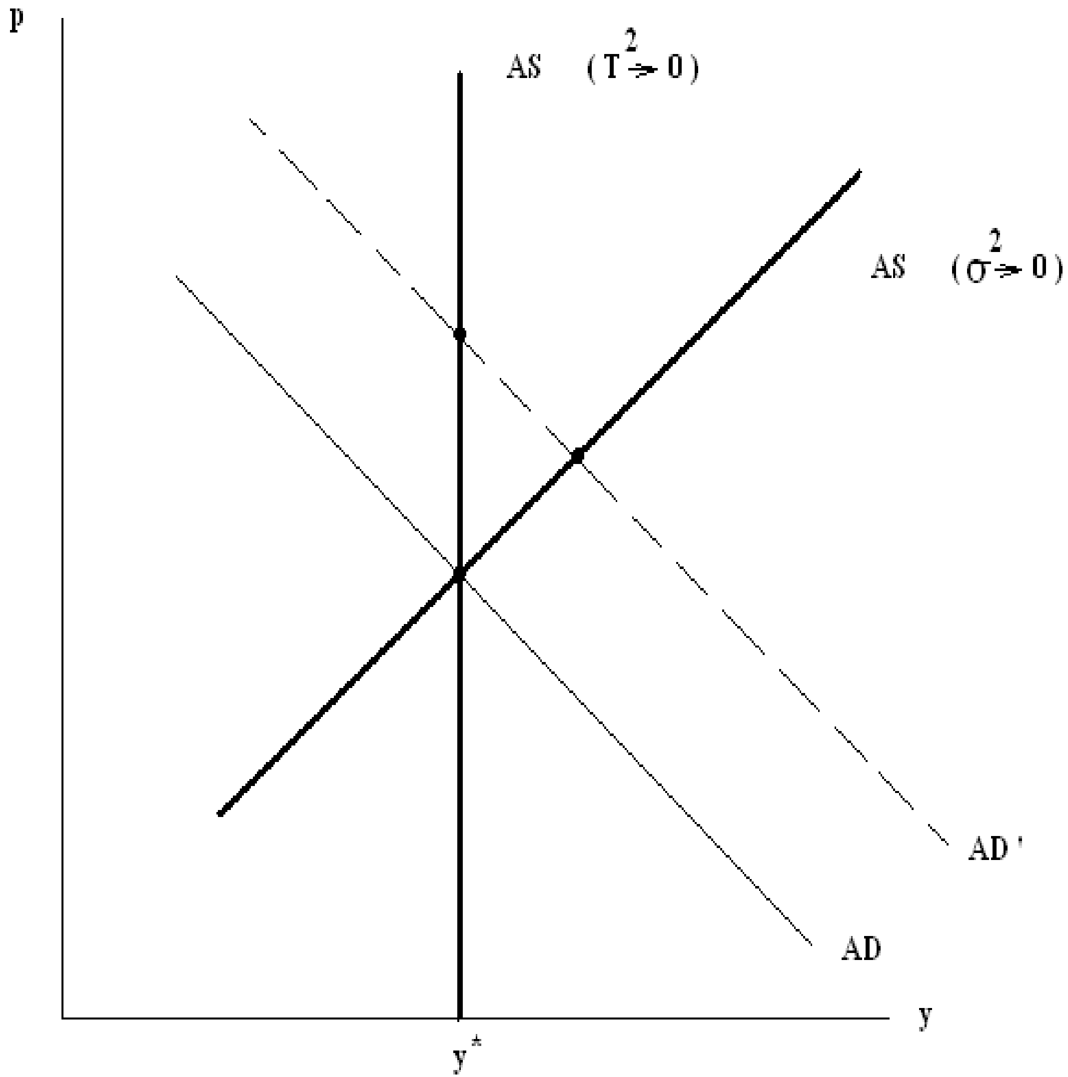
$$p = \underbrace{E(m) - y^*}_{\bar{p}} + \underbrace{\frac{1}{1+\gamma\theta}(m - E(m))}_{v}$$

equilibrium output:

$$y = y^* + \frac{\gamma\theta}{1+\gamma\theta}(m - E(m))$$

Implications

- ***output-inflation trade-off***
  - *microeconomic foundations* for Friedman's model (key role of informational imperfections in generating comovements between nominal and real variables)
  - adoption of *rational expectations* → *"surprise Phillips curve"*
- ***"Lucas critique"***
  - the relation between real and nominal variables depends also on "non structural" parameters (θ), that change as policies change


p
AS ($T^2 \to 0$)
AS ($\sigma^2 \to 0$)
AD '
AD
y
y*

- ***International evidence on the output.inflation trade-off***
  from equilibrium output

$$y_t = y^* + \frac{\gamma\theta}{1+\gamma\theta}(m_t - E(m_t))$$

$$\frac{\gamma\theta}{1+\gamma\theta} = \frac{\gamma\tau^2}{\sigma^2 + (1+\gamma)\tau^2}$$

decreasing in σ^2

Test: comparing economies with different levels of aggregate demand variability (σ^2), there should be a negative correlation between estimated σ^2 and estimated coefficients $\frac{\gamma\theta}{1+\gamma\theta}$

Estimation strategy:

- take annual changes in *nominal GDP* as proxy for the unexpected component of aggregate demand:

$$\Delta GDPN_t \equiv \Delta(y_t + p_t) \simeq m_t - E(m_t)$$

Table 3. Descriptive Statistics on Inflation and Output, Various Countries, Selected Periods, 1948–86

Country	Sample period	Real growth: Mean	Real growth: Standard deviation	Inflation: Mean	Inflation: Standard deviation	Nominal growth: Mean	Nominal growth: Standard deviation
Argentina	1963–81	0.0262	0.04253	0.5439	0.42064	0.5702	0.40685
Australia	1949–85	0.0416	0.02446	0.0677	0.05043	0.1094	0.04880
Austria	1950–86	0.0396	0.02615	0.0526	0.04745	0.0923	0.04329
Belgium	1950–85	0.0329	0.02238	0.0424	0.03028	0.0754	0.03343
Bolivia	1958–83	0.0376	0.03839	0.2012	0.29272	0.2388	0.26608

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		Real growth		*Inflation*		*Nominal growth*	
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Belgium	1950–85	0.0329	0.02238	0.0424	0.03028	0.0754	0.03343
Bolivia	1958–83	0.0376	0.03839	0.2012	0.29272	0.2388	0.26608

United Kingdom	1948–86	0.0243	0.01890	0.0664	0.04984	0.0909	0.04202
United States	1948–86	0.0315	0.02676	0.0415	0.02529	0.0731	0.03252
Venezuela	1950–85	0.0459	0.03757	0.0526	0.08237	0.0985	0.07665
Zaire	1950–84	0.0334	0.04248	0.2002	0.22374	0.2338	0.21167

Estimation strategy:

- take annual changes in *nominal GDP* as proxy for the unexpected component of aggregate demand:

$$\Delta GDPN_t \equiv \Delta(y_t + p_t) \simeq m_t - E(m_t)$$

- estimate, for each country i, the time-series equation

$$y_{i,t} = \beta_{0,i} + \beta_{1,i} t + \beta_{2,i} y_{i,t-1} + \delta_i \Delta GDPN_{i,t} + \eta_{i,t}$$

Table 4. Estimates of the Output-Inflation Trade-off, Various Countries, Selected Periods, 1948–86[a]

		Full sample		Data through 1972		Data after 1972	
Country	*Sample period*	*Trade-off parameter,* **(delta)**	*Standard error*	*Trade-off parameter,* τ	*Standard error*	*Trade-off parameter,* τ	*Standard error*
Argentina	1963–81	−0.0047	0.0335	−0.1179	0.1140	0.0021	0.0322
Australia	1949–85	0.1383	0.0862	0.3029	0.0858	0.3196	0.1937
Austria	1950–86	−0.0196	0.1069	−0.0830	0.1219	0.6823	0.2058
Belgium	1950–85	0.4967	0.1035	0.3897	0.1036	0.2081	0.2950
Bolivia	1958–83	−0.0525	0.0424	0.1418	0.1567	−0.0621	0.0276

United Kingdom	1948–86	−0.0199	0.0958	0.0793	0.1293	−0.0766	0.2197
United States	1948–86	0.6714	0.0771	0.7229	0.0598	0.8486	0.1915
Venezuela	1950–85	0.1146	0.0623	0.3252	0.1239	−0.0240	0.0784
Zaire	1950–84	0.0160	0.0414	−0.0188	0.0419	−0.0502	0.0984

Across country values

Estimation strategy:

- take annual changes in *nominal GDP* as proxy for the unexpected component of aggregate demand:

$$\Delta GDPN_t \equiv \Delta(y_t + p_t) \simeq m_t - E(m_t)$$

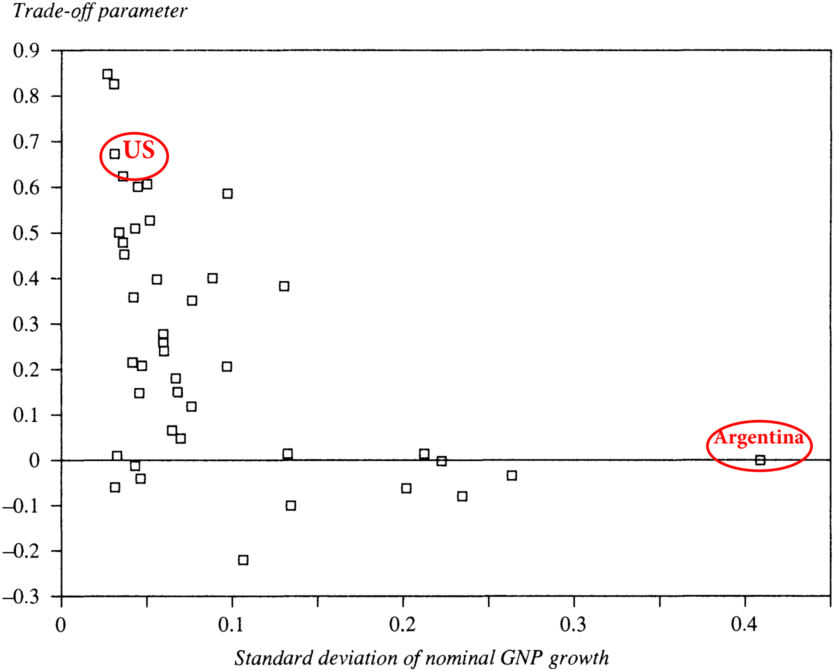
- estimate, for each country i, the time-series equation

$$y_{i,t} = \beta_{0,i} + \beta_{1,i} t + \beta_{2,i} y_{i,t-1} + \delta_i \Delta GDPN_{i,t} + \eta_{i,t}$$

- check the negative correlation between the output.inflation trade-off coefficient and demand variability ($\kappa_1 < 0$):

$$\delta_i = \kappa_0 + \kappa_1 \sigma_{\Delta GDPN_i} + \xi_i$$

Figure 2. The Output-Inflation Trade-off and the Variability of Demand



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- check the negative correlation between the output.inflation trade-off coefficient and demand variability ($\kappa_1 < 0$):

$$\delta_i = \kappa_0 + \kappa_1 \sigma_{\Delta GDPN_i} + \xi_i$$

result (43 countries, 1948-1986):

$$\delta_i = \underset{(0.06)}{0.39} - \underset{(0.48)}{1.64}\ \sigma_{\Delta GDPN_i}$$

New Classical Macroeconomics

Aim:

account for cyclical fluctuations as an *equilibrium* phenomenon (*equilibrium business cycle*

Main features of NCM models:

- *natural level of output* determined only by real forces;
- continuous and instantaneous *market-clearing* on all markets with perfectly flexible prices;
- *rational expectations*.

Main implication:

ineffectiveness of aggregate demand management policies that follow systematic rules (*policy ineffectiveness proposition*).

Typical NCM model (Sargent-Wallace 1976)

Demand: IS-LM structure

$$y_t = -b(i_t - E_{t-1}(p_{t+1} - p_t)) + v_{1t} \qquad \text{IS}$$

$$m_t - p_t = y_t - d\,i_t + v_{2t} \qquad \text{LM}$$

with $E_{t-1}v_{1t} = E_{t-1}v_{2t} = 0$.

Eliminating the nominal interest rate $i_t \Rightarrow$ *aggregate demand*:

$$y_t = \alpha(m_t - p_t) + \beta E_{t-1}(p_{t+1} - p_t) + v_t \qquad \text{AD}$$

with $\alpha \equiv \frac{b}{b+d}$, $\beta \equiv \frac{bd}{b+d}$, $v_t \equiv \left(\frac{1}{b+d}\right)(d\,v_{1t} - b\,v_{2t})$

Supply:

Lucas-type *aggregate supply* function:

$$y_t = \gamma (p_t - E_{t-1} p_t) + u_t \qquad \text{AS}$$

with $E_{t-1} u_t = 0$

Monetary policy

nominal quantity of money m_t set following a "*feedback rule*":

$$m_t = m_{t-1} - \delta y_{t-1} + \varepsilon_t \qquad \text{feedback rule}$$

with $E_{t-1} \varepsilon_t = 0$

Solution: *method of undetermined coefficients*

Equate AD and AS using the *feedback rule*:

$$p_t = \left(\frac{1}{\alpha + \gamma}\right)\left[(\gamma - \beta)E_{t-1}p_t + \beta E_{t-1}p_{t+1} + \alpha m_{t-1} - \delta\alpha y_{t-1} + \alpha\varepsilon_t + v_t - u_t\right]$$

Guess a linear solution for p_t :

$$p_t = \pi_1 m_{t-1} + \pi_2 y_{t-1} + \pi_3 \varepsilon_t + \pi_4 v_t + \pi_5 u_t$$

where $\pi_1, \ldots \pi_5$ are coefficients to be determined.

From

$$p_t = \pi_1 m_{t-1} + \pi_2 y_{t-1} + \pi_3 \varepsilon_t + \pi_4 v_t + \pi_5 u_t$$

derive $E_{t-1}p_t$ and $E_{t-1}p_{t+1}$:

$$\begin{aligned} E_{t-1}p_t &= \pi_1 m_{t-1} + \pi_2 y_{t-1} \\ E_{t-1}p_{t+1} &= \pi_1 E_{t-1}m_t + \pi_2 E_{t-1}y_t \\ &= \pi_1 m_{t-1} - \pi_1 \delta y_{t-1} \end{aligned}$$

Equate the *guess solution* to the equilibrium price (after substituting for expected values):

$$
\begin{aligned}
\pi_1 m_{t-1} + \pi_2 y_{t-1} + \pi_3 \varepsilon_t + \pi_4 v_t + \pi_5 u_t = {} & \frac{\gamma - \beta}{\alpha + \gamma}(\pi_1 m_{t-1} + \pi_2 y_{t-1}) \\
& + \frac{\beta}{\alpha + \gamma}(\pi_1 m_{t-1} - \pi_1 \delta y_{t-1}) \\
& + \frac{1}{\alpha + \gamma}(\alpha m_{t-1} - \alpha \delta y_{t-1} + \alpha \varepsilon_t + v_t - u_t)
\end{aligned}
$$

Equate coefficients on the same variable:

$$m_{t-1} \quad : \quad \pi_1 = \frac{\gamma - \beta}{\alpha + \gamma}\pi_1 + \frac{\beta}{\alpha + \gamma}\pi_1 + \frac{\alpha}{\alpha + \gamma}$$

$$\Rightarrow \quad \pi_1 = 1$$

$$y_{t-1} \quad : \quad \pi_2 = \frac{\gamma - \beta}{\alpha + \gamma}\pi_2 - \frac{\beta\delta}{\alpha + \gamma}\pi_1 - \frac{\alpha\delta}{\alpha + \gamma}$$

$$\Rightarrow \quad \pi_2 = -\delta$$

Equate coefficients on the same variable:

$$\varepsilon_t \quad : \quad \pi_3 = \frac{\alpha}{\alpha + \gamma}$$

$$v_t \quad : \quad \pi_4 = \frac{1}{\alpha + \gamma}$$

$$u_t \quad : \quad \pi_5 = -\frac{1}{\alpha + \gamma}$$

Final solution for the price level p_t:

$$p_t = m_{t-1} - \delta y_{t-1} + \frac{1}{\alpha + \gamma}(\alpha \varepsilon_t + v_t - u_t)$$

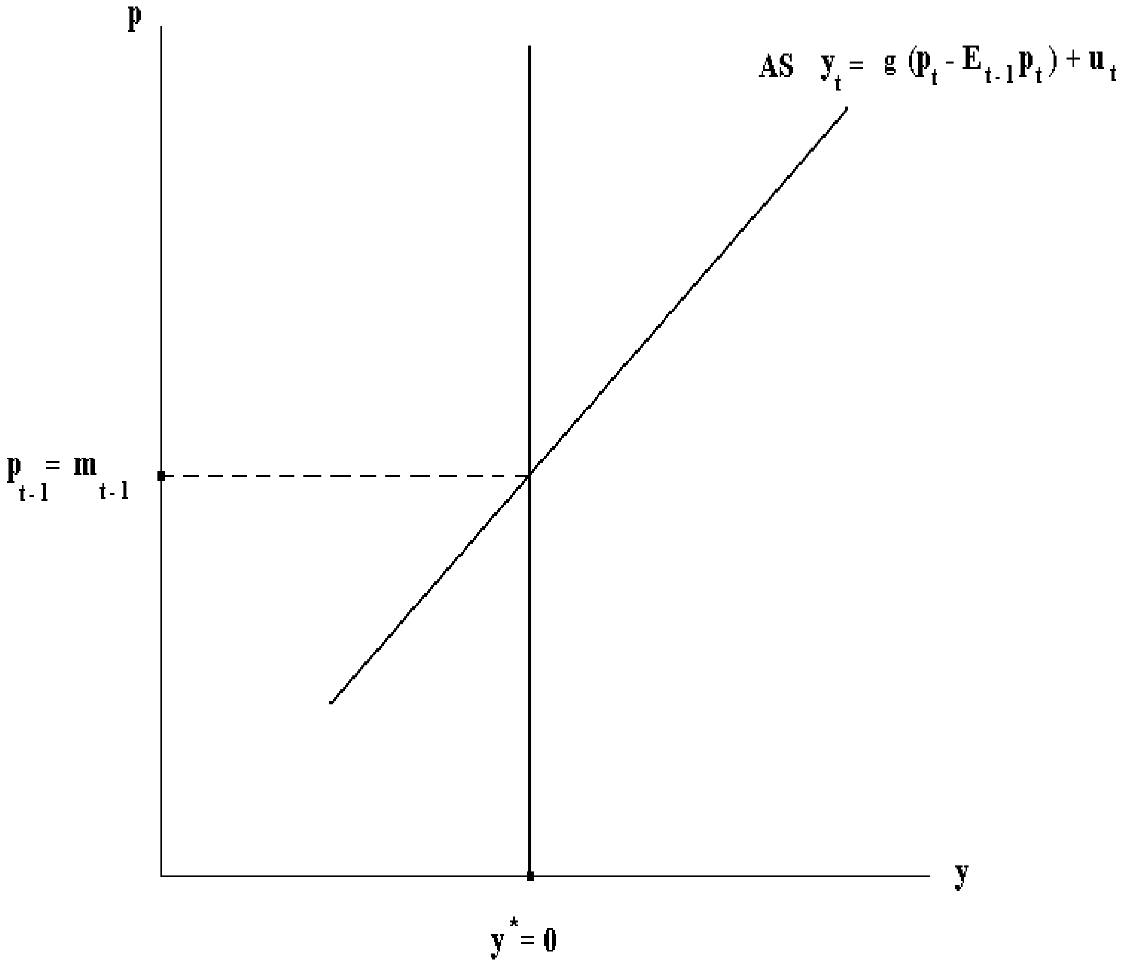
price "surprise":

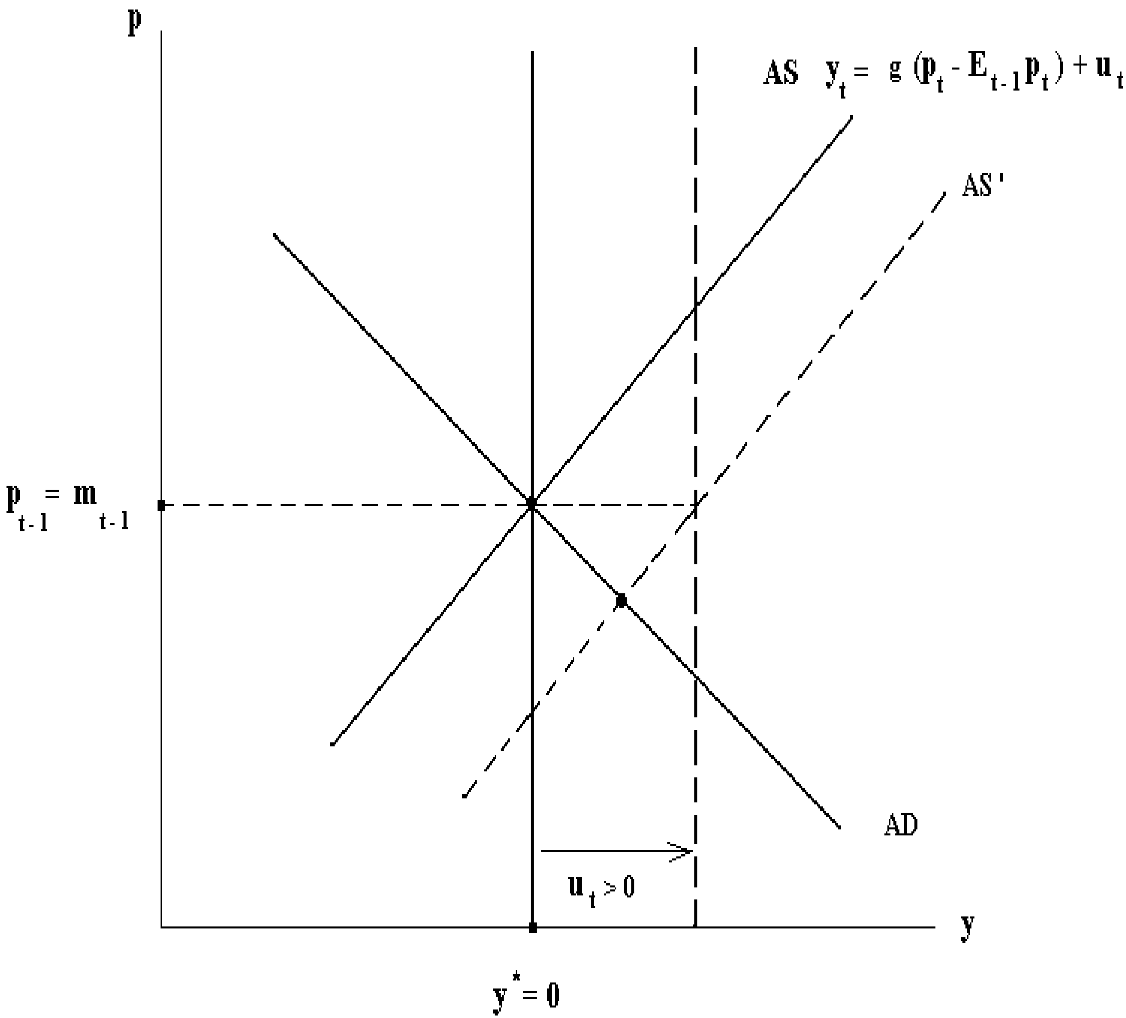
$$p_t - E_{t-1} p_t = \frac{1}{\alpha + \gamma}(\alpha \varepsilon_t + v_t - u_t)$$

solution for output:

$$y_t = \frac{\gamma}{\alpha + \gamma}(\alpha \varepsilon_t + v_t - u_t) + u_t$$

⇒ systematic *monetary policy* cannot be used to stabilize output (the "Phillips curve" in NCM models is vertical for expected policy $E_{t-1} m_t$ and shows a positive output-price correlation only for unexpected policy shocks ε_t)


p
AS $y_t = g(p_t - E_{t-1}p_t) + u_t$
$p_{t-1} = m_{t-1}$
y
$y^* = 0$


p
AS $y_t = g(p_t - E_{t-1}p_t) + u_t$
AS'
$p_{t-1} = m_{t-1}$
AD
$u_t > 0$
y
$y^* = 0$

Application of the "Lucas critique"

Consider the above NCM structure with the following feedback monetary rule:

$$m_t = \delta_1 m_{t-1} - \delta_2 y_{t-1} + \varepsilon_t$$

Solution for equilibrium output (same as above):

$$y_t = \frac{\alpha\gamma}{\alpha+\gamma}\varepsilon_t + \underbrace{\frac{\gamma}{\alpha+\gamma}v_t + \frac{\alpha}{\alpha+\gamma}u_t}_{\eta_t}$$

$$\equiv \frac{\alpha\gamma}{\alpha+\gamma}\varepsilon_t + \eta_t$$

To express y_t in terms of observable variables, substitute

$$\varepsilon_t = m_t - \delta_1 m_{t-1} + \delta_2 y_{t-1}$$

yielding output:

$$y_t = \frac{\alpha\gamma}{\alpha+\gamma} m_t - \delta_1 \frac{\alpha\gamma}{\alpha+\gamma} m_{t-1} + \delta_2 \frac{\alpha\gamma}{\alpha+\gamma} y_{t-1} + \eta_t$$

Equation to be estimated econometrically:

$$y_t = \beta_1 m_t + \beta_2 m_{t-1} + \beta_3 y_{t-1} + \eta_t$$

where β_2 and β_3 depend on policy parameters δ_1 and δ_2